Wits Maths Connect Secondary

A one-week programme for revision of negative numbers in Grade 9

Learners are introduced to negative numbers early in Grade 8. However, we know from experience and from our research that many Grade 9 learners still struggle to work with negative numbers. This impacts their work in many areas of the curriculum, particularly algebra and functions.

In this pack we propose a sequence of 5 one-hour lessons to revise negative numbers with Grade 9 learners. We refer to "negative numbers" rather than "integers" because our materials also include a small amount of work with negative fractions.

There are a range of tasks for the different days, including:

- typical integer questions from Grade 8
- substitution into algebraic expressions
- identifying errors in learners' responses to integer questions
- links to transformations of points in the plane
- two games that can be used to improve learners' ability to work with integers
- a short summative test

In addition to improving learners' ability to deal with integers, we want learners to:

- do a lot of examples to practice – both in class and at home
- develop their mathematical reasoning skills
- experience success
- enjoy learning mathematics through mathematical games

We are not suggesting that the section on integers be re-taught from scratch. Rather, we want to address the typical errors learners make and focus on helping them overcome these obstacles.

Linked to this programme are two mathematical games that help learners practice their skills in operating with negative numbers:

- The Integer Snakes and Ladders game (for addition and subtraction)
- The Integer Product game (for multiplication)

Note

In these materials we have made suggestions about typical questions for homework.
The actual homework tasks were developed in a workshop in November 2011, and are included in this pack. The following teachers and project team members participated in developing the homework tasks: Comfort Chigabo, Regina Essack , Andrew Halley, Sbongile Mashazi, Tawanda Mawire, Shadrack Moalusi, Ditshebo Molepo, Constance Motsiri, Pat Mpotu, Phumzile Mthiyane, Sitti Patahuddin, Craig Pournara and Faeeza Rawat

A final revision worksheet and the summative test are not provided in this pack.

Grade 9
Revision of negative numbers

Day 1		**Comments**
Focus	• Emphasising the basics of integers, notations and use of brackets • Identify learners' errors in simple questions involving adding and subtracting of positive and negative numbers • Adding and subtracting positive and negative numbers with 2 and 3 terms • Simple division of integers	
Tasks in class	1) Calculate the answers a) 4 – 7 b) 7 – 4 c) –4 + 7 d) –4 – 7 e) (–4)(–7) f) (–4) + (–7) g) (–4) – (–7) h) –4 + (–7)	Write (a) – (d) on the board before learners start. Once you have gone over the answers, write (e) - (h) on the board before learners continue. Do not erase (a) – (d)
	2) Calculate the answers a) 5 – 9 + 1 b) 5 + 9 – 1 c) –5 – 6 + 12 d) –5 – (6 + 12) e) –5 – (6 – 12) f) 1 – 2 + 3 – 4 + 5	Put all 6 questions on the board before learners start.
	3) Simplify a) $\frac{10}{-5}$ b) $\frac{-10}{-5}$ c) $\frac{-10}{5}$ d) $\frac{5}{10}$ e) $-\frac{10}{5}$	Put all 5 questions on the board before learners start. Learners don't easily accept that (a), (c) and (e) are equivalent (= -2)
	4) Calculate the answers a) $\frac{4+8}{-2}$ b) $\frac{3-9}{-2-4}$ c) $\frac{-1-3}{4+(-2)}$	These questions are similar to the calculations for gradient in the section on linear functions
Tasks for homework		15 questions that cover the same content as questions 1 – 4

Day 2		**Comments**
Focus	• Simple integer questions dealing with single operation (+, - ×, ÷) • More complex questions involving two operations • Learners producing "questions" for the given "answers" • Playing a mathematical game to reinforce proficiency with adding and subtracting integers	
Tasks in class	1) Choose numbers to put in the boxes to make the statements true. e.g. -1 + (-2) = -3 a) ☐ + ☐ = -3 b) ☐ – ☐ = -3 c) ☐ × ☐ = -6 d) $2 + \frac{\square}{\square} = -1$ e) ☐ + ☐ × ☐ = -4 f) ☐ – ☐ × ☐ = -9	Put one question on the board at a time. Ask learners to give answers. Collect at least 6 answers for each question and write them on the board. Deal with any incorrect answers. Push learners to find different kinds of answers e.g. subtracting negatives, using fractions, etc.
	2) Introduce learners to the Integer Snakes and Ladders game and get them to play the game in small groups.	Allocate at least 20 minutes for the game
Tasks for homework		15 questions involving mixed operations and brackets. See Day 1 Q2e, f; Day 2: Q1 e, f; Day 3 no 1.

Day 3		Comments
Focus	• Simplifying numerical expressions when there is a number and/or operation after brackets • Dealing with typical learner errors in integers • Substitution with negative numbers • Transformation of points in the Cartesian plane	
Tasks in class	1) Determine the answers to the following: a) (8 – 13) 5 b) (8 – 13) – 5 c) (8 – 13) + 5 d) (8 – 13) (-5) e) 5 (8 – 13) f) -5 (8 – 13)	Write all 6 questions on the board before learners start. Learners should attempt all 6 questions before answers are given. Learners have difficulty when there is a number on the right of the brackets. They need to learn what operation is associated with the different notations.
	2) There are five questions below, together with learner responses. Are the responses correct? If they are not correct, explain what error the learner has made, and correct the error. a) 4 + 6 – 8 + 2 = 10 – 10 = 0 b) 9 – 3 + 8 – 6 = 6 + 2 = 8 c) 13 – 20 = 7 d) 6 + 6(-6) = 6 + 0 = 6 e) (-2)(-4) – (-5) = 8 + 5 = 13	Write all 5 questions and responses on the board before learners start. If they are unsure about a question, they should move to the next question. We have deliberately used different layouts. Learners need to recognise that there is no problem with the layout. The problems lie in the operations that are incorrect in a, c, d, e. Ask learners for another way of calculating the answer for (b).
	3) You are given the following values: a = -4, b = 3, c = -5. Substitute these values into the expressions below and calculate the answers. a) $-3a^2$ b) $2b^3$ c) $5bc + \frac{1}{2}a$ d) ab + 6a e) $\frac{a+b}{2} - a + 4c$	This task starts to prepare learners for the algebraic work they will start the following week. Here they are not manipulating the expressions – they are only required to substitute and simplify the answer. If there is time, ask learners to generate some of their own examples and to answer them.
Tasks for homework	1) Given the point A(-4;5). Move the point as described below. Write down its new coordinates. Each question starts at the position (-4;5) a) up 4 left 5 b) down 7 left 2,5 c) right 1 down 2 d) left 3 up 6¼ right 4 e) left 8 down 5	This task reinforces work done in the previous section. Suggest that learners draw their own set of axes. Add the following: • 5 mixed examples dealing with typical errors such Q1, 2 • 5 substitution questions using different values and different letters similar to Q3

Day 4		**Comments**
Focus	• Working with typical learner errors involving brackets and multiplication • Focusing on signs of numbers and relative size of numbers • Substitution in algebraic expressions	
Tasks in class	1) There are 2 questions below, together with learners' responses. Identify the correct response. For the incorrect responses, say what error the learner has made. Question A: 7 – 3(-2) = a) 7 – 3(-2) = 4 (-2) = -8 b) 7 – 3(-2) = 7 – 6 = 1 c) 7 – 3(-2) = 7 + 6 = 13 Question B: (-4)(-5)(-½) = a) (-4)(-5)(-½) = -20½ b) (-4)(-5)(-½) = -9½ c) (-4)(-5)(-½)= -10 d) (-4)(-5)(-½) = 10	Write all the information for Question A before learners start. When the discussion on this question has been dealt with, write all the information for Question B.
	2) If a = -2, b = 5, c = -3, say which of the following expressions will give a negative answer. Try to do this without calculating the actual numerical answer. e.g. *ab* will be negative because *a* is positive and *b* is negative. When we multiply a positive number and negative number, the answer is negative. a) abc b) $(abc)^2$ c) a^2bc d) abc^2 e) $\frac{a}{c}$ f) -2b g) $\frac{c}{b}$ h) a – b i) a + b j) 2b + c Brain busters: • For what values of *p* will *p* + 4 be negative? • For what values of *m* and *n* will *m* – *n* be positive?	Encourage learners to focus on the sign of the numbers and not the actual value. Once they have decided which answers are negative, then can check by doing the substitutions. The brain busters will challenge all learners. They are mainly intended for strong learners.
Tasks for homework		Revision worksheet for test (answers should be provided on handout or somewhere in school that is accessible to learners, so they can check their answers before the test)

Day 5		**Comments**
Focus	• Test on work from the week, including marking test and feedback to learners • Play game if time permits	
Tasks in class	1) Complete the test in 10-15min. 2) Mark the test with learners and then take in scripts. 3) Allow learners to play Integer Snakes and Ladders game if there is time.	Test should contain 20 questions
Tasks for homework	Give some practice questions for homework for learners who have done poorly in the test (or ask them to redo the test).	