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supporting secondary maths

#equalitymatters

PRACTICE IN WORKING WITH LINEAR EQUATIONS

VERSION 1.0

Grades 7, 8 and 9

Solve for x: $3x - 4 = 5x + 6$

$$3x - 4 + 4 = 5x + 6 + 4$$
$$3x = 5x + 10$$
$$3x - 5x = 5x + 10 - 5x$$
$$-2x = 10$$
$$x = -5$$

Solve for x:

$$3x - 4 = 5x + 6$$
$$3x - 4 - 3x = 5x + 6 - 3x$$
$$-4 = 2x + 6$$
$$-2 = x + 3$$
$$\ldots$$

SOLVE

Equal sign

Inspection

COMMUTATIVE LAW

Multiplicative inverse

Solution

Additive inverse

T or F?

$5 + k = k + 5$

$5 - m = m - 5$

$5 + 4 = 9 - 2$

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

FIRSTRAND

NRF National Research Foundation

science & innovation
Department: Science and Innovation
REPUBLIC OF SOUTH AFRICA

#equaliTymatters: Practice in working with linear equations

These materials were produced by the Wits Maths Connect Secondary (WMCS) project at the University of the Witwatersrand.
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PRACTICE IN WORKING WITH LINEAR EQUATIONS

VERSION 1.0

About this booklet

This booklet contains 38 worksheets on linear equations, together with answers. These materials differ from our other materials in that we provide a pedagogical approach for the first-time teaching of equations. We do not assume that learners have already been taught equations. However, there are still many practice examples particularly in section 3.

We begin with numerical equations involving whole numbers only, and focus on the ideas of equality and balance to give learners a sense of solving equations without the distractions of algebraic notation. In these worksheets we use a place holder (☐) and a space (___) to represent the unknown quantity. Many of the worksheets are suitable for learners in Grade 7, maybe even Grade 6 learners.

We then move on to integer equations where we continue to use ☐ and ___ to represent the unknown. Here we reinforce the ideas introduced with whole number equations and extend the number range to integers. This helps learners to get more comfortable in working with negatives and subtraction without the complications of algebraic notation. We also push learner to pay attention to the *structure* of equations by posing questions about sets of numerical and algebraic equations that have minor variations.

> If $9-(-3)+(-2)=\square+2+4$, does ☐ have the same value in these 3 equations?
>
> $9-(-3)+(-2)-12=\square+2+4-12$
>
> $9-(-3)+(-2)+40-40=\square+2+4$
>
> $32+9-(-3)+(-2)=\square+2+4-32$

There are 14 worksheets that deal with algebraic equations and provide practice for learners. In these worksheets we emphasise the shift from equations with the variable on one side only, e.g. $2x-5=7$, to equations with the variable on both sides, e.g. $2x-5=7-x$. Local and international research shows that this shift is a substantial obstacle for learners to overcome, partly because it requires them to make use of inverses and inverse operations to solve equations. In addition, our research shows that learners' difficulties with linear equations frequently stem from their difficulties in simplifying algebraic expressions.

In addition to the sets of worksheets, we provide 12 pages of notes on the key ideas for solving equations as well as a section on what makes equations difficult for learners. For those who wish to jump straight to algebraic equations, the first two worksheets in Section 3 provide a recap of integers and algebraic simplification respectively.

Section	Content	No. of worksheets
	Notes and key ideas for solving equations	
1	Numerical equations with whole numbers	13
2	Numerical equations with integers	11
3	Algebraic equations	14

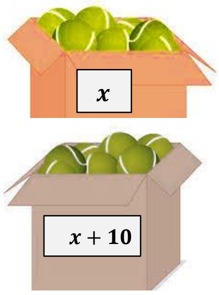
NOTES ON LINEAR EQUATIONS

In these notes we explain important concepts, terminology and procedures for solving linear equations. We also provide lots of practice in the form of worksheets. We have written these notes in simple language for Grade 8 and 9 learners.

The topic of *Equations* is a very important section of mathematics. We use equations in all topics of mathematics. For example, we may need to solve an equation to work out the sizes of angles in a triangle or the lengths of sides of a triangle. Linear equations lay the foundation for other kinds of equations in Grade 9 and beyond. We need to solve a linear equation to find the simple interest rate in financial maths, and we can use an equation to find the number of terms in a number pattern that were added to get a particular total. At the end of the notes we discuss some reasons why learners may have difficulty with linear equations.

1. What is an equation

An equation tells us about the numeric relationship between quantities. For example, say we have two boxes of balls and we know that there are 10 more balls in the one box than in the other. Let's use x to indicate the number of balls in the first box. Then there are $x + 10$ balls in the second box. To find the total number of balls we can make an equation: Total = $x + (x + 10)$, which gives: Total $= 2x + 10$. If we know that there are 46 balls altogether, then we can say $2x + 10 = 46$. When we solve this equation, we get $x = 18$. So we know there are 18 balls in one box and 28 balls in the other.



Before we define an equation in more detail, we need to introduce two other terms: *expression* and *statement*.

- An **expression** consists of terms that are combined using addition, subtraction, multiplication and division.
  Numeric expressions: $7 - 5$; $100 \div 5 + 1$; $6 \times 10 - 2 + 5$
  Algebraic expressions: $7x - 5$; $\frac{100p}{3} + 1$; $7x^2 + 3x - x - 2$
- A **statement** compares two expressions using relationship signs such as equal to ($=$), not equal to ($\neq$), greater than ($>$), less than ($<$), less than or equal to ($\leq$), etc. When we deal with equations, we use the equal sign.
  Numeric statements: $7 = 5 + 2$; $100 \div 5 < 25$; $-7 < 0$; $6 \times 10 - 2 + 5 = 63$
  Algebraic statements: $7x - 5 = 9$; $\frac{100p}{3} + 1 \geq 0$; $7x^2 + 3x - x - 2 \leq 6$
- A statement is **balanced** if the result on each side of the equal sign is the same. We use *result* in these materials to indicate the answer we get when we evaluate the expression on each side of the equal sign.
- A **balanced statement** is called an **equation**. So an equation consists of two expressions that produce the same result.

For numeric equations we will use a place holder (☐) or "space" (___) for the unknown value.
Examples:

$3 + \_\_ = 7 - 2$
$\square - 4 = 9 - 2$
$13 - 9 = \_\_ + 4 \times 2$
$7 - \square = 6 + 8$

We refer to the place holder, ☐, as "box".

For algebraic equations, we will use letters for the unknown value.
Examples:

$3 + x = 7 - 2$
$a - 4 = 9 - 2$
$13p - 9 = p + 4 \times 2$
$7 - 2k = 6 + 8k$
$2(n - 7) = 10$

- When there are no unknowns (such as letters or place holders) in a statement, it is easy to see if the statement is balanced. For example, $4 + 3 - 1 = 4 + 2$ is balanced because the result on the left of the equal sign is 6 and the result on the right of the equal sign is also 6.

But the statement $5 \times 2 - 7 = 5 \times 7 - 2$ is not balanced because the result on the left is 3 and the result on the right is 33. So we write $5 \times 2 - 7 \neq 5 \times 7 - 2$.

- When there are unknowns in a statement, we need to find the value/s that will make the statement balanced. This is called *solving an equation*. The value that makes the statement balanced is called the *solution*.

  Consider the example, $4x + 3 = 11$. This statement will be balanced if we can find a value for x that will give a result of 11 on the left side. It should be easy to see that x must equal 2. So the solution to the equation $4x + 3 = 11$ is $x = 2$.

  A statement that is balanced has the same result on both sides of the equal sign. So, a *balanced statement* is an equation.

  Consider another example, $2x - 1 = x + 3$. This is more difficult to solve without a special procedure. The solution is $x = 4$. If we substitute $x = 4$ on the left, we get $2(4) - 1 = 7$. If we substitute $x = 4$ on the right, we get $4 + 3 = 7$. So the statement is balanced when $x = 4$. It is not balanced when $x = 0$ or $x = 5$ or $x = -2$ or any other number you choose. You can check by substituting these values in both sides.

We have chosen to use *balanced* and *not balanced* rather than *true* or *false* when dealing with equations. This helps us to remember that working with equations is always about maintaining balance. So we won't speak about a *true statement* or a *false statement* but rather a statement that is balanced or not balanced.

Equations are essential for solving many problems. Here are two simple word problems:

	Word problem 1	**Word problem 2**
Verbal statement	I am thinking of a number. If you add 3 to my number, you get 10.	I am thinking of a number. If you subtract 3 from my number and then double it, you get 20.
Convert verbal statement into a numeric statement or an algebraic statement:	$\square + 3 = 10$ $n + 3 = 10$	$2(\square - 3) = 20$ $2(n - 3) = 20$
Solution	We can work out in our heads that $n = 7$. So the number I am thinking of is 7.	You may be able to see that $n = 13$. You can check this by substituting 13 into the left side to confirm that the result is 20.

2. How do we work with equations?

The most typical action to perform on an equation is to *solve* it, i.e. to find the solution. This applies to both numeric equations and algebraic equations. In this section we explain briefly the basics of solving equations using inspection and inverses. In section 3.5 we give more detailed explanations of solving equations using inverses.

2.1 Solving equations using inspection

When we solve equations using inspection, we do most of the work in our heads. Here are three examples:

$4 \times \square = 11 - 3$	$2(\square - 3) = 20$	$4x + 3 = 11$
The result on the right side of the equal sign is 8. This means we must get 8 on the left side too. So "4 times something must equal 8". So $\square = 2$.	We can reason that $2 \times 10 = 20$. This means that the bracket must have a value of 10. Which means something subtract 3 equals 10. This means $\square = 13$. Note this is the same as word problem 2 above.	We can reason: $11 - 3 = 8$. Then $8 \div 4 = 2$. So $x = 2$. The general approach is that you do the "opposite operation". On the left of the equal sign, we were adding 3 so we must subtract 3. We were multiplying by 4 so we must divide by 4.

Although inspection is a powerful method, it is only useful when the unknown is on one side of the equal sign. It is essential that we know how to solve equations using inverse operations.

2.2 Solving equations using inverses

The goal in solving equations using inverses is to collect the terms with variables on one side of the equal sign and the terms with constants on the other side. It does not matter which side you choose. We illustrate the method with brief explanations. We explain these processes in more detail in section 3.5.

<u>Example 1</u>: Solve for m: $m+5=7$	
We choose to collect all the constants on the right side Add -5 to both sides so that the constants on the left of the equal sign sum to zero The solution is 2. We can see that this is the correct solution by using inspection	$m+5+(-5)=7+(-5)$ $m+5-5=7-5$ $m=2$
<u>Example 2</u>: Solve for y: $3y-5=y+8$	
We choose to collect terms with letters on the left and constants on the right. Add $-y$ to both sides so that terms with variables sum to zero on right side Add 5 to both sides so that constant terms sum to zero on the left side We get $2y=13$ We want to determine the value of y, not $2y$, therefore we multiply both sides by the multiplicative inverse of 2. The solution is $\frac{13}{2}$	$3y-5+(-y)=y+8+(-y)$ $2y-5=8$ $2y-5+5=8+5$ $2y=13$ $2y\times\frac{1}{2}=13\times\frac{1}{2}$ $y=\frac{13}{2}$ or $6\frac{1}{2}$

In example 1 we added negative 5. We could also have said “subtract 5” because “adding a negative number” is the same as “subtracting a positive number”. In example 2, we multiplied by the multiplicative inverse of 2 which is $\frac{1}{2}$. We could also have said “divide by 2” because “dividing by 2” is the same as “multiplying by $\frac{1}{2}$”.

3. Important knowledge for understanding equations

There are seven important ideas that need to be understood well in order to solve linear equations successfully. There are: 1) seeing the equal sign as an equivalence, i.e. “is the same as”; 2) the idea of *balancing* an equation; 3) paying attention to structure; 4) commutative and associative properties; 5) inverses and inverse operations; 6) the meaning of a solution; and 7) the distributive law. We discuss each of these in more detail below. At the end of this section we provide examples of different ways to use inverse operations to solve one equation.

3.1 Seeing the equal sign as equivalence, i.e. “is the same as”

When we first encounter the equal sign in primary school, we treat it as “gives me”, e.g. $4+5=\square$. Here we say “4 add 5 *gives me* 9”. But when we have a statement with at least two terms on each side, e.g. $4+5=3+\square$, we need to reason as follows: “4 add 5 *is the same as* 3 add something”. The left side of the equal sign adds to 9 so the right side must also add to 9. This means the place holder ($\square$) must have a value of 6.
So we have $4+5=3+6$ and we say “4 add 5 *is the same as* 3 add 6”. The statement is balanced because we get the same result on both sides of the equal sign. We say the left side is equivalent to the right side which means that the sides have the same result but they don’t look the same.

Here is another example: $4 + 5 = \square - 2$. Once again, we must treat the equal sign as "is the same as". So we say "4 add 5 is the same as a number subtract 2". If the left side adds to 9, then the right side must also give a result of 9. This means the place holder must have a value of 11. So we have $4 + 5 = 11 - 2$ and we say "4 add 5 *is the same as* 11 subtract 2". We can also write this as an equation using a letter such as p: $4 + 5 = p - 2$. If we solve the equation, we will get $p = 11$. Here are three more numeric examples where we need treat the equal sign as "is the same as". Can you work out the value of $\square$?

$$3 + 4 = \square - 1 \qquad 2 + 7 \times 3 = \square \times 2 - 1 \qquad 80 \div 4 + 15 \times 2 = \square \times 100$$

3.2 The idea of balance in working with equations

The idea of balance is very important when we work with equations. Some teachers use the idea of balancing a scale and they might say things like "what you do on the left, you must do on the right". This can be a helpful reminder. Consider the statement, $4 + 5 = 3 + 6$. This is an equation because the result on the left is 9 and the result on the right is 9. If we subtract 2 from both sides we get: $4 + 5 - 2 = 3 + 6 - 2$. This will give a result of 7 on both sides. If we add 8 to both sides of the original equation, we will have $4 + 5 + 8 = 3 + 6 + 8$. The result will now be 17 on both sides. But if we add 8 to left side and subtract 8 from the right side we get $4 + 5 + 8 = 3 + 6 - 8$. Now the result on the left will be 17 but the result on the right will be 1. So the equation is no longer balanced, and we must write $4 + 5 + 8 \neq 3 + 6 - 8$ to show that the left side is not equal to the right side.

There is another way to maintain balance in an equation. Consider the example: $4 + 5 + 8 - 8 = 3 + 6$. Notice that we started with a balanced statement, then we added 8 and then subtracted 8 which means we have "added zero". This maintains the balance of the equation.

Now consider a numeric equation with a box: $4 + 6 = \square - 2$. The left side gives a result of 10 so we must put 12 in the box so that the right side to give a result of 10. If we add the same number to both sides, or subtract the same number from both sides, the numerical equation will still be balanced. But if we perform different operations on each side (or use different numbers with the same operation), the numeric equation will no longer be balanced. Here are three examples to illustrate these cases (LHS = left hand side, RHS = right hand side):

Original equation: $4 + 6 = \square - 2$		
Add 5 to both sides: $4 + 6 = \square - 2$ $4 + 6 + 5 = \square - 2 + 5$ LHS: $4 + 6 + 5 = 15$ RHS: $12 - 2 + 5 = 15$	Subtract 3 from both sides: $4 + 6 = \square - 2$ $4 + 6 - 3 = \square - 2 - 3$ LHS: $4 + 6 - 3 = 7$ RHS: $12 - 2 - 3 = 7$	Add 10 on left, subtract 10 on right: $4 + 6 = \square - 2$ $4 + 6 + 10 = \square - 2 - 10$ LHS: $4 + 6 + 10 = 20$ RHS: $12 - 2 - 12 = 0$ The equation is no longer balanced.

Note that we don't need to work out the value of $\square$ before we make changes to the original equation. If we know that the shaded parts are the same $4 + 6 + 5 = \square - 2 + 5$, when we add 5 to both sides, we maintain the balance. The same applies if we subtract 3 from both sides.

It works in the same way when we work with algebraic equations. Consider the example: $x - 4 = 7 + 5$. It is easy to work out that x must have a value of 16 to balance the equation. Here are four examples where we change the equation but still keep it balanced. Note that the parts of the original equation are shaded.

Add 1 to both sides:	$x - 4 + 1 = 7 + 5 + 1$
Subtract 6 from both sides:	$x - 4 - 6 = 7 + 5 - 6$
Multiply by 2 on both sides:	$2(x - 4) = 2(7 + 5)$
Add 6, then subtract 6 from the same side:	$x - 4 = 7 + 5 + 6 - 6$

We call these *equivalent equations* because they all come from the same original equation and they are all balanced. Another equivalent equation would be $x - 4 = 12$ or even $x = 16$.

When we solve equations, each line must be an *equivalent equation* that maintains balance. The phrase *equivalent equation* means that the equations don't look the same but the expressions on each side have the same value. This will happen if we have performed the same arithmetic operation on each side, or if we have manipulated the terms in the same way. We say the left side of the equal sign is equivalent to the right side which means that the sides have the same result if we substitute a value for the variable. See section 3.8.

Assume you are given the equation: $x - 3 = 12$. The five equations in column A are equivalent to this equation, but the five equations in column B are not. Can you see why? There are some hints to help you.

A: 5 equations that are equivalent to $x - 3 = 12$		**B: 5 equations that are NOT equivalent to $x - 3 = 12$**	
$x - 4 = 11$	subtract … from both sides	$x - 4 = 10$	subtract … on LHS, subtract … on RHS
$x + 10 = 25$		$x + 10 = 19$	
$2(x - 3) = 24$	multiply both sides by …	$2x - 3 = 24$	didn't multiply …. by 2
$x + 9 - 9 = 12 + 3$		$x + 9 = 12 + 3 - 9$	
$-x + 3 = -12$	multiply both sides by -1	$3 - x = 12$	

3.3 Paying attention to the structure of equations

In these materials you will also find questions with *sets of equations* (and sets of statements) that focus on the idea of balance. In such questions you are expected to pay attention to the *structure* of the equations, i.e. look at the expressions on each side of the equal sign and focus on what is the same and what is different. Here is a set of statements that all have $12 - 3$ on the left side and $4 + 5$ on the right of the equal sign. Now we make some changes to each side and then ask you to respond, depending on the question:

A. What value must we put in the box to keep the statement balanced: $12 - 3 + 7 = 4 + 5 + \square$
B. What value must we put in the box to keep the statement balanced: $12 - 3 + \square = 4 + 5 - 5$
C. Is this statement balanced: $12 - 3 - 6 = 4 + 5 - 6$
D. Is this statement balanced: $12 - 3 - 6 + 6 = 4 + 5$

In A and B above, you should have noticed that we have added/subtracted a number on one side of the equal sign. To maintain balance, we must perform the same operation with the same number on the other side.
In D, notice that we have "added zero" on the left side of the equal sign and we have not made any changes on the right side.

There are also some more challenging examples like the ones below. Here too you need to focus on the structure. For example, look at the numbers, symbols and operations on each side of the equal sign, and look for repetition of these groups of number/operations, etc.

You are told that $\square + ☺ = 13$,

a) What is the value of: $\square + ☺ + 1 = \cdots$ (Hint: what did we add on the left side?)
b) Is this statement TRUE or FALSE: $\square + ☺ - 5 = 13 - 5$
c) Is this statement TRUE or FALSE: $\square + ☺ - 5 = 8$
d) Is this statement balanced: $\square + ☺ + 25 - 20 = 13$
e) Is this statement balanced: $\square + ☺ + m = 13 + m$

Note: We don't know the individual values of ☐ or ☺. Of course, we can make up some combinations like $\square = 10$ and $☺ = 3$.

3.4 Commutative and associative properties

The *commutative property* and *associative property* are helpful in solving equations because they allow us to change the appearance of the equation, without changing the relationships between the components. We will first revisit these properties for whole numbers and integers. Then we will use them to manipulate expressions in algebraic equations.

3.4.1 Adding and subtracting whole numbers

The commutative property *for whole numbers states that when we add two numbers, the order does not matter, e.g.* $4 + 5 = 5 + 4$*. When we add more than two whole numbers, we apply the associative property.*

This means we can change their order, for example, $4 + 5 + 6 = 4 + 6 + 5 = 5 + 4 + 6$. We often do this when we want to combine some terms, like we know that $4 + 6 = 10$, so we want to add them first.

But these properties do not apply to subtraction. For example, $5 - 4 \neq 4 - 5$ (the result on the left is 1, while the result on the right is -1).

3.4.2 Adding and subtracting integers

When we add and subtract integers, it is easy to get confused with the extra symbols in the expressions. If we add 5 and -2, we write $5 + (-2)$. Using the commutative property, we can write: $5 + (-2) = (-2) + 5$. If we see this as an equation, then both sides give the same result, i.e. 3. If we have $-7 + 3$, this can be rewritten as $3 + (-7)$ using the commutative property.

When we subtract 5 and -2, the commutative property will not hold. For example, compare $5 - (-2)$ and $(-2) - 5$. The left side gives a result of 7 but the result on the right side is -7.

Although the commutative property doesn't hold for subtraction, we can convert a subtraction operation into the addition of a negative number. Then the commutative property will hold. For example, $4 - 6$ can be rewritten as $4 + (-6)$. Now this sum can be rewritten as $-6 + 4$ because addition is commutative. It is important to note that we will still get the same result (of -2) whether we use $4 - 6$ or $4 + (-6)$ or $-6 + 4$.

Here is an example using the associative property for adding integers: $(-6) + 5 + (-3)$ and $5 + (-6) + (-3)$:

$(-6) + 5 + (-3)$ can be written as: $-6 + 5 - 3 = -4$
$5 + (-6) + (-3)$ can be written as: $5 - 6 - 3 = -4$

This shows that the order of adding does not matter. Notice that *we started by adding the three numbers*. This is important because this is where the associative property applies. It allows us to change the order of the numbers. However, when we simplified each expression and so that it has no brackets, some addition operations changed to subtraction. But we still got the same result.

3.4.3 Adding and subtracting terms with variables in equations

We use the commutative and associative properties when solving algebraic equations. For example, consider $4 + 5k + 7 = 9$. We know that addition is commutative so we can rewrite $5k + 7$ as $7 + 5k$, and rewrite the equation as $4 + 7 + 5k = 9$. This helps us in adding the like terms, 4 and 7.

Similarly, the commutative property can be used in terms with a negative constant or coefficient.
For example, the equation $4 - 5k - 7 = 9$ can be rewritten as $4 + (-5k) + (-7) = 9$ and therefore we can write $4 + (-7) + (-5k) = 9$.
This means we can rewrite the original equation directly because $4 - 5k - 7 = 9$ is the same as $4 - 7 - 5k = 9$. In these materials, we have separate worksheets for numeric equations with whole numbers and numeric equations with integers. Both set of worksheets will provide practice in applying the commutative and associative properties.

3.5 Inverses and inverse operations

In section 2.2 we described briefly how inverses and inverse operations are used to solve equations. In this section we give more detail.

We use two inverses when working with equations: the additive inverse and the multiplicative inverse. There are two pairs of inverse operations: 1) addition and subtraction; and 2) multiplication and division. Inverse operations "undo" each other. For example, if we start with 12 and then add 5, we get 17. If we then subtract 5 from 17, we get back to 12. So the subtraction "reversed" the effect of adding.

Similarly, if we start with 12 and multiply by 3, we get 36. If we then divide 36 by 3, we get back to 12. So dividing by 3 reversed the effect of multiplying by 3. If we start with 12 and divide by 2, we get 6. Now multiply 2 by 6 and we get 12 again. So multiplying by 2 has reversed the effect of dividing by 2.

3.5.1 Additive inverse

The *additive identity* when working with numbers and algebraic expressions is zero. We use the idea of additive identity to find the *additive inverse* of a number. The additive inverse of a number is another number that must be added to make the sum (or the result of addition) to be zero. For example, the additive inverse of 5, is -5. We know this because $5 + (-5) = 0$. The additive inverse of -7 is 7 because $-7 + 7 = 0$. We ask ourselves: "what must I add to this number to get zero?"

The same applies with variables. The additive inverse of x is $-x$ because $x + (-x) = 0$. The additive inverse of $-3p$ is $3p$ because $-3p + 3p = 0$. We can see that the sum is zero by simplifying the like terms but we can also check by substituting values for x and p.

Here are three examples using substitution:

Let $x = 4$, then $-x = -4$	Let $p = 2$,	Let $p = -4$,
So $x + (-x) = 4 + (-4)$	then $-3p = -3(2) = -6$	then $-3p = -3(-4) = 12$
$= 4 - 4 = 0$	and $3p = 3(2) = 6$	and $3p = 3(-4) = -12$
	So $-3p + 3p = -6 + 6 = 0$	So $-3p + 3p = 12 + (-12) = 12 - 12 = 0$

3.5.2 Multiplicative inverse

We also use multiplicative inverses in solving equations. Remember that 1 is the multiplicative identity of a number because when you multiply any number by 1, the product is the original number, e.g. $5 \times 1 = 5$ and $-7 \times 1 = -7$. We use the multiplicative identity to find the *multiplicative inverse* of a number. To determine the multiplicative inverse of a number we way "what number must I multiply by to get a product of 1?".
For example, the multiplicative inverse of 5 is $\frac{1}{5}$ because $5 \times \frac{1}{5} = 1$. Also, the multiplicative inverse of $\frac{1}{5}$ is 5. So we can say that 5 and $\frac{1}{5}$ are multiplicative inverses of each other. The multiplicative inverse of -2 is $-\frac{1}{2}$ because $(-2)\left(-\frac{1}{2}\right) = 1$. In section 3.7 we give further examples to illustrate the use of the multiplicative inverse.

3.6 The meaning of "solution"

In section 1, we defined the *solution* of an equation as the value of the variable that makes the left side of the equation equal to the right side of the equation. When we want to check the solution, it is important to separate the left side from the right side to test whether they are equal. There are three examples below, labelled A – C.

Sometimes we can easily find the solution by inspection (e.g. A). This usually means we can find the answer by using mental arithmetic. When there are variables on both sides of the equal sign, it is more difficult to find the solution and so we use inverse operations (e.g. B). Solutions can be integers or fractions. It is also possible to have a solution of zero (e.g. C).

	Equation	Solution	Checking the solution
A	$2x - 3 = 15$	$x = 9$	LHS: $2(9) - 3 = 15$ RHS: 15
B	$2x + 15 = 3 + x$ $2x + 15 - 15 = 3 + x - 15$ $2x = -12 + x$ $2x - x = -12 + x - x$ $x = -12$	$x = -12$	LHS: $2(-12) + 15 = -9$ RHS: $3 + (-12) = -9$
C	$4x + 6 = 2(x + 1) + 4$	$x = 0$	LHS: $4(0) + 6 = 6$ RHS: $2(0 + 1) + 4 = 6$

It is possible to get linear equations where there is no solution, e.g. $4x + 6 = 4(x + 1) + 4$. It is also possible to get linear equations that are true for all values of the variable, e.g. $1 + 2x + 3 = 2(x + 2)$. However, we have not included such examples in our materials.

In the maths class, we sometimes refer to the answer (of any maths problem) as the "solution". In these materials we talk about someone's *response* to a question, and to their *answers*. When we use the word *solution*, we specifically mean the value that makes the equation true, i.e. the value that makes the result on the left to be the same as the result on the right.

3.7 Distributive law

We apply the distributive law when we multiply a monomial (a single term) by an expression containing two or more unlike terms. For example, $2(x+5)$ means the 2 must be multiplied by each term in the bracket. Note that the expression could also be written as: $(x+5)2$. In both cases the 2 is multiplied by the binomial to give $2x+10$. A typical example of an equation with brackets is: $2(x+5)=-3(4+x)-2$. To solve this equation, we must apply the distributive law.

We must apply the distributive law on the left side and the right side. On the left, 2 is multiplied by x and 5. On the right, -3 is multiplied by 4 and x. Note that -2 is not multiplied by the terms in the brackets, i.e. we read -2 as "subtract 2".	$2(x+5)=-3(4+x)-2$ $2x+10=-12-3x-2$ $2x+10=-14-3x$
Add additive inverse of $-3x$ to both sides.	$2x+10+3x=-14-3x+3x$ $5x+10=-14$
Add additive inverse of 10 to both sides.	$5x+10+(-10)=-14+(-10)$ $5x=-24$
Divide both sides by 5 (or apply multiplicative inverse of 5).	$\frac{5x}{5}=-\frac{24}{5}$ $x=-\frac{24}{5}$ or $-4\frac{4}{5}$

3.8 Examples of different strategies for using inverse operations to solve equations

We have already explained the procedures for solving linear equations and how to use additive and multiplicative inverses. In this section, we focus on one example that has letters on both sides, $3x-4=5x+6$. We show four different ways to solve this equation by applying the inverse operations in different orders and by collecting the variable on the left side in three cases, and on the right side in one case. In response 1 we explicitly show *adding* the additive inverse when it is a negative number. In responses 2, 3 and 4, we show it as subtraction. Each "new" equation is an equivalent equation to the equation in the previous line. It looks different but it doesn't change the value of the solution. Note that response 4 involves factorising. This is not a typical response, but it shows that the multiplicative inverse can be used at different stages of the process to find the solution.

Response 1		**Response 2**	
Collect variable on right Additive inverse of $3x$ Additive inverse of 6 Multiplicative inverse of 2	$3x-4=5x+6$ $3x-4+(-3x)=5x+6+(-3x)$ $-4=2x+6$ $-4+(-6)=2x+6+(-6)$ $-10=2x$ $\frac{-10}{2}=\frac{2x}{2}$ $-5=x$	Collect variable on left Additive inverse of $5x$ Additive inverse of -4 Multiplicative inverse of -2	$3x-4=5x+6$ $3x-4-5x=5x+6-5x$ $-2x-4=6$ $-2x-4+4=6+4$ $-2x=10$ $\frac{-2x}{-2}=\frac{10}{-2}$ $x=-5$
Response 3		**Response 4**	
Collect variable on left Additive inverse of -4 Additive inverse of $5x$ Multiplicative inverse of -2	$3x-4=5x+6$ $3x-4+4=5x+6+4$ $3x=5x+10$ $3x-5x=5x+10-5x$ $-2x=10$ $\frac{-2x}{-2}=\frac{10}{-2}$ $x=-5$	Collect variable on left Additive inverse of $5x$ Factorise on left, taking out common factor of -2, then multiplicative inverse of -2 Additive inverse of 2	$3x-4=5x+6$ $3x-4-5x=5x+6-5x$ $-2x-4=6$ $-2(x+2)=6$ $\frac{-2(x+2)}{-2}=\frac{6}{-2}$ $x+2=-3$ $x+2-2=-3-2$ $x=-5$

4. What makes equations difficult

Many learners have difficulty in solving equations because they are still getting used to other new aspects of high school maths – like working with integers and algebraic symbols. In this section we discuss six of the typical difficulties that learners have with these sections and we provide a summary of the essential aspects that need to be mastered to succeed in solving linear equations. The subsections are: 1) the view of the equal sign; 2) limitations of solving by inspection; 3) negatives and subtraction; 4) fractions and division; 5) algebraic notation; and 6) working with brackets.

4.1 Viewing the equal sign as "do-something" instead of having an equivalence view

If learners can only view the equal sign as a "do something" signal, they will have difficulty in making sense of equations. For example, if asked to fill the blank in a numerical equation like $7 + 2 =$ ___ $+3$, some learners give the answer of 9 because they focus on $7 + 2 =$ ___, but they ignore the other part of the equation. When learners do this, it tells us that they are interpreting the equal sign as something "to do" or "gives me". They need to view the equal sign as indicating equivalence between the left side and the right side of the statement. In the example, they need to work out what must be put in place of the blank so that the result on the right side *is the same as* the result on the left side, i.e. $7 + 2 = \mathbf{6} + 3$. See section 3.1 where this is discussed in more detail.

4.2 Recognising the limitations of solving by inspection

Solving equations by inspection is a very important skill but it only works well for equations with the variable on one side, such as $2x - 5 = 1$ and $4 - x = 20$. When the variable appears on both sides of the equal sign, we need to use inverses to solve the equation, e.g. $2x - 5 = 3 - x$. Many learners still try to solve this kind of equation by inspection and then they get stuck.

So this difficulty that learners experience is partly self-inflicted. It stems from a resistance to learn to solve equations using inverses because they can successfully solve easy equations using inspection.

4.3 Difficulties with negatives and subtraction

Learners encounter negative numbers for the first time in Grade 7. Until then, the minus symbol ($-$) has only one meaning: the *operation* of *subtraction*. When learners encounter negative numbers, the minus symbol takes on another meaning: it can represent a *sign* (negative) or an *operation* (subtraction). This can be very confusing. Consider the following example:

$$3 + (\underset{\text{sign}}{-}5) = 3 \overset{\text{operation}}{-} 5 = \underset{\text{sign}}{-}2$$

Reading from left to right, the first minus symbol represents a *sign* (*negative* five). The second minus symbol represents the operation *subtraction* (three subtract five). The third minus symbol represents a *sign* (*negative* two).

For this reason, we recommend the use of precise language when working with integers:
For operations, we say: *add* and *subtract*
For signs, we say: *positive* and *negative*

We avoid using *plus* and *minus* because these words don't tell us whether we are dealing with a sign or an operation. So we rather say:

Negative six add (positive) nine:	$-6 + 9 = 3$	
Five subtract positive seven:	$5 - (+7) = -2$	(equals negative two)
Negative five add negative seven:	$-5 + (-7) = -12$	(equals negative twelve)
Negative five subtract negative seven:	$-5 - (-7) = 2$	(equals positive two)

4.4 Working with fractions and division

In Grades 8 and 9, we mainly use fractions as coefficients of the variable. For example, $\frac{3}{2}x + 4 = 8$. We also encounter fractions as the multiplicative inverses of integers, for example $\frac{1}{4}$ is the multiplicative inverse of 4.

In the examples in section 3.5, we multiplied both sides of the equation by the multiplicative inverse, for example:

$$2y = 13$$
$$2y \times \tfrac{1}{2} = 13 \times \tfrac{1}{2}$$
$$y = \tfrac{13}{2}$$

Most times when we need to apply the multiplicative inverse, we actually focus on division. For example, instead of multiplying by $\frac{1}{2}$, we can divide by 2:

$$2y = 13$$
$$\tfrac{2y}{2} = \tfrac{13}{2}$$
$$y = \tfrac{13}{2}$$

We can do this because multiplying by $\frac{1}{2}$ is the same as dividing by 2.

Remember: when we divide by the coefficient of the variable, we are actually applying the multiplicative inverse to both sides of the equation.

4.5 Difficulties with algebraic symbols

Our research has shown that many difficulties that learners have with equations stem from their difficulties in working with algebraic expressions. Here we discuss three ways in which algebraic notation and terminology can be confusing.

4.5.1 Sometimes a symbol represents a sign, sometimes it represents an operation

We have already noted that the minus symbol can represent a sign or an operation. Similarly, the plus sign can represent a sign or an operation. Consider the expression: $4 - 3x$. We say "4 subtract $3x$". This sounds as if the minus symbol does not belong to $3x$. We say the expression has two terms that are separated by the operation of subtraction. This also suggests that the minus symbol does not belong to the $3x$. But then we say the terms are 4 and $-3x$ (four and negative three x) which means the minus symbol *is* connected to the $3x$. We also say "the coefficient of x is negative 3". Once again, this indicates that the minus symbol belongs to the 3.

This is confusing because sometimes we are separating the minus symbol from the 3 and sometimes we are attaching it to the 3. Part of learning algebra involves learning when to combine the minus (or plus) symbol with the letter or number and when to separate it from the letter or number.

4.5.2 Errors in operating on like and unlike terms

We can add and subtract *like* terms. We cannot add and subtract *unlike* terms. Many learners combine unlike terms. This is often called the *conjoining* error. Typical examples of this error are: $7 + x = 7x$; $2g + h = 2gh$; $3x - 3 = x$. We also see errors with *like* terms such as $3x - x = 3$ (instead of $2x$)

When working with equations, we aim to collect terms with variables on one side of the equal sign, and terms with constants on the other side of the equal sign. To do this we must apply additive inverses. See section 3.5.1 for more details on additive inverses.

4.6 Difficulties with brackets

Brackets have many different uses in mathematics. We describe some of these in detail in the ***x*.act** materials. When working with linear equations, we generally use brackets in four ways:

i. To substitute numbers into expressions: we usually do this when checking the solution of an equation. See example A below.
ii. To separate signs and operations: we usually do this when adding an additive inverse that is a negative number. e.g. given the equation $x + 3 = 2x + 5x$, we write: $+3 + (-3) = 2x + 5 + (-3)$.
iii. When applying the distributive law to a given equation, as shown in section 3.7.
iv. To multiply an entire side of an equation by a number, e.g. C.

A. Checking the solution	**B. Applying the additive inverse**	**C. Multiplying the entire side of an equation by a number**
Given: $2x + 5 = x - 3$ LHS: $2(-8) + 5 = -16 + 5 = -11$ RHS: $(-8) - 3 = -11$	$2x + 5 = x - 3$ $2x + 5 + (-5) = x - 3 + (-5)$ $2x + 5 - 5 = x - 3 - 5$ $2x = x - 8$ etc.	$\frac{1}{2}x = x - 3$ $2 \times \frac{1}{2}x = (x - 3) \times 2$ $x = 2x - 6$ etc.
We have substituted -8 in place of x	We add -5 so we use a bracket to separate the *addition operation* from the *negative sign.*	We multiplied the entire right side by 2. We need to use brackets to ensure that all terms on the right side are multiplied by 2 (distributive law).

Note that some of the ways of working with brackets were also dealt with in sections 4.1, 4.2 and 4.3.

Worksheet 1.1: Numerical equations with whole numbers

This worksheet focuses on solving numerical equations involving addition. There are two whole numbers on each side of the equal sign.

Questions

1) Thabo solved these equations:

A. $5+3=4+\square$ Answer: $\square=4$

B. $5+3=\square+6$ Answer: $\square=8$

C. $7+\square=4+4$ Answer: $\square=4$

D. $8+\square=5+3$ Answer: $\square=0$

Some of his answers are wrong.

a) Correct any mistakes Thabo has made.

b) Why do you think he made the mistakes?

> When solving a linear equation, we find a value to balance the result on each side of the equal sign.

2)

a) What is the additive inverse of -4? How do you know?

b) What is the additive inverse of 6? How do you know?

3) Look at the equation $5+7=\square\ +4$. Helena and Nisha solve the equation in different ways. Both girls find that the solution is 8.

Read their methods carefully. Make sure you can link the words and the statements.

<u>Helena uses *solving by inspection*.</u>	
Helena first works out the value on the side without the $\square$:	$5+7=\square\ +4$
she adds the 5 and 7 to get 12.	
She now knows $\square\ +4$ must be 12.	$12=\square\ +4$
Helena thinks 'What must I add to 4 to get 12?' to get a value for $\square$.	
Helena gets:	$\square=8$

<u>Nisha uses *solving using additive inverses.*</u>	$5+7=\square\ +4$
Nisha's first step is to simplify $5+7$ just like Helena did:	$12=\square\ +4$
Nisha then subtracts 4 from both sides of the equation to get the $\square$ on its own. (She uses the *additive inverse* of 4)	$12-4=\square\ +4-4$ $8=\ \square+0$
Nisha then simplifies both sides of the equation to get a value for $\square$.	
She gets:	$8=\square$
Which is the same as:	$\square=8$

Look at the equations below:

A. $6+3=\square+5$ B. $3+3=\square+1$ C. $\square+7=9+2$

a) Quickly work out the answer using Helena's method. Write down only the answers.

b) Now try to solve the equations using Nisha's method which uses *additive inverses.*

c) Go back to your response for equation A. Write what you did in each step and say why you did it.

4) If we know that $7+\Delta=0$, then which of the following statements are TRUE? Give reasons.

A. The value of Δ is 7.

B. Δ can be any whole number.

C. The value of Δ is 0.

D. The value of Δ is -7.

E. Δ and 7 are additive inverses of each other.

F. Δ is the additive identity for addition.

Worksheet 1.1: Numerical equations with whole numbers

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1) Thabo solved these equations:
A. $5+3=4+\square$ Answer: $\square=4$
B. $5+3=\square+6$ Answer: $\square=8$
C. $7+\square=4+4$ Answer: $\square=4$
D. $8+\square=5+3$ Answer: $\square=0$

Some of his answers are wrong.
a) Correct any mistakes Thabo has made.
b) Why do you think he made the mistakes?</td><td>1)
a) B. $\square=2$
b) The left side was 8 so he made $\square=8$
a) C. $\square=1$
b) 4 was added on the right in the same position as $\square$ on the left side so he replaced $\square$ with 4.</td></tr>
<tr><td>2)
a) What is the additive inverse of -4? How do you know?
b) What is the additive inverse of 6? How do you know?</td><td>2)
a) 4 because $-4+4=0$
b) -6 because $6+(-6)=0$</td></tr>
<tr><td>3) Look at the equation $5+7=\square+4$. Helena and Nisha solve the equation in different ways. Both girls find that the solution is 8.
Read their methods carefully. Make sure you can link the words and the statements.
Look at the equations below:
A. $6+3=\square+5$
B. $3+3=\square+1$
C. $\square+7=9+2$
a) Quickly work out the answer using Helena's method. Write down only the answers.
b) Now try to solve the equations using Nisha's method which uses additive inverses.
c) Go back to your response for equation A. Write what you did in each step and say why you did it.</td><td>3)
a) A. $\square=4$ B. $\square=5$ C. $\square=4$
b)
A. $6+3=\square+5$
$9=\square+5$
$9-5=\square+5-5$
$4=\square+0$
$4=\square$
C. $\square+7=9+2$
$\square+7=11$
$\square+7-7=11-7$
$\square+0=4$
$\square=4$
B. $3+3=\square+1$
$6=\square+1$
$6-1=\square+1-1$
$5=\square+0$
$5=\square$
c) Simplify $6+3$ to see what $\square+5$ must equal.
Subtract 5 from both sides to get $\square$ on own.
Simplify both sides to get a result for $\square$.</td></tr>
<tr><td>4) If we know that $7+\Delta=0$, then which of the following statements are TRUE? Give reasons.
A. The value of Δ is 7.
B. Δ can be any whole number.
C. The value of Δ is 0.
D. The value of Δ is -7.
E. Δ and 7 are additive inverses of each other.
F. Δ is the additive identity for addition.</td><td>4) Statements which are TRUE:
D: The value of Δ is -7 because $7-7=0$

E: Δ and 7 are additive inverses of each other.
Additive inverses add up to 0.</td></tr>
</table>

Worksheet 1.2: Numerical equations with whole numbers

This worksheet focuses on solving numerical equations. Most equations involve subtraction and there are two whole numbers on each side of the equal sign.

> A statement that is balanced has the same result on both sides of the equal sign. So, a *balanced statement* is an equation.

Questions

1) Solve the equations by inspection:

   a) $5 - 0 = 9 - \square$
   b) $\square - 2 = 9 - 6$
   c) $7 - 5 = \square - 4$
   d) $9 + 3 = \square + 4$

2) Helena and Nisha solve the equation $6 - 3 = \square - 4$ in different ways:
   Read them carefully and make sure you can see the links between the words and the statements.

Helena uses *inspection*, as you did in Q1.	
Helena subtracts 3 from 6 to get:	$6 - 3 = \square - 4$
Helena then thinks,	
'What subtract 4 gives 3?'	$3 = \square - 4$
She gets:	$\square = 7$

Nisha uses *additive inverses.*	$6 - 3 = \square - 4$
Nisha subtracts 3 from 6 to get:	$3 = \square - 4$
Nisha adds 4 to both sides to get the $\square$ on its own.	$3 + 4 = \square - 4 + 4$
	$3 + 4 = \square - 0$
Nisha then simplifies further to get:	$7 = \square$
which is the same as:	$\square = 7$

   Look at the equations below:

   A. $6 - 3 = \square - 2$
   B. $\square - 4 = 10 - 3$
   C. $\square - 7 = 13 - 11$

   a) Solve them using Helena's method. Just write the answers.
   b) Now try to solve the equations using Nisha's method which uses *additive inverses*.
   c) Write down what you did in each step of A in Q2b and why you did it.

3) If you know that $8 - 4 = 7 - 3$, use it to solve these equations:

   a) $8 - 4 - 7 = \square$
   b) $8 - 4 + 3 = \square$
   c) $7 - 3 + 4 = \square$
   d) $8 - 4 + 3 - 7 = \square$

4) If $\square - \triangle = 29 - 5$,
   give *four* sets of values for $\square$ and $\triangle$ that will balance the statement.

5) Look at these four statements and answer the questions that follow:

   A. $2 + 3 = 4 + 1$
   B. $2 + 3 + 5 = 4 + 1$
   C. $2 + 3 - 1 + 1 = 4 + 1$
   D. $2 + 3 - 3 = 4 + 1$

   a) Highlight $2 + 3$ and $4 + 1$ in each statement. This will help you see the structure of the statements.
   b) Which statements are equations?
   c) Rewrite the statements that are not equations with the $\neq$ sign.

6) Here is a list of six statements:

   A. $\lozenge - 2 = \oslash$
   B. $\oslash + 2 = \lozenge$
   C. $\lozenge + 6 = \oslash + 2 + 6$
   D. $\lozenge + 5 = \oslash + 2$
   E. $\lozenge + 5 = \oslash + 7$
   F. $\lozenge - 2 + 3 = \oslash + 3$

   You are now told that $\lozenge = \oslash + 2$. Use this information to decide which statements are balanced.
   Write 'EQUATION' for those statements that are balanced.

Worksheet 1.2: Numerical equations with whole numbers

Answers

Questions	Answers
1) Solve the equations by inspection: a) $5-0=9-\square$ c) $7-5=\square-4$ b) $\square-2=9-6$ d) $9+3=\square+4$	1) a) $\square=4$ c) $\square=6$ b) $\square=5$ d) $\square=8$
2) Helena and Nisha Look at the equations below: A. $6-3=\square-2$ B. $\square-4=10-3$ C. $\square-7=13-11$ a) Solve them using Helena's method. Just write the answers. b) Now try to solve the equations using Nisha's method which uses *additive inverses*. c) Write down what you did in each step of A in Q2b and why you did it.	2) a) A. $\square=5$ B. $\square=11$ C. $\square=9$ b) A. $6-3=\square-2$ $3=\square-2$ $3+2=\square+2-2$ $5=\square+0$ $5=\square$ B. $\square-4=10-3$ $\square-4=7$ $\square-4+4=7+4$ $\square-0=11$ $\square=11$ C. $\square-7=13-11$ $\square-7=2$ $\square-7+7=2+7$ $\square+0=9$ $\square=9$ c) Simplify $6-3$ to see what the right side must equal. Subtract 2 from both sides to get $\square$ on own. Simplify both sides to get the result for $\square$
3) If you know that $8-4=7-3$, use it to solve these equations: a) $8-4-7=\square$ b) $8-4+3=\square$ c) $7-3+4=\square$ d) $8-4+3-7=\square$	3) a) $\square=-3$ Subtracted 7 on both sides b) $\square=7$ Added 3 to both sides c) $\square=8$ Added 4 to both sides d) $\square=0$ Added 3 and subtracted 7 on both sides
4) If $\square-\triangle=29-5$, give *four* sets of values for $\square$ and $\triangle$ that will balance the statement.	4) Many possible answers will result in a difference of 24 e.g. 25 and 1; 26 and 2; 39 and 15
5) Look at these four statements and answer the questions that follow: A. $2+3=4+1$ B. $2+3+5=4+1$ C. $2+3-1+1=4+1$ D. $2+3-3=4+1$ a) Highlight $2+3$ and $4+1$ in each statement. This will help you see the structure of the statements. b) Which statements are equations? c) Rewrite the statements that are not equations with the $\neq$ sign.	5) a) Highlighting of $2+3$ and $4+1$ b) A. $2+3=4+1$ C. $2+3-1+1=4+1$ c) B. $2+3+5\neq4+1$ D. $2+3-3\neq4+1$
6) Here is a list of six statements: A. $\Diamond-2=\oslash$ D. $\Diamond+5=\oslash+2$ B. $\oslash+2=\Diamond$ E. $\Diamond+5=\oslash+7$ C. $\Diamond+6=\oslash+2+6$ F. $\Diamond-2+3=\oslash+3$ You are now told that $\Diamond=\oslash+2$. Use this information to decide which statements are balanced. Write 'EQUATION' for those statements that are balanced.	6) Given: $\Diamond=\oslash+2$ A. $\Diamond-2=\oslash$ EQUATION B. $\oslash+2=\Diamond$ EQUATION C. $\Diamond+6=\oslash+2+6$ EQUATION D. $\Diamond+5=\oslash+2$ Not an equation E. $\Diamond+5=\oslash+7$ Not an equation F. $\Diamond-2+3=\oslash+3$ EQUATION

Worksheet 1.3: Numerical equations with whole numbers

This worksheet focuses on solving whole number numerical equations that have addition on both sides, or subtraction on both sides of the equal sign.

Questions
1) Solve these equations by inspection: a) $5+3=7+\square$ b) $\square-2=8-5$ c) $10-\square=12-3$ d) $4+\square=5+7$
2) Select the correct option for $\square$: a) $10-\ 2=\square-4$ i) 8 ii) 12 iii) 14 iv) 16 b) $7+\ \square=9+2$ i) 0 ii) 2 iii) 3 iv) 4
3) Solve the equations using *inspection* and then using *additive inverses*. a) $10-3=\square-1$ b) $\square+3=8+9$ c) $\square+6=7+3$ d) $12-4=\square\ -6$
4) A balanced statement has the same result one each side of the equal sign. e.g. $9+3=10+2$. The result on each side of the equal sign is 12. If the result on each side of the equal sign is not the same, the statement is not balanced. e.g. $9+3=10+2-3$ The result on the left of the equal sign is 12 but the result on the right is 9. So, the statement is not balanced. We write $9+3\neq10+2-3$. We say that “the left side *is not equal to* the right side”. Look at these three statements: A. $12-3=4+5$ B. $12-3-5=4+5$ C. $12-3+6=4+5-6$ a) Decide which statements are not balanced. Re-write them using the $\neq$ sign. b) Make one change to the right side so that the statement becomes balanced.
5) The equation $8-4=\square-3$ has two whole numbers on each side. Use this information to decide whether the following equations are balanced or NOT balanced. If they are not balanced, re-write them with the $\neq$ sign. a) $8-4+3=\square+3$ b) $\square-3+4=8$ c) $8=4+\square-3$ d) $\square=8-4-3$

Worksheet 1.3: Numerical equations

Answers

Questions	**Answers**
1) Solve these equations by inspection: a) $5 + 3 = 7 + \square$ b) $\square - 2 = 8 - 5$ c) $10 - \square = 12 - 3$ d) $4 + \square = 5 + 7$	1) a) $\square = 1$ b) $\square = 5$ c) $\square = 1$ d) $\square = 8$
2) Select the correct option for $\square$: a) $10 - 2 = \square - 4$ i) 8 ii) 12 iii) 14 iv) 16 b) $7 + \square = 9 + 2$ i) 0 ii) 2 iii) 3 iv) 4	2) a) $\square = 12$ option ii) b) $\square = 4$ option iv)
3) Solve the equations using *inspection* and then using *additive inverses*. a) $10 - 3 = \square - 1$ b) $\square + 3 = 8 + 9$ c) $\square + 6 = 7 + 3$ d) $12 - 4 = \square - 6$	3) Inspection / Additive inverses hint a) $\square = 7$ Add 1 to both sides b) $\square = 14$ Subtract 3 from both sides c) $\square = 4$ Subtract 6 from both sides d) $\square = 14$ Add 6 to both sides
4) A balanced statement has the same result on both sides of the equal sign. e.g. $9 + 3 = 10 + 2$ The result on each side of the equal sign is 12 If the result on both sides of the equal sign is not the same, the statement is not balanced e.g. $9 + 3 = 10 + 2 - 3$ The result on the left of the equal sign is 12 but the result on the right is 9 Look at these three statements: A. $12 - 3 = 4 + 5$ B. $12 - 3 - 5 = 4 + 5$ C. $12 - 3 + 6 = 4 + 5 - 6$ a) Decide which statements are not balanced. Re-write them using the $\neq$ sign. b) Make one change to the right side so that the statement becomes balanced.	4) a) B. $12 - 3 - 5 \neq 4 + 5$ C. $12 - 3 + 6 \neq 4 + 5 - 6$ b) B. Subtract 5 from the right side: $12 - 3 - 5 = 4 + 5 \mathbf{- 5}$ C. Add 6 to the right side: $12 - 3 - 6 = 4 + 5 \mathbf{+ 6}$
5) The equation $8 - 4 = \square - 3$ has two whole numbers on each side. Use this information to decide whether the following equations are balanced or NOT balanced. If they are not balanced, re-write them with the $\neq$ sign. a) $8 - 4 + 3 = \square + 3$ b) $\square - 3 + 4 = 8$ c) $8 = 4 + \square - 3$ d) $\square = 8 - 4 + 3$	5) Given: $8 - 4 = \square - 3$ a) $8 - 4 + 3 \neq \square + 3$ NOT balanced b) $\square - 3 + 4 = 8$ balanced c) $8 = 4 + \square - 3$ balanced d) $\square \neq 8 - 4 + 3$ NOT balanced

Worksheet 1.4: Numerical equations with whole numbers

This worksheet focuses on solving equations involving: 1) addition only; and 2) subtraction only. There are two or more whole numbers on each side of the equal sign.

Questions
1) Three equations are given below. Shane's answers are given next to each equation. Copy the equations and answers. If Shane's answer is correct, give it a tick ✓. If Shane's answer is incorrect, give it a cross ✗ and give the correct answer. a) $1+3+2=\square+2+1$ Answer: $\square=6$ b) $8-5-\square=14-7-6$ Answer: $\square=2$ c) $9-4-\square=10-4-4$ Answer: $\square=2$
2) Solve the following equations: a) $7+3=\square+7$ b) $\square+4=6+3$ c) $12-2=\square-9$ d) $\square-4=10-5$ If you are not told which method to use to solve an equation, you can use inspection **or** additive inverses.
3) Below is a set of four equations: A. $12-3-5-0=\square-1$ B. $12-3-5-1=\square-1$ C. $12-3-5-2=\square-1$ D. $12-3-5-3=\square-1$ Look at the highlighted part in equation A. Note that these numbers appear in all the equations. a) Highlight another part which is the same in all the equations. b) Solve all the equations. c) Compare your answers to the four equations. What is the relation between the values of $\square$ in each case? What causes this relation?
4) Below is a set of four equations: A. $5+3+2=\square+4$ B. $5+3+2+7=\square+4+7$ C. $40+5+3+2=\square+4+40$ D. $3+2+5+12=12+\square+4$ To predict the answers: Look for what is the same and different in the equations and write what you expect the value of $\square$ to be. a) Highlight the parts which are the same for each equation. b) Predict TRUE or FALSE: The value of the box will NOT be the same for all the equations. Give reasons for your response. c) Solve all the equations. d) Was your prediction in Q4b correct? Why/why not?
5) You are given the equation: $\Diamond+4=\oslash-2$. This equation has two unknown values, $\Diamond$ and $\oslash$. When you balance the statements below, both unknown values must appear in the equation. They could be on different sides of the equal sign, or they could be on the same side as in Q5b. a) $4+\Diamond=$ b) $\Diamond+4-\oslash=$ c) $\oslash-2+0=$ d) $\Diamond+4+2=$ e) $\oslash-2-4=$ f) $\oslash=$

Worksheet 1.4: Numerical equations with whole numbers

Answers

Questions	**Answers**
1) Three equations are given below. Shane's answers are given next to each equation. Copy the equations and answers. If Shane's answer is correct, give it a tick ✓. If Shane's answer is incorrect, give it a cross ✗ and give the correct answer. a) $1 + 3 + 2 = \square + 2 + 1$ Answer: $\square = 6$ b) $8 - 5 - \square = 14 - 7 - 6$ Answer: $\square = 2$ c) $9 - 4 - \square = 10 - 4 - 4$ Answer: $\square = 2$	1) a) Answer: $\square = 6$ ✗ $\square = 3$ Shane used the left side as his answer b) Answer: $\square = 2$ ✓ c) Answer: $\square = 2$ ✗ $\square = 3$ Shane used the right side as his answer
2) Solve the following equations: a) $7 + 3 = \square + 7$ b) $\square + 4 = 6 + 3$ c) $12 - 2 = \square - 9$ d) $\square - 4 = 10 - 5$	2) Inspection — Additive inverse hint a) $\square = 3$ — Subtract 7 from each side b) $\square = 5$ — Subtract 4 from each side c) $\square = 19$ — Add 9 to each side d) $\square = 9$ — Add 4 to each side
3) Below is a set of four equations: A. $12 - 3 - 5 - 0 = \square - 1$ B. $12 - 3 - 5 - 1 = \square - 1$ C. $12 - 3 - 5 - 2 = \square - 1$ D. $12 - 3 - 5 - 3 = \square - 1$ Look at the highlighted part in equation A. Note that these numbers appear in all the equations. a) Highlight another part which is the same in all the equations. b) Solve all the equations. c) Compare your answers to the four equations. What is the relation between the values of $\square$ in each case? What causes this relation?	3) a) $\square - 1$ b) A. $\square = 5$ B. $\square = 4$ C. $\square = 3$ D. $\square = 2$ c) $\square$ is 1 less each time. This is because on the left of the equal sign we subtract one extra each time, e.g. 0 then 1 etc.
4) Below is a set of four equations: A. $5 + 3 + 2 = \square + 4$ B. $5 + 3 + 2 + 7 = \square + 4 + 7$ C. $40 + 5 + 3 + 2 = \square + 4 + 40$ D. $3 + 2 + 5 + 12 = 12 + \square + 4$ a) Highlight the parts which are the same for each equation. b) Predict TRUE or FALSE: The value of the box will NOT be the same for all the equations. Give reasons for your response. c) Solve all the equations. d) Was your prediction in Q4b correct? Why/why not?	4) a) $5 + 3 + 2$ and $\square + 4$ b) FALSE: The value of the box will be the same because the same number is added on each side of each equation. c) $\square = 6$ for all equations d) Prediction was correct. All the equations have the same value for the box. [If we subtract 7 from each side of B. It is the same as adding 0, The same applies to subtracting 40 in C. and 4 in D. This makes each equation go back to A's equation]
5) You are given the equation: $\Diamond + 4 = \oslash - 2$. This equation has two unknown values, $\Diamond$ and $\oslash$. When you balance the statements below, both unknown values must appear in the equation. They could be on different sides of the equal sign, or they could be on the same side as in Q5b. a) $4 + \Diamond =$ b) $\Diamond + 4 - \oslash =$ c) $\oslash - 2 + 0 =$ d) $\Diamond + 4 + 2 =$ e) $\oslash - 2 - 4 =$ f) $\oslash =$	5) Given: $\Diamond + 4 = \oslash - 2$ a) $4 + \Diamond = \oslash - 2$ b) $\Diamond + 4 - \oslash = -2$ c) $\oslash - 2 + 0 = \Diamond + 4 + 0$ d) $\Diamond + 4 + 2 = \oslash$ e) $\oslash - 2 - 4 = \Diamond + 4 - 4$ f) $\oslash = \Diamond + 4 + 2$

Worksheet 1.5 Numerical equations with whole numbers

This worksheet focuses on solving equations that contain a mixture of addition and subtraction. There are two or more whole numbers on each side of the equal sign.

Questions

1) Match the columns:

COLUMN A
a) $9 + 4 - \square = 10 + 4 - 7$
b) $7 - 4 + 6 = 8 + 6 - \square$
c) $3 + 9 - 5 = 11 - 4 - \square$

COLUMN B	
I.	$\square = 0$
II.	$\square = 5$
III.	$\square = 6$
IV.	$\square = 7$

2) Solve these four equations by inspection:
   a) $5 + \square = 10 - 3$
   b) $9 - 2 = \square + 2$
   c) $8 - \square = 7 - 1$
   d) $\square - 4 = 9 - 8$

3) Solve these equations using additive inverses:
   a) $8 + 3 - 5 = \square + 2$
   b) $\square + 4 - 2 = 6 + 7 - 3$
   c) $11 - 1 - 3 = \square + 4 + 2$

To predict the answers: Look for what is the same and different in the equations and write what you expect the value of $\square$ to be.

4) Given: $4 + 3 + 6 = \square + 5$
   a) Look for what is the same and different in this set of four equations:
      A. $4 + 3 + 6 - 4 = \square + 5$
      B. $4 + 3 + 6 - 5 = \square + 5$
      C. $4 + 3 + 6 - 6 = \square + 5$
      D. $4 + 3 + 6 - 7 = \square + 5$
   b) Do you expect the values for $\square$ to be the same for each equation? Give a reason for your answer.
   c) Solve the equations.
   d) Was your prediction in Q4b correct?

5) Given: $9 + 3 + 2 = \square + 2 + 4$
   a) Solve each equation:
      A. $9 + 3 + 2 - 1 = \square + 2 + 4 - 1$
      B. $9 + 3 + 2 + 2 = \square + 2 + 4 + 2$
   b) You should have got the same answers to equations A and B. Why does this happen?
   c) Predict the answers to the following:
      C. $9 + 3 + 2 - 12 = \square + 2 + 4 - 12$
      D. $9 + 3 + 2 + 40 - 40 = \square + 2 + 4$
      E. $32 - 32 + 9 + 3 + 2 = \square + 2 + 4$
      F. $3 + 2 + 9 - 14 = 2 + 4 + \square - 14$
   d) Use any method to check your predictions.

6) Here is a set of five statements:
   A. $\Diamond - 2 + 5 = \oslash - 2$
   B. $\Diamond - 2 + 5 = \oslash + 5$
   C. $\Diamond = \oslash + 2$
   D. $\Diamond - 2 + 3 = \oslash$
   E. $\Diamond - \oslash - 1 = 1$

   You are now told that $\Diamond - 2 = \oslash$
   Use the equation $\Diamond - 2 = \oslash$ to decide which of the statements (A to E) are balanced and why.

Worksheet 1.5: Numerical equations whole numbers

Answers

Questions and answers

1) Match the columns:

COLUMN A	
a)	$9 + 4 - \square = 10 + 4 - 7$
b)	$7 - 4 + 6 = 8 + 6 - \square$
c)	$3 + 9 - 5 = 11 - 4 - \square$

COLUMN B	
I.	$\square = 0$
II.	$\square = 5$
III.	$\square = 6$
IV.	$\square = 7$

Answers

a) III $\square = 6$ b) II $\square = 5$ c) I $\square = 0$

2) Solve these four equations by inspection:
   a) $5 + \square = 10 - 3$ **Answer** $\square = 2$
   b) $9 - 2 = \square + 2$ **Answer** $\square = 5$
   c) $8 - \square = 7 - 1$ **Answer** $\square = 2$
   d) $\square - 4 = 9 - 8$ **Answer** $\square = 5$

3) Solve these equations using additive inverses:
   a) $8 + 3 - 5 = \square + 2$
   b) $\square + 4 - 2 = 6 + 7 - 3$
   c) $11 - 1 - 3 = \square + 4 + 2$

Answers
   a) $\square = 4$ subtract 2 from each side
   b) $\square = 8$ subtract 2 from each side
   c) $\square = 1$ subtract 6 from each side

4) Given: $4 + 3 + 6 = \square + 5$
   a) Look for what is the same and different in this set of four equations:
      A. $4 + 3 + 6 - 4 = \square + 5$
      B. $4 + 3 + 6 - 5 = \square + 5$
      C. $4 + 3 + 6 - 6 = \square + 5$
      D. $4 + 3 + 6 - 7 = \square + 5$
   b) Do you expect the values for $\square$ to be the same for each equation? Give a reason for your answer.
   c) Solve the equations.
   d) Was your prediction in Q4b correct?

Answers
   a) $4 + 3 + 6$ and $\square + 5$ is the same in each equation. What is different is that a different number is subtracted from the left of each equation each time.
   b) $\square$ will not to be the same. We subtract a different number each time. This means the value in $\square$ will have to change to ensure the equations remain balanced.
   c) A. $\square = 4$ B. $\square = 3$ C. $\square = 2$ D. $\square = 1$
   d) Yes. The answers were different

Questions and answers

5) Given: $9 + 3 + 2 = \square + 2 + 4$
   a) Solve each equation:
      A. $9 + 3 + 2 - 1 = \square + 2 + 4 - 1$
      B. $9 + 3 + 2 + 2 = \square + 2 + 4 + 2$
   b) You should have got the same answers to equations A and B. Why does this happen?
   c) Predict the answers to the following:
      C. $9 + 3 + 2 - 12 = \square + 2 + 4 - 12$
      D. $9 + 3 + 2 + 40 - 40 = \square + 2 + 4$
      E. $32 - 32 + 9 + 3 + 2 = \square + 2 + 4$
      F. $3 + 2 + 9 - 14 = 2 + 4 + \square - 14$
   d) Use any method to check your predictions.

Answers
   a) A. $\square = 8$ B. $\square = 8$
   b) Same answers because the $9 + 3 + 2$ on the left and the $\square + 2 + 4$ on the right stay the same and we subtract the same value from (or add the same value to) the left and right.
   c) $\square = 8$ for all equations
   d) Check A: using inverses
      $2 = \square - 6$
      $2 + 6 = \square - 6 + 6$
      $8 = \square$
      or inspection $2 = \square - 6$ so $8 = \square$

6) Here is a set of five statements:
   A. $\lozenge - 2 + 5 = \oslash - 2$
   B. $\lozenge - 2 + 5 = \oslash + 5$
   C. $\lozenge = \oslash + 2$
   D. $\lozenge - 2 + 3 = \oslash$
   E. $\lozenge - \oslash - 1 = 1$

You are now told that $\lozenge - 2 = \oslash$
Use the equation $\lozenge - 2 = \oslash$ to decide which of the statements (A to E) are balanced and why.

Answers Given $\lozenge - 2 = \oslash$

B: The same number is added to each side so when it is subtracted from each side we get $\lozenge - 2 = \oslash$ which we were given, so the statement is balanced **or** substitute $\oslash$ for $\lozenge - 2$ into $\lozenge - 2 + 5 = \oslash + 5$ and get $\oslash + 5 = \oslash + 5$ so the statement is balanced.

C: Subtract 2 from each side to get $\lozenge - 2 = \oslash$ **or** substitute $\lozenge - 2$ for $\oslash$ into $\lozenge = \oslash + 2$ and get $\lozenge = \lozenge - 2 + 2$ which simplifies to $\lozenge = \lozenge$ hence the statement is balanced.

E: Subtract 1 from each side to get $\lozenge - \oslash - 2 = 0$ then add $\oslash$ to each side to get $\lozenge - 2 = \oslash$

Note: substitution for $\oslash$ gets complicated [substitute $\lozenge - 2 = \oslash$ for $\oslash$ into E. and get $\lozenge - (\lozenge - 2) - 1 = 1$
Which involves integers and a solution of $1 = 1$.
This type of solution is discussed in algebraic equations]

Worksheet 1.6: Numerical equations with whole numbers

This worksheet focuses on the $\square$ in different positions in equations with whole numbers. The position of the $\square$ makes some equations more difficult than others. As in Worksheets 1.1 to 1.5, our aim is to get $\square$ on its own.

Questions

1) Look at this equation: $5+6=9+\square$. Note that it has $\square$ at the end of the equation.
   a) Solve the equation by inspection.
   b) Jack solved $5+6=9+\square$ using additive inverses.
   Copy his response and answer the questions:

Jack's response	**Questions**
$5+6=9+\square$	i) Is Jack correct to re-write $9+\square$ as $\square+9$? Why?
$11=\square+9$	ii) How does Jack get 11?
$11-9=\square+9-9$	iii) Why does Jack subtract 9 from each side?
$2=\square+0$	iv) How does Jack get 2?
	v) And how does he get 0?
$2=\square$	vi) Does Jack get the correct answer?

2) Here is another equation: $7-3=9-\square$. Note that it has "subtract $\square$" at the end of the equation.
   a) Solve the equation by inspection.
   b) Mbali solved the equation using additive inverses.
   Copy her response and answer the questions:

Mbali's response	**Questions**
$7-3=9-\square$	
$4=9-\square$	i) How does Mbali get 4?
$4+\square=9-\square+\square$	ii) What has Mbali done on each side?
$4+\square=9+0$	iii) Why is there a 0?
$\square+4=9$	iv) Why does Mbali change $\square+4$ to $4+\square$?
$\square=9-4$	v) How does Mbali get -4 on the right of the equal sign?
$\square=5$	vi) This should be the same answer you got for Q2a. Is it?
	vii) Go back to the first line of Mbali's response. Could she have/ changed $9-\square$ to $\square-9$? Explain.

3) Solve these equations using additive inverses. Refer to Jack and Mbali's responses if you need help.
   a) $5-3=9-\square$ also write what you did in each step in Q3a, and why you did it.
   b) $7-5=1+\square$
   c) $16-\square=12-4$
   d) $2+\square=7-3$ also write what you did in each step in Q3d, and why you did it.
   e) $9+3=18-\square$

Worksheet 1.6: Numerical equations whole numbers

Answers

Questions	Answers
1) Look at this equation: $5+6=9+\square$. Note that it has $\square$ at the end of the equation. a) Solve the equation by inspection. b) Jack solved $5+6=9+\square$ using additive inverses. Copy his response and answer the questions: (see table below)	1) a) $\square=2$ b) i) Yes, because addition is commutative ii) Adds 5 and 6 iii) To get $\square$ on its own iv) By subtracting 9 from 11 v) $9-9=0$ vi) Yes

Jack's response	Questions
$5+6=9+\square$	i) Is Jack correct to re-write $9+\square$ as $\square+9$? Why?
$11=\square+9$	ii) How does Jack get 11?
$11-9=\square+9-9$	iii) Why does Jack subtract 9 from each side?
$2=\square+0$	iv) How does Jack get 2?
	v) And how does he get 0?
$2=\square$	vi) Does Jack get the correct answer?

Questions	Answers
2) Here is another equation: $7-3=9-\square$. Note that it has "subtract $\square$" at the end of the equation. a) Solve the equation by inspection. b) Mbali solved the equation using additive inverses. Copy her response and answer the questions: (see table below)	2) a) $\square=5$ b) i) Subtracted 3 from 7 ii) Added box iii) $-\square+\square=0$ iv) Addition is commutative. i.e. $4+\square=\square+4$ and it is easier to see what to do to each side v) Added the additive inverse of 4 to each side vi) Yes vii) No because subtraction is not commutative i.e. $9-\square\neq\square-9$

Mbali's response	Questions
$7-3=9-\square$	
$4=9-\square$	i) How does Mbali get 4?
$4+\square=9-\square+\square$	ii) What has Mbali done on each side?
$4+\square=9+0$	iii) Why is there a 0?
$\square+4=9$	iv) Why does Mbali change $\square+4$ to $4+\square$?
$\square=9-4$	v) How does Mbali get -4 on the right of the equal sign?
$\square=5$	vi) This should be the same answer you got for Q2a. Is it?
	vii) Go back to the first line of Mbali's response. Could she have changed $9-\square$ to $\square-9$? Explain.

Questions	Answers
3) Solve these equations using additive inverses. Refer to Jack and Mbali's responses if you need help. a) $5-3=\ 9-\square$ also write what you did in each step in Q3a, and why you did it. b) $7-5=1+\square$ c) $16-\square=12-4$ d) $2+\square=7-3$ also write what you did in each step in Q3d, and why you did it. e) $9+3=18-\square$ **Answers to Q3a and Q3d 's steps** a) Subtract 3 from 5 to simplify the side with no $\square$. Add box to each side to get rid of -$\square$.. Subtract 2 from each side to get $\square$ on its own d) Subtract 3 from 7 to simplify the side with no $\square$. Rewrite $2+\square$ as $\square+2$ so $\square$ is before the equal sign. Subtract 2 from each side to get $\square$ on its own.	3) **Final answers to Q3a to Q3e** a) $\square=7$ b) $\square=1$ c) $\square=8$ d) $\square=2$ e) $\square=6$ **See steps for Q3a and Q3d below the question.**

Worksheet 1.7: Numerical equations with whole numbers

This worksheet focuses on solving equations that contain a mixture of addition and subtraction. There are two or more whole numbers on each side of the equal sign with ☐ in a variety of positions. As in Worksheets 1.1 to 1.6, our aim is to get ☐ on its own.

Questions
1) Look at this equation: $7 + 8 = 20 - \square$. Note that it has ☐ at the end of the equation. a) Solve the equation by using additive inverses. b) Based on your answer to Q1a answer the following questions: i) Could we rewrite $20 - \square$ as $\square - 20$? Explain. ii) How do you get $+\square$ on the left of the equal sign?
2) Given: $10 + 13 + 4 = \square + 11$ a) Look for what is the same and different in this set of four equations: A. $10 + 13 + 4 - 4 = \square + 11$ B. $10 + 13 + 4 - 5 = \square + 11$ C. $10 + 13 + 4 - 6 = \square + 11$ D. $10 + 13 + 4 - 7 = \square + 11$ b) Do you expect the results for ☐ to be the same for each equation? Give a reason for your answer. c) Solve the equations. d) Was your prediction in Q2b correct?
3) Solve these equations using additive inverses. a) $10 - 3 = 9 - \square$ b) $11 - \square = 12 - 4$ c) $2 + \square = 5 + 3$ d) $9 - 5 = 2 + \square$ e) $12 + 3 = 18 - \square$
4) Here is a set of five statements: A. $\lozenge - 4 + 7 = \oslash - 4$ B. $\lozenge - 4 + 7 = \oslash + 7$ C. $\lozenge = \oslash + 4$ D. $\lozenge - 4 + 3 = \oslash$ E. $\lozenge - \oslash = 4 + 7$ You are now told that $\lozenge - 4 = \oslash$ Use the equation $\lozenge - 4 = \oslash$ to decide which statements (A to E) are **not** balanced. Say or show why they are not balanced and write them using a $\neq$ sign.

Worksheet 1.7: Numerical equations with whole numbers

Answers

Questions	Answers
1) Look at this equation: $7+8=20-\square$. Note that it has $\square$ at the end of the equation. a) Solve the equation by using additive inverses. b) Based on your answer to Q1a answer the following questions: i) Could we rewrite $20-\square$ as $\square-20$? Explain. ii) How do you get $+\square$ on the left of the equal sign?	1) a) $7+8=20-\square$ $15=20-\square$ $15+\square=20-\square+\square$ $\square+15-15=20-15$ $\square=5$ b) i) No. Subtraction is not commutative i.e. $20-\square\neq\square-20$ ii) add $\square$ to each side of the equation
2) Given: $10+13+4=\ \square\ +11$ a) Look for what is the same and different in this set of four equations: A. $10+13+4-4=\ \square\ +11$ B. $10+13+4-5=\ \square\ +11$ C. $10+13+4-6=\ \square\ +11$ D. $10+13+4-7=\ \square\ +11$ b) Do you expect the results for $\square$ to be the same for each equation? Give a reason for your answer. c) Solve the equations. d) Was your prediction in Q4b correct?	2) a) $10+13+4$ and $\square\ +11$ is the same in each equation. What is subtracted on the left of the equation is one more each time. So -4 in the first, -5 in the second, etc. b) No. The left side gets smaller by one each time so the value for $\square$ would also get smaller each time. c) A. $\square=12$ B. $\square=11$ C. $\square=10$ D. $\square=9$ d) Yes
3) Solve these equations using additive inverses. a) $10-3=\ 9-\square$ b) $11-\square=12-4$ c) $2+\square=5+3$ d) $9-5=2+\square$ e) $12+3=18-\square$	3) a) $\square=2$ b) $\square=3$ c) $\square=6$ d) $\square=2$ e) $\square=3$
4) Here is a set of five statements: A. $\Diamond-4+7=\oslash-4$ B. $\Diamond-4+7=\oslash+7$ C. $\Diamond=\oslash+4$ D. $\Diamond-4=\oslash-4$ E. $\Diamond-\oslash=4+7$ You are now told that $\Diamond-4=\oslash$ Use the equation $\Diamond-4=\oslash$ to decide which statements (A to E) are **not** balanced. Say or show why they are not balanced and write them using a $\neq$ sign.	4) Given $\Diamond-4=\oslash$ A On the left of the equal sign: 7 is added to $\Diamond-4$. On the right 4 is subtracted from $\oslash$. The left and the right have not been changed in the same way $\Diamond-4+7\neq\Diamond-4$ and $\oslash\neq\oslash+7$ **or** subtracting 7 from each side we get $\Diamond-4$ on the left and $\oslash-11\neq\oslash$ **or** Substitute $\oslash$ for $\Diamond-4$ on the left side of A to get $\oslash+7$ and $\oslash+7\neq\oslash-4$ <u>Conclusion</u>: the statement is not balanced: $\Diamond-4+7\neq\oslash-4$ D Left: 0 is added to $\Diamond-4$. Right: 4 is subtracted from $\oslash$, changes on each side are different **or** by adding 4 to both sides we get $\Diamond$ on the left and $\oslash$ on the right and $\Diamond\neq\oslash$ **or** substitute $\oslash$ for $\Diamond-4$ on the left to get $\oslash$ and $\oslash\neq\oslash-4$ <u>Conclusion</u>: the statement is not balanced: $\Diamond-4\neq\oslash-4$ E Simplifying the right side, we get $\Diamond-\oslash=11$ but we were given $\Diamond-4=\oslash$ which is the same as $\Diamond-\oslash=4$ and $11\neq4$ <u>Conclusion</u>: the statement is not balanced: $\Diamond-\oslash\neq4+7$

Worksheet 1.8: Numerical equations with whole numbers

This worksheet focuses on solving equations involving multiplication where there are two whole numbers on each side of the equal sign. Some answers are not whole numbers.

Questions

1) Give the multiplicative inverse of each number:
   a) 3 b) 2 c) 5 d) 7

   When we multiply *multiplicative inverses*, we get a product of 1. e.g. $4 \times \frac{1}{4} = 1$

2) Work out the value of ☐ to balance these statements:
   a) $4 \times \frac{1}{\square} = 4 \times 1$ b) $8 \times \frac{3}{\square} = 8 \times 1$ c) $5 \times \frac{6}{\square} = 5 \times 1$

3) Marti and Musa solve the equation $6 \times 2 = \square \times 4$ in different ways:

Marti uses i*nspection:*	$6 \times 2 = \square \times 4$
Marti first simplifies the equation	$12 = \square \times 4$
She then thinks: "What multiplied by 4 gives 12?"	
And she gets:	$\square = 3$

Musa uses *multiplicative inverses:*	$6 \times 2 = \square \times 4$
Musa also simplifies the equation	$12 = \square \times 4$
He then multiplies both sides by $\frac{1}{4}$ to get ☐ on its own.	$12 \times \frac{1}{4} = \square \times 4 \times \frac{1}{4}$
Musa then simplifies $12 \times \frac{1}{4}$ like this: $\frac{12}{1} \times \frac{1}{4} = \frac{12}{4} = 3$	$12 \times \frac{1}{4} = \square \times 1$
And gets:	$3 = \square$
	So $\square = 3$

Solve these equations using *multiplicative inverses* in the way Musa did. Then check your answers using *inspection*.
   a) $4 \times 6 = \square \times 3$
   b) $\square \times 5 = 10 \times 4$

4) Marti made up this question to try Musa's method. She found the answer was a fraction. Copy her response and answer the questions:

$4 \times 8 = 5 \times \square$	
$4 \times 8 = \square \times 5$	a) Marti rewrote $5 \times \square$ as $\square \times 5$. Explain why she can do this.
$4 \times 8 \times \frac{1}{5} = \square \times 5 \times \frac{1}{5}$	b) Why does Marti multiply both sides by the multiplicative inverse of 5?
$\frac{4}{1} \times \frac{8}{1} \times \frac{1}{5} = \square$	c) What is Marti doing here?
$\frac{32}{5} = \square$	d) Is Marti's answer a common fraction or an improper fraction?

5) Solve the following equations using *multiplicative inverses*:
   a) $\square \times 6 = 5 \times 12$ b) $\square \times 6 = 5 \times 5$
   c) $9 \times 2 = 3 \times \square$ d) $4 \times \square = 7 \times 5$
   e) $3 \times 5 = \square \times 2$

Worksheet 1.8: Numerical equations whole numbers

Answers

Questions	Answers
1) Give the multiplicative inverse of each number: a) 3 b) 2 c) 5 d) 7	1) a) $\frac{1}{3}$ c) $\frac{1}{5}$ b) $\frac{1}{2}$ d) $\frac{1}{7}$
2) Work out the value of ☐ to balance these statements: a) $4 \times \frac{1}{\square} = 4 \times 1$ b) $8 \times \frac{3}{\square} = 8 \times 1$ c) $5 \times \frac{6}{\square} = 5 \times 1$	2) a) $\square = 1$ b) $\square = 3$ c) $\square = 6$
3) Marti and Musa solve the equation $6 \times 2 = \square \times 4$ in different ways: Marti uses *inspection:* $6 \times 2 = \square \times 4$ Marti first simplifies the equation $12 = \square \times 4$ She then thinks: "What multiplied by 4 gives 12?" And she gets: $\square = 3$ Musa uses *multiplicative inverses:* $6 \times 2 = \square \times 4$ Musa also simplifies the equation $12 = \square \times 4$ He then multiplies both sides by $\frac{1}{4}$ to get ☐ on its own. $12 \times \frac{1}{4} = \square \times 4 \times \frac{1}{4}$ $12 \times \frac{1}{4} = \square \times 1$ Musa then simplifies $12 \times \frac{1}{4}$ like this: $\frac{12}{1} \times \frac{1}{4} = \frac{12}{4} = 3$ And gets: $3 = \square$ So, $\square = 3$ Solve these equations using *multiplicative inverses* in the way Musa did. Then check your answers using *inspection*. a) $4 \times 6 = \square \times 3$ b) $\square \times 5 = 10 \times 4$	3) a) $4 \times 6 = \square \times 3$ $24 = \square \times 3$ $24 \times \frac{1}{3} = \square \times 3 \times \frac{1}{3}$ $8 = \square$ b) $\square \times 5 = 10 \times 4$ $\square \times 5 = 40$ $\square \times 5 \times \frac{1}{5} = 40 \times \frac{1}{5}$ $\square = 8$
4) Marti made up this question to try Musa's method. She found the answer was a fraction. Copy her response and answer the questions: $4 \times 8 = 5 \times \square$ $4 \times 8 = \square \times 5$ $4 \times 8 \times \frac{1}{5} = \square \times 5 \times \frac{1}{5}$ $\frac{4}{1} \times \frac{8}{1} \times \frac{1}{5} = \square$ $\frac{32}{5} = \square$ a) Marti rewrote $5 \times \square$ as $\square \times 5$. Explain why she can do this. b) Why does Marti multiply both sides by the multiplicative inverse of 5? c) What is Marti doing here? d) Is Marti's answer a common fraction or an improper fraction?	4) a) Multiplication is commutative, i.e. $5 \times \square = \square \times 5$. b) To isolate the box. $5 \times \frac{1}{5} = 1$ c) Converting 4 and 8 to fractions d) Improper fraction
5) Solve the following equations using *multiplicative inverses*: a) $\square \times 6 = 5 \times 12$ d) $\square \times 6 = 5 \times 5$ b) $9 \times 2 = 3 \times \square$ e) $4 \times \square = 7 \times 5$ c) $3 \times 5 = \square \times 2$	5) **Answers** a) $\square = 10$ d) $\square = \frac{25}{6}$ b) $\square = 6$ e) $\square = \frac{35}{4}$ c) $\square = \frac{15}{2}$

Worksheet 1.9: Numerical equations with whole numbers

This worksheet focuses on solving equations involving division. There are two whole numbers on each side of the equal sign.

A statement that is balanced has the same result on each side of the equal sign.
So, a *balanced statement* is an equation.

Questions

1) If twelve is divided by three, we can write it as $12 \div 3$ or $\frac{12}{3}$ or $12 \times \frac{1}{3}$.
   Write these divisions in two other ways:
   a) $20 \div 5$ b) $9 \div 2$ c) $3 \div 7$ d) $4 \div 4$

2) Write these multiplications as divisions:
   a) $7 \times \frac{1}{2}$ b) $9 \times \frac{1}{4}$ c) $\frac{1}{5} \times 8$ d) $\frac{1}{6} \times 6$

3) Solve using i*nspection*. The first one has been done for you:
   a) $18 \div 3 = \square \div 6$
   $6 = \square \div 6$ Think: "What divided by 6 gives 6?"
   $\square = 36$
   b) Now try it this way: $\frac{18}{3} = \frac{\square}{6}$
   c) $\square \div 8 = 4 \div 2$ You can choose the method used in Q3a or in Q3b.

4) Give the multiplicative inverse of each number:
   a) $\frac{1}{6}$ b) $\frac{1}{2}$ c) $\frac{1}{7}$ d) $\frac{2}{3}$

When we multiply *multiplicative inverses,* we get a product of 1. e.g. $\frac{1}{4} \times 4 = 1$

5)
   a) Solve this equation using *inspection*: $16 \div 2 \times 3 = \square \div 4$
   b) Andrew solves the equation: $16 \div 2 \times 3 = \square \div 4$ using *multiplicative inverses*. Copy his response and answer the questions:

$16 \div 2 \times 3 = \square \div 4$

$16 \times \frac{1}{2} \times 3 = \square \times \frac{1}{4}$ a) What has Andrew done in this step?

$24 = \square \times \frac{1}{4}$ b) Show how Andrew got 24 on the left side.

$24 \times 4 = \square \times \frac{1}{4} \times 4$ c) Why did Andrew multiply by 4 on each side of the equal sign?

$96 = \square$

6) Solve the following equations using *multiplicative inverses* and then check using *inspection*:
   a) $\frac{\square}{6} = \frac{12}{3}$ b) $\frac{\square}{5} = \frac{12}{4}$ c) $30 \div 4 = \frac{\square}{2}$ d) $\frac{15}{3} = \square \div 5$ e) $\frac{12}{4} = \frac{\square}{3}$

7) Here is a set of three statements:
   A. $15 \div 3 \times 3 = 15 \times 1$
   B. $15 \div 3 = 5 \div 5 \times 1$
   C. $\frac{23}{3} \times 3 = 15 \times 1$

   a) Which statements are **not** balanced? Re-write them using a $\neq$ sign.
   b) Say why they are not balanced.

Worksheet 1.9: Numerical equations whole numbers

Answers

Questions	**Answers**
1) If twelve is divided by three, we can write it as $12 \div 3$ or $\frac{12}{3}$ or $12 \times \frac{1}{3}$. Write these divisions in two other ways: a) $20 \div 5$ c) $3 \div 7$ b) $9 \div 2$ d) $4 \div 4$	1) a) $\frac{20}{5}$ or $20 \times \frac{1}{5}$ c) $\frac{3}{7}$ or $3 \times \frac{1}{7}$ b) $\frac{9}{2}$ or $9 \times \frac{1}{2}$ d) $\frac{4}{4}$ or $4 \times \frac{1}{4}$
2) Write these multiplications as divisions: a) $7 \times \frac{1}{2}$ c) $\frac{1}{5} \times 8$ b) $9 \times \frac{1}{4}$ d) $\frac{1}{6} \times 6$	2) a) $\frac{7}{2}$ b) $\frac{9}{4}$ c) $\frac{8}{5}$ d) $\frac{6}{6}$
3) Solve using i*nspection*. The first one has been done for you: a) $18 \div 3 = \square \div 6$ $6 = \square \div 6$ Think: "What divided by 6 gives 6?" $\square = 36$ b) Now try it this way: $\frac{18}{3} = \frac{\square}{6}$ c) $\square \div 8 = 4 \div 2$ You can choose the method used in Q3a or in Q3b.	3) b) $\frac{18}{3} = \frac{\square}{6}$ $6 = \frac{\square}{6}$ $\square = 36$ c) $\square \div 8 = 4 \div 2$ $\square \div 8 = 2$ $\square = 16$ **or** $\frac{\square}{8} = \frac{4}{2}$ $\frac{\square}{8} = 2$ $\square = 16$
4) Give the multiplicative inverse of each number: a) $\frac{1}{6}$ b) $\frac{1}{2}$ c) $\frac{1}{5}$ d) $\frac{2}{3}$	4) a) 6 b) 2 c) 5 d) $\frac{3}{2}$
5) **Question and answers:** Andrew solves the equation: $16 \div 2 \times 3 = \square \div 4$ using *multiplicative inverses*. $16 \div 2 \times 3 = \square \div 4$ $16 \times \frac{1}{2} \times 3 = \square \times \frac{1}{4}$ $24 = \square \times \frac{1}{4}$ $24 \times 4 = \square \times \frac{1}{4} \times 4$ $96 = \square$ a) What has Andrew done in this step? b) Show how Andrew got 24 on the left side. c) Why did Andrew multiply by 4 on each side of the equal sign?	**Answers** a) Rewritten the division as a multiplication b) 8×3 or $48 \times \frac{1}{2}$ c) 4 is multiplicative inverse of $\frac{1}{4}$ and $\frac{1}{4} \times 4 = 1$ so $\square$ will be on its own.
6) Solve the following equations using *multiplicative inverses* and then check using *inspection*: a) $\frac{\square}{6} = \frac{12}{3}$ c) $30 \div 4 = \frac{\square}{2}$ e) $\frac{12}{4} = \frac{\square}{3}$ b) $\frac{\square}{5} = \frac{12}{4}$ d) $\frac{15}{3} = \square \div 5$	6) **Answers** **Multiplicative inverses hint** — **Results** a) $\times$ each side by 6 — $\square = 24$ b) $\times$ each side by 5 — $\square = 15$ c) $\frac{30}{4} \times 2 = \frac{\square}{2} \times 2$ — $\square = 15$ d) $\times$ each side by 5 — $\square = 25$ e) $\times$ each side by 3 — $\square = 9$
7) Here is a set of three statements: A. $15 \div 3 \times 3 = 15 \times 1$ B. $15 \div 3 = 5 \div 5 \times 1$ C. $\frac{23}{3} \times 3 = 15 \times 1$ a) Which statements are **not** balanced? Re-write them using a $\neq$ sign b) Say why they are not balanced.	a) B. $15 \div 3 \neq 5 \div 5 \times 1$ C. $\frac{23}{3} \times 3 \neq 15 \times 1$ b) $5 \neq 1$ $23 \neq 15$

Worksheet 1.10: Numerical equations with whole numbers

This worksheet focuses on the equal sign as a balance for numerical equations. The equations have a mixture of multiplication and division where there are two or more whole numbers on each side of the equal sign.

Questions

1) Write the question and select the correct number for $\square$.
   a) $5 \times 2 \times 3 = \square \times 6$
      A. 10 B. 30 C. 5 D. 6
   b) $6 \times \square \div 2 = 4 \times 3$
      A. 2 B. 4 C. 12 D. 24
   c) $\square \times 5 \times 5 = 5 \times 10 \times 1$
      A. 25 B. 20 C. 5 D. 2

2) Copy and solve by inspection:
   a) $\frac{\square}{2} \times 5 \times 2 = 3 \times 5$
   b) $5 \times \frac{6}{4} \times 4 = \square$

3)
   a) Use the equation $10 \times 4 = 40$, to complete the following:
      A. $10 \times 4 \times 2 = 40 \times \square$
      B. $40 \times \frac{1}{5} = 4 \times 10 \div \square$
      C. $10 \times 4 \div \square = 40 \div 5$
   b) Which answers in Q3 a are the same? Explain your answer.

4) Solve these equations by inspection. Rewrite the division as multiplication if you find it is easier.
   a) $5 \times \frac{6}{4} \times 4 = \square$
   b) $5 \times 6 \div 4 \times 4 = \square$
   c) $\frac{\square}{2} \times 4 \times 2 = 3 \times 4$
   d) $\square \div 2 \times 4 \times 2 = 3 \times 4$

5) Solve these equations using inverses.
   a) $7 \times 3 \times 2 = \square \times 6$
   b) $\square \div 2 \times 5 = 1 \times 10 \div 2$
   c) $\square \times 6 \div 4 = 10 \times 3 \div 2$
   d) $10 \div 5 \times 2 = 16 \times \square \div 2$
   e) $9 \times \square \times 2 = 3 \times 5 \times 2$

6) Copy and give the value for $\square$ in each equation.
   A. $5 \times 4 \times 3 = \square \times 4 \times 3$
   B. $5 \times 4 \times 3 = \square \times 2 \times 3$
   C. $5 \times 4 \times 3 = \square \times 1 \times 6$
   D. $5 \times 4 \times 3 = 5 \times 2 \times \square$
   E. $5 \times 4 \times 3 = 5 \times 1 \times \square$

   F. $5 \times \square \times 3 = 3 \times 4 \times 5$
   G. $10 \times \square \times 3 = 5 \times 4 \times 3$
   H. $5 \times \square \times 6 = 4 \times 5 \times 3$

7) Look at the set of eight equations in Q6 and answer the questions:
   a) Note that $\square$ is sometimes on the right as in A to E. What is the product of the numbers on the left in A to E?
   b) Sometimes $\square$ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H?
   c) So, what should the product be on the sides with a $\square$ in A to E? And in F to H?

8) Use the numbers 4, 5 and 6 to make multiplication equations:
   a) two equations with $\square$ on the right
      e.g. $4 \times 5 \times 6 = \square \times 6 \times 20$
   b) two equations with $\square$ on the left
      e.g. $\square \times 6 \times 20 = 6 \times 5 \times 4$

Worksheet 1.10: Numerical equations with whole numbers

Answers

Questions and answers

1) Write the question and select the correct number for ☐.

a) $5 \times 2 \times 3 = \square \times 6$
   A. 10 B. 30 C. 5 D. 6

b) $6 \times \square \div 2 = 4 \times 3$
   A. 2 B. 4 C. 12 D. 24

c) $\square \times 5 \times 5 = 5 \times 10 \times 1$
   A. 25 B. 20 C. 5 D. 2

Answers

a) C b) B c) D

2) Copy and solve by inspection:

a) $\frac{\square}{2} \times 5 \times 2 = 3 \times 5$

b) $5 \times \frac{6}{4} \times 4 = \square$

Answers

a) $\square = 3$

b) $\square = 30$

3)

a) Use the equation $10 \times 4 = 40$, to complete the following:
   A. $10 \times 4 \times 2 = 40 \times \square$
   B. $40 \times \frac{1}{5} = 4 \times 10 \div \square$
   C. $10 \times 4 \div \square = 40 \div 5$

b) Which answers In Q3a are the same? Explain your answer.

Answers

a)
   A. $\square = 2$
   B. $\square = 5$
   C. $\square = 5$

b) B and C: Multiplying by $\frac{1}{5}$ is the same as dividing by 5

4) Solve these equations by inspection. Rewrite the division as multiplication if you find it is easier.

a) $5 \times \frac{6}{4} \times 4 = \square$

b) $5 \times 6 \div 4 \times 4 = \square$

c) $\frac{\square}{2} \times 4 \times 2 = 3 \times 4$

d) $\square \div 2 \times 4 \times 2 = 3 \times 4$

Answers for ☐

a) 30 b) 30 c) 3 d) 3

5) Solve these equations using inverses.

a) $7 \times 3 \times 2 = \square \times 6$

b) $\square \div 2 \times 5 = 1 \times 10 \div 2$

c) $\square \times 6 \div 4 = 10 \times 3 \div 2$

d) $10 \div 5 \times 2 = 16 \times \square \div 2$

e) $9 \times \square \times 2 = 3 \times 5 \times 2$

Answers

a) $\square = 7$ multiply each side by $\frac{1}{6}$

b) $\square = 2$ multiply each side by $\frac{2}{1}$ and $\frac{1}{5}$ or by $\frac{2}{5}$

c) $\square = 10$ multiply each side by $\frac{4}{6}$

d) $\square = \frac{1}{2}$ multiply each side by $\frac{2}{16}$

e) $\square = \frac{5}{3}$ multiply each side by $\frac{1}{18}$

6) Copy and give the value for ☐ in each equation.

A. $5 \times 4 \times 3 = \square \times 4 \times 3$
B. $5 \times 4 \times 3 = \square \times 2 \times 3$
C. $5 \times 4 \times 3 = \square \times 1 \times 6$
D. $5 \times 4 \times 3 = 5 \times 2 \times \square$
E. $5 \times 4 \times 3 = 5 \times 1 \times \square$

F. $5 \times \square \times 3 = 3 \times 4 \times 5$
G. $10 \times \square \times 3 = 5 \times 4 \times 3$
H. $5 \times \square \times 6 = 4 \times 5 \times 3$

Answers for ☐

A. 5 C. 10 E. 12 G. 2
B. 10 D. 6 F. 4 H. 2

7) Look at the set of eight equations in Q6 and answer the questions:

a) Note that ☐ is sometimes on the right as in A to E. What is the product of the numbers on the left in A to E?

b) Sometimes ☐ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H?

c) So, what should the product be on the sides with a ☐ in A to E? And in F to H?

Answers

a) 60
b) 60
c) 60

8) Use the numbers 4, 5, and 6 to make multiplication equations:

a) two equations with ☐ on the right
   e.g. $4 \times 5 \times 6 = \square \times 6 \times 20$

b) two equations with ☐ on the left
   e.g. $\square \times 6 \times 20 = 6 \times 5 \times 4$

Answers will differ

a) $4 \times 5 \times 6 = \square \times 2 \times 10$ and
   $2 \times 10 \times \square = 6 \times 5 \times 4$

b) $\square \times 3 \times 10 = 6 \times 5 \times 4$ and
   $6 \times 4 \times 5 = 6 \times 20 \times \square$

Worksheet 1.11: Numerical equations

This worksheet focuses on the equal sign as a balance for numerical equations. The questions involve a mixture of multiplication and division where there are two or more whole numbers on each side of the equal sign.

Questions

1) Write the question and select the correct number for $\square$.
   a) $9 \times 3 \times 2 = \square \times 6$
      A. 6 B. 9 C. 27 D. 54
   b) $8 \times \square \div 2 = 4 \times 3$
      A. 12 B. 4 C. 3 D. 2
   c) $\square \times 6 \times 6 = 6 \times 12 \times 1$
      A. 6 B. 2 C. 36 D. 12

2) Copy and solve by inspection:
   a) $\frac{\square}{3} \times 5 \times 3 = 3 \times 5$
   b) $9 \times \frac{6}{7} \times 7 = \square$

3) If $12 \times 4 = 48$ then use it to complete the following:
   a) $12 \times 4 \times 2 = 48 \times \square$
   b) $48 \times \frac{1}{12} = 4 \times 12 \div \square$
   c) $12 \times 4 \div \square = 48 \div 12$
   d) Which answers are the same? Why?

4) Solve these equations by inspection. Rewrite the division as multiplication if you find it is easier.
   a) $9 \times \frac{3}{2} \times 2 = \square$
   b) $11 \times 6 \div 3 \times 3 = \square$
   c) $\frac{\square}{3} \times 4 \times 3 = 9 \times 4$
   d) $\square \div 7 \times 4 \times 7 = 6 \times 2$

5) Solve these equations using inverses.
   a) $\square \div 2 \times 7 = 1 \times 14 \div 2$
   b) $8 \times 3 \times 2 = \square \times 6$
   c) $\square \times 8 \div 4 = 10 \times 4 \div 2$
   d) $12 \div 6 \times 3 = 24 \times \square \div 2$
   e) $10 \times \square \times 2 = 3 \times 5 \times 2$

6) Copy and write down the value for $\square$ in each equation.
   A. $7 \times 2 \times 3 = \square \times 2 \times 3$
   B. $7 \times 2 \times 3 = \square \times 1 \times 3$
   C. $7 \times 2 \times 3 = \square \times 1 \times 6$
   D. $7 \times 2 \times 3 = 7 \times 2 \times \square$
   E. $7 \times 2 \times 3 = 7 \times 1 \times \square$

   F. $7 \times \square \times 3 = 3 \times 2 \times 7$
   G. $14 \times \square \times 3 = 7 \times 2 \times 3$
   H. $7 \times \square \times 6 = 2 \times 7 \times 3$

7) Look at the set of eight equations in Q6 and answer these questions:
   a) Note that $\square$ is sometimes on the right of the equal sign as in A to E. What is the product of the numbers on the left in A to E?
   b) Sometimes $\square$ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H?
   c) So, what should the product be on the sides with $\square$ in A to E? And in F to H?

8) Use the numbers 8, 9 and 10 to make multiplication equations with:
   a) two equations with $\square$ on the right
      e.g. $8 \times 9 \times 10 = 8 \times \square \times 1$
   b) two equations with $\square$ on the left
      e.g. $8 \times \square \times 2 = 10 \times 8 \times 9$

Worksheet 1.11: Numerical equations

Answers

Questions and answers

1) Write the question and select the correct number for ☐.

a) $9\times 3\times 2 = \square\times 6$
A. 6 B. 9 C. 27 D. 54

b) $8\times\square\div 2 = 4\times 3$
A. 12 B. 4 C. 3 D. 2

c) $\square\times 6\times 6 = 6\times 12\times 1$
A. 6 B. 2 C. 36 D. 12

Answers
a) B b) C c) B

6) Copy and write down the value for ☐ in each equation.

A. $7\times 2\times 3 = \square\times 2\times 3$
B. $7\times 2\times 3 = \square\times 1\times 3$
C. $7\times 2\times 3 = \square\times 1\times 6$
D. $7\times 2\times 3 = 7\times 2\times\square$
E. $7\times 2\times 3 = 7\times 1\times\square$

F. $7\times\square\times 3 = 3\times 2\times 7$
G. $14\times\square\times 3 = 7\times 2\times 3$
H. $7\times\square\times 6 = 2\times 7\times 3$

Answers for ☐
A. 7 C. 7 E. 6 G. 1
B. 14 D. 3 F. 2 H. 1

2) Copy and solve by inspection:

a) $\frac{\square}{3}\times 5\times 3 = 3\times 5$
b) $9\times\frac{6}{7}\times 7 = \square$

Answers
a) $\square = 3$ b) $\square = 54$

3) If $12\times 4 = 48$ then use it to complete the following:

a) $12\times 4\times 2 = 48\times\square$
b) $48\times\frac{1}{12} = 4\times 12\div\square$
c) $12\times 4\div\square = 48\div 12$
d) Which answers are the same? Why?

Answers
a) $\square = 2$
b) $\square = 12$
c) $\square = 12$
d) Q3b and Q3c. Multiplying by $\frac{1}{12}$ is the same as dividing by 12.

7) Look at the set of eight equations in Q6 and answer these questions:

a) Note that ☐ is sometimes on the right of the equal sign as in A to E. What is the product of the numbers on the left in A to E?
b) Sometimes ☐ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H?
c) So, what should the product be on the sides with ☐ in A to E? And in F to H?

Answers
a) 42
b) 42
c) 42 and 42

4) Solve these equations by inspection. Rewrite the division as multiplication if you find it is easier.

a) $9\times\frac{3}{2}\times 2 = \square$
b) $11\times 6\div 3\times 3 = \square$
c) $\square\div 7\times 4\times 7 = 6\times 2$
d) $\frac{\square}{3}\times 4\times 3 = 9\times 4$

Answers for ☐
a) 27 b) 66 c) 3 d) 9

8) Use the numbers 8, 9 and 10 to make multiplication equations:

a) two equations with ☐ on the right
e.g. $8\times 9\times 10 = 8\times\square\times 1$
b) two equations with ☐ on the left
e.g. $8\times\square\times 2 = 10\times 8\times 9$

Answers will differ, here are two of each:
a) $8\times 9\times 10 = 4\times\square\times 2$ and
$3\times\square\times 3 = 8\times 9\times 10$
b) $4\times\square\times 2 = 10\times 8\times 9$ and
$10\times 8\times 9 = 5\times 2\times\square$

5) Solve these equations using inverses.

a) $8\times 3\times 2 = \square\times 6$
b) $\square\div 2\times 7 = 1\times 14\div 2$
c) $\square\times 8\div 4 = 10\times 4\div 2$
d) $12\div 6\times 3 = 24\times\square\div 2$
e) $10\times\square\times 2 = 3\times 5\times 2$

Answers for ☐
a) 8 c) 10 e) $\frac{3}{2}$
b) 2 d) $\frac{1}{2}$

Note: $\times\frac{2}{1}$ then $\times\frac{1}{7}$ is the same as $\times\frac{2}{7}$

Worksheet 1.12: Numerical equations with whole numbers

This worksheet focuses on solving numerical equations involving addition, subtraction, multiplication, and division of large numbers. There are two or more whole numbers on each side of the equal sign.

Questions	
1) The equation $4630 + 490 = \square + 450$ has two whole numbers on each side. Without doing any calculations, use this information to balance each of the following statements: a) $4630 + 490 - 450 =$ b) $\square + 450 - 490 =$ c) $4630 =$ d) $\square =$	4) Given that $362 + 29 = \square + 31$, which of the following statements are true? a) Because 31 is 2 more than 29, ☐ is 2 more than 362 b) Because 31 is 2 more than 29, ☐ is 2 less than 362 c) $\square = 362$
2) Below is a set of four equations. A. $4863 + 2121 = \square + 2120$ B. $4863 + 2121 + 70 = \square + 2120 + 70$ C. $40 + 4863 + 2121 = \square + 2120 + 40$ D. $2121 + 4863 + 64 = 64 + \square + 2120$ a) Highlight the parts which are the same for each equation. b) Will the value of the box be the same for all the equations? Give reasons for your response. c) Solve all the equations. d) Was your answer in Q4b correct? Why/why not?	5) Solve these equations using inverses: a) $74 \times 30 \times 20 = \square \times 600$ b) $\frac{\square}{40} \times 4 \times 20 = 3 \times 4$ c) $\square \times 60 \div 4 = 10 \times 150 \div 10$ d) $200 \div 5 \times 2 = 400 \times \square \div 5$
3) Solve these equations using inverses. a) $512 + 347 = 510 + \square$ b) $794 - 56 = \square - 52$ c) $1600 - \square = 1200 - 300$ d) $1200 - \square + 50 = 1000 - 300 + 50$	6) Give a value for ☐ to balance these statements: a) $1564 \times \frac{\square}{100} = 1564 \times 1$ b) $\frac{\square}{215} \times 215 = 307 \times 3$ c) $2222 \times \square = 1111$ d) $5000 \times \frac{\square}{1\,428} = 2500 \times 2$

Worksheet 1.12: Numerical equations with whole numbers

Answers

Questions	Answers
1) The equation $4630 + 490 = \square + 450$ has two whole numbers on each side. Without doing any calculations, use this information to balance each of the following statements: a) $4630 + 490 - 450 =$ b) $\square + 450 - 490 =$ c) $4630 =$ d) $\square =$	1) a) $\square$ b) 4630 c) $\square + 450 - 490$ d) $4630 + 490 - 450$
2) Below is a set of four equations. A. $4863 + 2121 = \square + 2120$ B. $4863 + 2121 + 70 = \square + 2120 + 70$ C. $40 + 4863 + 2121 = \square + 2120 + 40$ D. $2121 + 4863 + 64 = 64 + \square + 2120$ a) Highlight the parts which are the same for each equation. b) Will the value of the box be the same for all the equations? Give reasons for your response. c) Solve all the equations. d) Was your answer in Q4b correct? Why/why not?	2) a) $4863 + 2121 = \square + 2120$ b) Yes, because we are adding the same value to each side c) $\square = 4864$ for all equations d) Yes. The answer stayed the same
3) Solve these equations using inverses. a) $512 + 347 = 510 + \square$ b) $794 - 56 = \square - 52$ c) $1600 - \square = 1200 - 300$ d) $1200 - \square + 50 = 1000 - 300 + 50$	3) a) $\square = 349$, subtract 510 from each side b) $\square = 798$, add 52 to each side c) $\square = 700$, add $\square$ on each side, subtract 900 on each side d) $\square = 500$, add $\square$ on each side, subtract 750 on each side
4) Given that $362 + 29 = \square + 31$, which of the following statements are true? a) Because 31 is 2 more than 29, $\square$ is 2 more than 362 b) Because 31 is 2 more than 29, $\square$ is 2 less than 362 c) $\square = 362$	4) b) is true
5) Solve these equations using inverses: a) $74 \times 30 \times 20 = \square \times 600$ b) $\frac{\square}{40} \times 4 \times 20 = 3 \times 4$ c) $\square \times 60 \div 4 = 10 \times 150 \div 10$ d) $200 \div 5 \times 2 = 400 \times \square \div 5$	5) a) $\square = 74$ b) $\square = 6$ c) $\square = 10$ d) $\square = 1$
6) Give a value for $\square$ to balance these statements: a) $1564 \times \frac{\square}{100} = 1564 \times 1$ b) $\frac{\square}{215} \times 215 = 307 \times 3$ c) $2222 \times \square = 1111$ d) $5000 \times \frac{\square}{1\,428} = 2500 \times 2$	6) a) $\square = 100$ b) $\square = 307 \times 3$ c) $\square = \frac{1}{2}$ d) $\square = 1428$

Worksheet 1.13: Numerical equations with whole numbers

This worksheet focuses on solving numerical equations involving addition, subtraction, multiplication, and division of large numbers. There are two or more whole numbers on each side of the equal sign.

Questions	
1) The equation $8\,570 - 4\,123 = \square - 4\,124$ has two whole numbers on each side. Without doing any calculations, use this information to balance each of the following statements: a) $8\,570 - 4\,123 + 4\,124 =$ b) $\square - 4\,124 + 4\,123 =$ c) $8570 =$ d) $\square =$	4) Given that $953 + 24 = \square + 30$, which of the following statements are true? a) Because 30 is 6 more than 24, ☐ is 6 more than 953. b) Because 30 is 6 more than 24, ☐ is 6 less than 953. c) $\square = 977$
2) Below is a set of four equations. A. $1\,234 - 56 - 78 - 0 = \square - 0$ B. $1\,234 - 56 - 78 - 10 = \square - 10$ C. $1\,234 - 56 - 78 - 20 = \square - 20$ D. $1\,234 - 56 - 78 - 30 = \square - 30$ a) Highlight the parts which are the same for each equation. b) Will the value of the box be the same for all the equations? Why or why not? c) Knowing that $1\,234 - 56 - 78 = 1\,100$, without doing a calculation, what is the value of the box in each equation? d) Was your answer in Q2b correct?	5) Solve these equations using inverses: a) $99 \times \square \times 2 = 33 \times 6 \times 3$ b) $\square \div 18 \times 22 = 11 \div 18$ c) $50 \times \frac{\square}{20} = 1 \times 1\,000 \div 20$ d) $400 \div 2 \times 5 = 400 \times \square \div 2$
3) Solve these equations using inverses. a) $2\,130 + \square = 7\,130 + 3\,130$ b) $\square + 915 = 472 + 910$ c) $432 + 227 + 100 = 510 + \square + 100$ d) $824 + 53 - 200 = \square + 52 - 200$	6) Give a value for ☐ to balance these statements: a) $\frac{832}{15} \times \square = 832 \times 1$ b) $873 \times \frac{201}{\square} = 67 \times 3$ c) $6\,666 \times \square = 2\,222$ d) $\frac{231}{51} \times \square = 20 \div 20$

Worksheet 1.13: Numerical equations with whole numbers

Answers

Questions	Answers
1) The equation $8\,570 - 4\,123 = \square - 4\,124$ has two whole numbers on each side. Without doing any calculations, use this information to balance each of the following statements: a) $8\,570 - 4\,123 + 4\,124 =$ b) $\square - 4\,124 + 4\,123 =$ c) $8\,570 =$ d) $\square =$	1) a) $\square$ b) $8\,570$ c) $\square - 4\,124 + 4\,123$ d) $8\,570 - 4\,123 + 4\,124$
2) Below is a set of four equations. A. $1\,234 - 56 - 78 - 0 = \square - 0$ B. $1\,234 - 56 - 78 - 10 = \square - 10$ C. $1\,234 - 56 - 78 - 20 = \square - 20$ D. $1\,234 - 56 - 78 - 30 = \square - 30$ a) Highlight the parts which are the same for each equation. b) Will the value of the box be the same for all the equations? Why or why not? c) Knowing that $1\,234 - 56 - 78 = 1\,100$, without doing a calculation, what is the value of the box in each equation? d) Was your answer in Q2b correct?	2) a) $1\,234 - 56 - 78$ and $\square$ b) Yes, because we are subtracting the same value from each side c) $\square = 1\,100$ d) Yes
3) Solve these equations using inverses. a) $2\,130 + \square = 7\,130 + 3\,130$ b) $\square + 915 = 472 + 910$ c) $432 + 227 + 100 = 510 + \square + 100$ d) $824 + 53 - 200 = \square + 52 - 200$	3) a) $\square = 8\,130$ subtract 2 130 from each side b) $\square = 467$ subtract 915 from each side c) $\square = 149$ subtract 100 and 510 from each side d) $\square = 825$ add 200 and subtract 52 from each side
4) Given that $953 + 24 = \square + 30$, which of the following statements are true? a) Because 30 is 6 more than 24, $\square$ is 6 more than 953. b) Because 30 is 6 more than 24, $\square$ is 6 less than 953. c) $\square = 977$	4) b) is true
5) Solve these equations using inverses: a) $99 \times \square \times 2 = 33 \times 6 \times 3$ b) $\square \div 18 \times 22 = 11 \div 18$ c) $50 \times \frac{\square}{20} = 1 \times 1\,000 \div 20$ d) $400 \div 2 \times 5 = 400 \times \square \div 2$	5) a) $\square = 3$ multiply by $\frac{1}{99\times2}$ on each side b) $\square = \frac{1}{2}$ multiply by $\frac{18}{2}$ on each side c) $\square = 20$ multiply by $\frac{20}{50}$ on each side d) $\square = 5$ multiply by $\frac{2}{400}$ on each side
6) Give a value for $\square$ to balance these statements: a) $\frac{832}{15} \times \square = 832 \times 1$ b) $873 \times \frac{201}{\square} = 67 \times 3$ c) $6\,666 \times \square = 2\,222$ d) $\frac{231}{51} \times \square = 20 \div 20$	6) a) $\square = 15$ b) $\square = 873$ c) $\square = \frac{1}{3}$ d) $\square = \frac{51}{231}$

Worksheet 2.1: Numerical equations with integers

This worksheet focuses on solving numerical equations involving addition. There are two integers on each side of the equal sign.

When solving a linear equation, we find a value to balance the result on each side of the equal sign.

Questions

1) Thabo solved these equations:

A. $5 + 3 = 3 + \square$ Answer: $\square = -5$

B. $(-5) + 3 = \square + 3$ Answer: $\square = -8$

C. $(-3) + \square = 5 + (-3)$ Answer: $\square = 5$

D. $(-5) + \square = (-3) + (-5)$ Answer: $\square = -3$

Some of his answers are wrong.

a) Correct any mistakes Thabo has made.
b) Why do you think he made the mistakes?

2)
a) What is the additive inverse of -7? How do you know?
b) What is the additive inverse of 3? How do you know?

3) Look at the equation $5 + (-8) = \square\ + 4$. Shahid and Mona solve the equation in different ways. Both learners find that the solution is -7.
Read their methods carefully. Make sure you can link the words and the statements.

<u>Shahid uses *solving by inspection*.</u>	
Shahid first works out the value on the side without the $\square$:	
He adds 5 and -8 to get -3.	$5 + (-8) = \square\ + 4$
He now knows $\square + 4$ must equal -3.	$5 - 8 = \square + 4$
Shahid thinks 'What must I add to 4 to get -3?' This is the value of $\square$.	$-3 = \square\ + 4$
Shahid gets:	$\square = -7$

<u>Mona uses *solving using additive inverses.*</u>	$5 + (-8) = \square + 4$
Mona's first step is to simplify $5 + (-8)$ just like Shahid did:	$5 - 8 = \square\ + 4$
	$-3 = \square\ + 4$
Mona then subtracts 4 from each side of the equation to get $\square$ on its own.	$-3 - 4 = \square\ + 4 - 4$
(She uses the *additive inverse* of 4)	$-7 = \square + 0$
Mona then simplifies each side of the equation to get a value for $\square$.	
She gets:	$-7 = \square$
Which is the same as:	$\square = -7$

Look at the equations below:

A. $6 + (-3) = \square + 5$ B. $(-3) + (-3) = \square + 1$ C. $\square + (-7) = (-9) + 2$

a) Quickly work out the answers using Shahid's method. Write down only the answers.
b) Now try to solve the equations using Mona's method which uses *additive inverses.*
c) Look at your response to equation B. Write what you did in each step and say why you did it.

4) If we know that $(-5) + \Delta = 0$, then which of the following statements are TRUE? Give reasons.

a) The value of Δ is -5.
b) Δ can be any whole number.
c) The value of Δ is 0.
d) The value of Δ is 5.
e) Δ and -5 are additive inverses of each other.
f) Δ is the additive identity for addition.

Worksheet 2.1: Numerical equations with integers

Answers

Questions	**Answers**
1) Thabo solved these equations: A. $5 + 3 = 3 + \square$ Answer: $\square = -5$ B. $(-5) + 3 = \square + 3$ Answer: $\square = -8$ C. $(-3) + \square = 5 + (-3)$ Answer: $\square = 5$ D. $(-5) + \square = (-3) + (-5)$ Answer: $\square = -3$ Some of his answers are wrong. a) Correct any mistakes Thabo has made. b) Why do you think he made the mistakes?	1) a) b) A. $\square = 5$ He wrote the additive inverse of 5. B. $\square = -5$ He added 5 and 3 and then attached the negative or he thought the result on the right must give -5.
2) a) What is the additive inverse of -7? How do you know? b) What is the additive inverse of 3? How do you know?	2) a) $+7$ because $-7 + 7 = 0$ b) -3 because $-3 + 3 = 0$
3) Look at the equation $5 + (-8) = \square + 4$. Shahid and Mona solve the equation in different ways. Both learners find that the solution is -7. Read their methods carefully. Make sure you can link the words and the statements. Look at the equations below: A. $6 + (-3) = \square + 5$ B. $(-3) + (-3) = \square + 1$ C. $\square + (-7) = (-9) + 2$ a) Quickly work out the answers using Shahid's method. Write down only the answers. b) Now try to solve the equations using Mona's method which uses *additive inverses.* c) Look at your response to equation B. Write what you did in each step and say why you did it.	3) a) A. $\square = -2$ B. $\square = -7$ C. $\square = 0$ b) A. $6 + (-3) = \square + 5$ $3 - 5 = \square + 5 - 5$ $-2 = \square$ B. $(-3) + (-3) = \square + 1$ $-6 = \square + 1$ $-6 - 1 = \square + 1 - 1$ $-7 = \square$ C. $\square + (-7) = (-9) + 2$ $\square + (-7) = -7$ $\square = (-7) + 7$ $\square = 0$ c) Add -3 and -3 to see what $\square + 1$ must equal. Subtract 1 from each side to get $\square$ on its own. Simplify each side to get a result for $\square$.
4) If we know that $(-5) + \Delta = 0$, then which of the following statements are TRUE? Give reasons. A. The value of Δ is -5. B. Δ can be any whole number. C. Δ is the additive identity for addition. D. The value of Δ is 0. E. The value of Δ is 5. F. Δ and -5 are additive inverses of each other.	4) E: The value of Δ is 5 because $-5 + 5 = 0$ F: Δ and -5 are additive inverses of each other. Additive inverses add up to 0.

Worksheet 2.2: Numerical equations with integers

This worksheet focuses on solving numerical equations. Most equations involve subtraction and there are integers on each side of the equal sign.

A statement that is balanced has the same result on both sides of the equal sign. So, a *balanced statement* is an equation.

Questions

1) Solve the equations by inspection.
Remember: Subtracting a negative number is the same as adding a positive number, e.g. $2-(-3)=2+3$
   a) $7-0=8-\square$
   b) $\square-(-2)=9-6$
   c) $(-7)-5=\square+(-4)$
   d) $9+(-3)=\square-(-5)$

2) Lina and Nisha solve the equation $6-(-3)=\square-4$ in different ways.
Read them carefully and make sure you can see the links between the words and the statements.

<u>Lina uses *inspection*</u>, as you did in Q1.

	$6-(-3)=\square-4$
Lina adds 3 and 6 to get:	$9=\square-4$
Lina then thinks, 'What subtract 4 gives 9?' She gets:	$\square=13$

<u>Nisha uses *additive inverses.*</u>

Nisha subtracts -3 from 6 to get:	$6-(-3)=\square-4$ $9=\square-4$
Nisha adds 4 to on each to get the $\square$ on its own.	$9+4=\square-4+4$
Nisha then simplifies further to get:	$9+4=\square-0$ $13=\square$
which is the same as:	$\square=13$

Look at the equations below:
   A. $-6-3=\square-2$
   B. $\square-4=10-(-3)$
   C. $\square-(-7)=13-11$

   a) Solve them using Lina's method. Just write the answers.
   b) Now try to solve the equations using Nisha's method which uses *additive inverses*.
   c) Write down what you did in each step of equation A in Q2b and why you did it.

3) If you know that $-8-(-4)=-7+3$, use it to solve these equations:
   a) $-8-(-4)+7=\square$
   b) $-8-(-4)-3=\square$
   c) $7-3+4=\square$
   d) $\square=-7+3+8-(-4)$

4) If $-\square-\triangle=-7-5$,
give *four* sets of values for $\square$ and $\triangle$ that will balance the statement.

5) Look at these four statements and answer the questions that follow:
   A. $2+(-3)=(-4)+3$
   B. $2+(-3)+5=(-4)+3$
   C. $2+(-3)-1+1=(-4)+3$
   D. $2+(-3)-2=(-4)+3$

Highlight $2+(-3)$ and $(-4)+3$ in each equation. This will help you see the structure of the statements.
   a) Which statements are equations?
   b) Rewrite the statements that are not equations with the $\neq$ sign.

6) Here is a list of six statements:
   A. $\Diamond+2=\oslash$
   B. $\oslash+2=\Diamond$
   C. $\Diamond+6=\oslash-2+6$
   D. $\Diamond+5=\oslash+3$
   E. $\Diamond+5=\oslash+7$
   F. $\Diamond+2-3=\oslash+3$

You are now told that $\Diamond=\oslash+(-2)$.
Use this information to decide which statements are balanced.
Write 'EQUATION' for those statements that are balanced.

Worksheet 2.2: Numerical equations with integers

Answers

Questions	Answers
1) Solve the equations by inspection. Remember: Subtracting a negative number is the same as adding a positive number, e.g. $2-(-3)=2+3$ a) $7-0=8-\square$ b) $\square-(-2)=9-6$ c) $(-7)-5=\square+(-4)$ d) $9+(-3)=\square-(-5)$	1) Results a) 1 b) 1 c) -8 d) 1
2) Lina and Nisha solve the equation $6-(-3)=\ \square\ -4$ in different ways: Read them carefully and make sure you can see the links between the words and the statements. Look at the equations below: A. $-6-3=\square-2$ B. $\square-4=10-(-3)$ C. $\square-(-7)=13-11$ a) Solve them using Lina's method. Just write the answers. b) Now try to solve the equations using Nisha's method which uses *additive inverses*. c) Write down what you did in each step of equation A in Q2b and why you did it.	2) a) A. $\square=-7$ B. $\square=17$ C. $\square=-5$ b) A. $6-3=\square-2$ $3=\square-2$ $3+2=\square+2-2$ $5=\square+0$ $5=\square$ B. $\square-4=10-(-3)$ $\square-4=13$ $\square-4+4=13+4$ $\square-0\ =17$ $\square=17$ C. $\square-(-7)=13-11$ $\square+7=2$ $\square+7-7=2-7$ $\square+0=-5$ $\square=-5$ c) Simplify $10-(-3)=13$ to see what the left side must equal; add 4 on each side to get $\square$ on own.
3) If you know that $-8-(-4)=-7+3$, use it to solve these equations: a) $-8-(-4)+7=\square$ b) $-8-(-4)-3=\square$ c) $7-3+4=\square$ d) $\square=-7+3+8+(-4)$	3) Use $-8-(-4)=-7+3$ i.e. $-8+4=-7+3$ a) $\square=3$ Add 7 on each side b) $\square=-7$ Subtract 3 from each side c) $\square=8$ Add 7 and subtract 3 on each side d) $\square=0$ Add 8 and add (-4) to each side
4) If $-\square-\triangle=-7-5$, give *four* sets of values for $\square$ and $\triangle$ that will balance the statement.	4) Many possible answers will result in a difference of -12 e.g. 7 and 5; 8 and 4; -10 and 2; 0 and 12
5) Look at these four statements and answer the questions that follow: A. $2+(-3)=(-4)+3$ B. $2+(-3)+5=(-4)+3$ C. $2+(-3)-1+1=(-4)+3$ D. $2+(-3)-2=(-4)+3$ Highlight $2+(-3)$ and $(-4)+3$ in each equation. This will help you see the structure of the statements. a) Which statements are equations? b) Rewrite the statements that are not equations with the $\neq$ sign.	5) a) A. $2+(-3)=(-4)+3$ C. $2+(-3)-1+1=(-4)+3$ D. $2+(-3)-2=(-4)+1$ b) B. $2+(-3)+5\neq(-4)+3$
6) Here is a list of six statements: A. $\lozenge+2=\oslash$ D. $\lozenge+5=\oslash+3$ B. $\oslash+2=\lozenge$ E. $\lozenge+5=\oslash+7$ C. $\lozenge+6=\oslash-2+6$ F. $\lozenge+2-3=\oslash+3$ You are now told that $\lozenge=\oslash+(-2)$. Use this information to decide which statements are balanced. Write 'EQUATION' for those statements that are balanced.	6) Given: $\lozenge=\oslash+(-2)$ A. EQUATION B. Not an equation C. EQUATION D. EQUATION E. Not an equation F. Not an equation

Worksheet 2.3: Numerical equations with integers

This worksheet focuses on solving numerical equations that have addition on each side, or subtraction on each side of the equal sign. The equations involve integers.

Questions

1) Solve these equations by inspection:

a) $7 + 3 = \square + 6$
b) $7 - 3 = 6 - \square$
c) $7 - 3 = \square - 6$
d) $7 - 3 = 7 + \square$
e) $\square - 2 = 5 - 8$
f) $10 - \square = 12 - (-2)$
g) $-4 + \square = 5 + (-7)$

2) Select the correct option for $\square$:

a) $-10 - 2 = \square - 4$
i) 8 ii) -12 iii) -8 iv) -16

b) $7 + \square = -9 + 2$
i) 0 ii) -7 iii) 7 iv) -14

3) Solve these four equations using *inspection* and then using *additive inverses*.

a) $-10 - (-3) = \square - 1$
b) $\square + 3 = (-8) + 9$
c) $\square + (-6) = (-7) + (-3)$
d) $12 - (-4) = \square\ - 6$

4) *Remember*: A balanced statement has the same result on each side of the equal sign.
e.g. $-9 + 3 = -10 + 4$.
The result on each side of the equal sign is -6.

If the result on each side of the equal sign is not the same, the statement is not balanced.
e.g. $-9 + 3 = -10 + 4 - 3$.
The result on the left of the equal sign is -6 but the result on the right is -9.
Therefore, the statement is not balanced, and we write $-9 + 3 \neq -10 + 4 - 3$. We say the "the left side *is not equal to* the right side".

Look at these three statements:
A. $11 + (-2) = 4 - (-5)$
B. $11 + (-2) - 5 = 4 - (-5)$
C. $11 + (-2) + (-3) = 4 - (-5) - 3$

a) Decide which statements are not balanced. Re-write them using the $\neq$ sign.
b) Show how you can balance the statement/s you wrote in Q4a by changing the right side.

5) The equation $-8 - (-4) = \square - 3$ has two integers on each side.
Use this information to decide whether the following equations are balanced or NOT balanced.
If they are not balanced, re-write them with the $\neq$ sign.

a) $-8 - (-4) + 3 = \square + 3$
b) $\square - 3 - 4 = -8$
c) $-8 = \square - 3 + (-4)$
d) $\square = -8 + 4 + 3$

Worksheet 2.3: Numerical equations with integers

Answers

Questions	Answers
1) Solve these equations by inspection: a) $7+3=\square+6$ b) $7-3=6-\square$ c) $7-3=\square-6$ d) $7-3=7+\square$ e) $\square-2=5-8$ f) $10-\square=12-(-2)$ g) $-4+\square=5+(-7)$	1) a) $\square=4$ b) $\square=2$ c) $\square=10$ d) $\square=-3$ e) $\square=-1$ f) $\square=-4$ g) $\square=2$
2) Select the correct option for $\square$: a) $-10-2=\square-4$ i) 8 ii) -12 iii) -8 iv) -16 b) $7+\square=-9+2$ i) 0 ii) -7 iii) 7 iv) -14	2) a) iii) -8 b) viii) -14
3) Solve the equations using *inspection* and then using *additive inverses*. a) $-10-(-3)=\square-1$ b) $\square+3=(-8)+9$ c) $\square+(-6)=(-7)+(-3)$ d) $12-(-4)=\square\ -6$	3) Inspection — Additive inverses hint a) $\square=-6$ — Add 1 to each side b) $\square=-2$ — Subtract 3 from each side c) $\square=-4$ — Subtract (-6) from each side or add 6 to each side d) $\square=22$ — Add 6 to each side
4) Look at these three statements: A. $11+(-2)=4-(-5)$ B. $11+(-2)-5=4-(-5)$ C. $11+(-2)+(-3)=4-(-5)-3$ a) Decide which statements are not balanced. Re-write them using the $\neq$ sign. b) Show how you can balance the statement/s you wrote in Q4a by changing the right side.	4) a) B. $11+(-2)-5\neq 4-(-5)$ b) B. Subtract 5 from the right side: $11+(-2)-5=4-(-5)-5$
5) The equation $-8-(-4)=\square-3$ has two integers on each side. Use this information to decide whether the following equations are balanced or NOT balanced. If they are not balanced, re-write them with the $\neq$ sign. a) $-8-(-4)+3=\square+3$ b) $\square-3-4=-8$ c) $-8=\square-3+(-4)$ d) $\square=-8+4+3$	5) a) $-8-(-4)+3\neq\square+3$ NOT balanced b) $\square-3-4=-8$ Balanced c) $-8=\square-3+(-4)$ Balanced d) $\square=-8+4+3$ Balanced

Worksheet 2.4: Numerical equations with integers

This worksheet focuses on solving equations involving: 1) addition only; or 2) subtraction only. There are two or more integers on each side of the equal sign.

Questions
1) Three equations are given below. Lee's answers are given next to each equation. Copy the equations and answers. If Lee's answer is correct, give it a tick ✓. If Lee's answer is incorrect, give it a cross ✖ and give the correct answer. a) $1+(-3)+2=\square+2+1$ Answer: $\square=6$ b) $8-(-5)-\square=14-7-(-6)$ Answer: $\square=3$ c) $-3-4-\square=-10-(-4)+4$ Answer: $\square=-9$
2) Solve the following equations: a) $7-(-3)=\square+7$ b) $\square+4=6+(-3)$ c) $12-2=\square+9$ d) $\square-4=10-(-5)$ If you are not told which method to use to solve an equation, you can use inspection **or** additive inverses.
3) Below is a set of four equations: A. $11-(-3)+(-5)-0=\square-1$ B. $11-(-3)+(-5)-1=\square-1$ C. $11-(-3)+(-5)-2=\square-1$ D. $11-(-3)+(-5)-3=\square-1$ Look at the highlighted part in equation A. Note that this expression appears in all the equations. a) Highlight another part which is the same in all the equations. b) Solve all the equations. c) Compare your answers to the four equations. What is the relation between the values of $\square$ in each case? What causes this relation?
4) Below is a set of four equations: A. $-5+3+(-2)=\square+(-8)$ B. $-5+3+(-2)+7=\square+(-8)+7$ C. $40+(-5)+3+(-2)=\square+(-8)+40$ D. $3+(-2)+(-5)-12=-12+\square+(-8)$ To predict the answers: Look for what is the same and different in the equations and write what you expect the value of $\square$ to be. a) Highlight the parts which are the same for each equation. b) Predict TRUE or FALSE: The value of the box will NOT be the same for all the equations. Give reasons for your response. c) Solve all the equations. d) Was your prediction in Q4b correct? Why/why not?
5) You are given the equation: $\lozenge+(-4)=-\oslash-2$. This equation has two unknown values, $\lozenge$ and $\oslash$. When you balance the statements below, both unknown values must appear in the equation. They could be on different sides of the equal sign, or they could be on the same side as in Q5b. a) $(-4)+\lozenge=$ b) $\lozenge+(-4)+\oslash=$ c) $-\oslash-2+0=$ d) $\lozenge+(-4)+2=$ e) $-\oslash-2-(-4)=$ f) $\oslash=$

Worksheet 2.4: Numerical equations with integers

Answers

Questions	Answers
1) Three equations are given below. Lee's answers are given next to each equation. Copy the equations and answers. If Lee's answer is correct, give it a tick ✓. If Lee's answer is incorrect, give it a cross ✗ and give the correct answer. a) $1+(-3)+2=\square+2+1$ Answer: $\square=6$ b) $8-(-5)-\square=14-7-(-6)$ Answer: $\square=3$ c) $-3-4-\square=-10-(-4)+4$ Answer: $\square=-9$	1) a) Answer: $\square=6$ ✗ $\square=-3$ b) Answer: $\square=3$ ✗ $\square=0$ c) Answer: $\square=-9$ ✗ $\square=-5$
2) Solve the following equations: a) $7-(-3)=\square+7$ b) $\square+4=6+(-3)$ c) $12-2=\square+9$ d) $\square-4=10-(-5)$	2) Inspection — Additive inverse hint a) $\square=3$ — Subtract 7 from each side b) $\square=-1$ — Subtract 4 from each side c) $\square=1$ — Subtract 9 from each side d) $\square=19$ — Add 4 to each side
3) Below is a set of four equations: A. $11-(-3)+(-5)-0=\square-1$ B. $11-(-3)+(-5)-1=\square-1$ C. $11-(-3)+(-5)-2=\square-1$ D. $11-(-3)+(-5)-3=\square-1$ Look at the highlighted part in equation A. Note that this expression appears in all the equations. a) Highlight another part which is the same in all the equations. b) Solve all the equations. c) Compare your answers to the four equations. What is the relation between the values of $\square$ in each case? What causes this relation?	3) a) $\square-1$ b) A. $\square=10$ B. $\square=9$ C. $\square=8$ D. $\square=7$ c) The value of box decreases by 1 each time. This happens because on the left side we decrease the value by 1 each time too.
4) Below is a set of four equations: A. $-5+3+(-2)=\square+(-8)$ B. $-5+3+(-2)+7=\square+(-8)+7$ C. $40+(-5)+3+(-2)=\square+(-8)+40$ D. $3+(-2)+(-5)-12=-12+\square+(-8)$ a) Highlight the parts which are the same for each equation. b) Predict TRUE or FALSE: The value of the box will NOT be the same for all the equations. Give reasons for your response. c) Solve all the equations. d) Was your prediction in Q4b correct? Why/why not?	4) a) $-5+3+(-2)=$ and $\square+(-8)$ b) FALSE. The value of the box will be the same because the same number is added to or subtracted from each side of each equation. c) $\square=4$ for all equations d) Prediction was correct because $\square=4$ for all equations. Dependent on learners' response in 4b.
5) You are given the equation: $\Diamond+(-4)=-\oslash-2$. This equation has two unknown values, $\Diamond$ and $\oslash$. When you balance the statements below, both unknown values must appear in the equation. They could be on different sides of the equal sign, or they could be on the same side as in Q5b. a) $(-4)+\Diamond=$ b) $\Diamond+(-4)+\oslash=$ c) $-\oslash-2+0=$ d) $\Diamond+(-4)+2=$ e) $-\oslash-2-(-4)=$ f) $\oslash=$	5) Given: $\Diamond+(-4)=-\oslash-2$ a) $(-4)+\Diamond=-\oslash-2$ b) $\Diamond+(-4)+\oslash=-2$ c) $-\oslash-2+0=\Diamond+(-4)$ d) $\Diamond+(-4)+2=-\oslash$ e) $-\oslash-2-(-4)=\Diamond$ f) $\oslash=-2-\Diamond-(-4)$

Worksheet 2.5 Numerical equations with integers

This worksheet focuses on solving equations that contain a mixture of addition and subtraction. There are two or more integers on each side of the equal sign.

Questions

1) Match the columns:

COLUMN A
a) $9 + (-4) - \square = 10 + (-4) - 7$
b) $7 - 4 + (-6) = 8 - 6 + \square$
c) $-3 + 9 - (-5) = 11 - (-4) - \square$

COLUMN B	
I.	$\square = -5$
II.	$\square = 4$
III.	$\square = 6$
IV.	$\square = -1$

2) Solve these four equations by inspection:
   a) $-5 + \square = -10 - 3$
   b) $9 - (-2) = \square + (-2)$
   c) $8 - \square = 7 - (-1)$
   d) $\square - (-4) = -9 + (-8)$

3) Solve these equations using additive inverses:
   a) $8 + (-3) - 5 = \square + (-2)$
   b) $\square + 4 - (-2) = 6 - 7 - 3$
   c) $-11 + (-1) - (-3) = \square + (-4) + (-2)$

> To predict the answers: Look for what is the same and different in the equations and write what you expect the value of $\square$ to be.

4) Given: $-4 + 3 + (-6) = \square + (-5)$
   a) Look for what is the same and what is different in this set of four equations:
      A. $-4 + 3 + (-6) - 4 = \square + (-5)$
      B. $-4 + 3 + (-6) - 5 = \square + (-5)$
      C. $-4 + 3 + (-6) - 6 = \square + (-5)$
      D. $-4 + 3 + (-6) - 7 = \square + (-5)$
   b) Do you expect the values for $\square$ to be the same for each equation? Give a reason for your answer.
   c) Solve the equations.
   d) Was your prediction in Q4b correct?

5) Given: $9 - (-3) + (-2) = \square + 2 + 4$
   a) Solve each equation:
      A. $9 - (-3) + (-2) - 1 = \square + 2 + 4 - 1$
      B. $9 - (-3) + (-2) + 2 = \square + 2 + 4 + 2$
   b) You should have got the same answers to equations A and B. Why does this happen?
   c) Predict the answers to the following:
      C. $9 - (-3) + (-2) - 12 = \square + 2 + 4 - 12$
      D. $9 - (-3) + (-2) + 40 - 40 = \square + 2 + 4$
      E. $32 - 32 + 9 - (-3) + (-2) = \square + 2 + 4$
      F. $-(-3) + (-2) + 9 - 14 = 2 + 4 + \square - 14$
   d) Use any method to check your predictions.

6) Here is a set of five statements:
   A. $\Diamond - (-2) + 2 = \oslash - 2$
   B. $\Diamond - (-2) + 7 = \oslash + 7$
   C. $\Diamond = \oslash + 2$
   D. $\Diamond - (-2) + (-3) = \oslash + (-3)$
   E. $\Diamond - \oslash - (-1) = 1$

   You are now told that $\Diamond - (-2) = \oslash$
   Use the equation $\Diamond - (-2) = \oslash$ to decide which of the statements (A to E) are balanced and why.

Worksheet 2.5: Numerical equations with integers

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1) Match the columns:

<table><tr><th>COLUMN A</th></tr>
<tr><td>a) $9+(-4)-\square = 10+(-4)-7$</td></tr>
<tr><td>b) $7-4+(-6) = 8-6+\square$</td></tr>
<tr><td>c) $-3+9-(-5) = 11-(-4)-\square$</td></tr></table>
<table><tr><th>COLUMN B</th></tr>
<tr><td>I. $\square=-5$</td></tr>
<tr><td>II. $\square=4$</td></tr>
<tr><td>III. $\square=6$</td></tr>
<tr><td>IV. $\square=-1$</td></tr></table></td>
<td>1)
a) III
b) I
c) II</td></tr>
<tr><td>2) Solve these four equations by inspection:
a) $-5+\square=-10-3$
b) $9-(-2)=\square+(-2)$
c) $8-\square=7-(-1)$
d) $\square-(-4)=-9+(-8)$</td>
<td>2)
a) $\square=-8$
b) $\square=13$
c) $\square=0$
d) $\square=-21$</td></tr>
<tr><td>3) Solve these equations using additive inverses:
a) $8+(-3)-5=\square+(-2)$
b) $\square+4-(-2)=6-7-3$
c) $-11+(-1)-(-3)=\square+(-4)+(-2)$</td>
<td>3)
a) $\square=2$ subtract (-2) from each side
b) $\square=-10$ subtract 6 from each side
c) $\square=-3$ subtract (-6) or add 6 from each side</td></tr>
<tr><td>4) Given: $-4+3+(-6)= \square+(-5)$
a) Look for what is the same and what is different in this set of four equations:
A. $-4+3+(-6)-4= \square+(-5)$
B. $-4+3+(-6)-5= \square+(-5)$
C. $-4+3+(-6)-6= \square+(-5)$
D. $-4+3+(-6)-7= \square+(-5)$
b) Do you expect the values for $\square$ to be the same for each equation? Give a reason for your answer.
c) Solve the equations.
d) Was your prediction in Q4b correct?</td>
<td>4)
a) $-4+3+(-6)$ and $\square+(-5)$ are the same. What is different is that a different number is subtracted from the left of each equation each time.
b) No $\square$ will not to be the same. because we are subtracting different values from each side
c) A. $\square=-6$ B. $\square=-7$ C. $\square=-8$ D. $\square=-9$
d) Yes. The answers were different</td></tr>
<tr><td>5) Given: $9-(-3)+(-2)=\square+2+4$
a) Solve each equation:
A. $9-(-3)+(-2)-1=\square+2+4-1$
B. $9-(-3)+(-2)+2=\square+2+4+2$
b) You should have got the same answers to equations A and B. Why does this happen?
c) Predict the answers to the following:
C. $9-(-3)+(-2)-12=\square+2+4-12$
D. $9-(-3)+(-2)+40-40=\square+2+4$
E. $32-32+9-(-3)+(-2)=\square+2+4$
F. $-(-3)+(-2)+9-14=2+4+\square-14$
d) Use any method to check your predictions.</td>
<td>5)
a) A. $\square=4$ B. $\square=4$
b) We subtracted or added the same value on each side of the given equation.
c) $\square=4$ for all equations
d) Check A: using inverses
$-2=\square-6$
$-2+6=\square-6+6$
$4=\square$
or inspection: $-2=\square-6$ so $4=\square$</td></tr>
<tr><td>6) Here is a set of five statements:
A. $\Diamond-(-2)+2=\oslash-2$
B. $\Diamond-(-2)+7=\oslash+7$
C. $\Diamond=\oslash+2$
D. $\Diamond-(-2)+(-3)=\oslash+(-3)$
E. $\Diamond-\oslash-(-1)=1$
You are now told that $\Diamond-(-2)=\oslash$
Use the equation $\Diamond-(-2)=\oslash$ to decide which of the statements (A to E) are balanced and why.</td>
<td>6) Given $\Diamond-(-2)=\oslash$
B. The same number is added to each side so when it is subtracted from each side we get $\Diamond-(-2)=\oslash$ (which we were given), so the statement is balanced **or** substitute $\oslash$ for $\Diamond-(-2)$ into $\Diamond-(-2)+7=\oslash+7$ and get $\oslash+7=\oslash+7$ so the statement is balanced.
D: Subtract (-3) from each side to get $\Diamond-(-2)=\oslash$ (Which we were given), so the statement is balanced.
Note: substitution for $\oslash$ gets complicated [substitute $\Diamond-(-2)=\oslash$ for $\oslash$ into D. and get $0=0$ This type of solution is mostly beyond Grade 8 and 9 level</td></tr>
</table>

Worksheet 2.6: Numerical equations with integers

This worksheet focuses on the $\square$ in different positions in the equations with integers. The position of the $\square$ makes some equations more difficult than others. As in Worksheets 2.1 to 2.5, our aim is to get $\square$ on its own.

Questions

1) Look at this equation: $-5+8=-9+\square$. Note that $\square$ is at the end of the equation.
   a) Solve the equation by inspection.
   b) Kate solved $-5+8=-9+\square$ using additive inverses.
   Copy her response and answer the questions:

Kate's response	Questions
$-5+8=-9+\square$	i) Is Kate correct to re-write $-9+\square$ as $\square+(-9)$? Why?
$3=\square+(-9)$	ii) How does Kate get 3?
$3+9=\square+(-9)+9$	iii) Why does Kate add 9 on each side?
$12=\square+0$	iv) How does Kate get 0?
	v) And how does she get 12?
$12=\square$	vi) Does Kate get the correct answer?

2) Here is another equation: $-7-(+3)=-11-\square$. Note that it has "subtract $\square$" at the end of the equation.
   a) Solve the equation by inspection.
   b) Tina solved the equation using additive inverses.
   Copy her response and answer the questions:

Tina's response	Questions
$-7-(+3)=-11-\square$	
$-10=-11-\square$	i) How does Tina get -10?
$-10+\square=-11-\square+\square$	ii) What has Tina done on each side?
$-10+\square=-11+0$	iii) How did Tina get 0?
$\square=-11+10$	iv) How does Tina get $+10$ on the right of the equal sign?
$\square=-1$	v) This should be the same answer you got for Q2a. Is it?

3) Solve these equations using additive inverses. Refer to Kate and Tina's responses if you need help.
   a) $5-(-3)=9-\square$ also write what you did in each step in Q3a, and why you did it.
   b) $-7-(-5)=1-\square$
   c) $16-\square=-1-(-19)$
   d) $-2+\square=-7-3$ also write what you did in each step in Q3d, and why you did it.
   e) $9+(-3)=-8-\square$

Worksheet 2.6: Numerical equations with integers

Answers

Questions	Answers

Questions

1) Look at this equation: $-5+8=-9+\square$. Note that $\square$ is at the end of the equation.
   a) Solve the equation by inspection.
   b) Kate solved $-5+8=-9+\square$ using additive inverses. Copy her response and answer the questions:

Kate's response	Questions
$-5+8=-9+\square$	i) Is Kate correct to re-write $-9+\square$ as
$3=\square+(-9)$	$\square+(-9)$? Why?
	ii) How does Kate get 3?
$3+9=\square+(-9)+9$	iii) Why does Kate add 9 on each side?
$12=\square+0$	iv) How does Kate get 0?
	v) And how does she get 12?
$12=\square$	vi) Does Kate get the correct answer?

Answers

1)
a) $\square=12$
b)
   i) Yes, because addition is commutative
   ii) Adds -5 and 8
   iii) To get $\square$ on its own.
   iv) $(-9)+9=0$
   v) $9+3=12$
   vi) Yes

Questions

2) Here is another equation: $-7-(+3)=-11-\square$.
   Note that it has "subtract $\square$" at the end of the equation.
   a) Solve the equation by inspection.
   b) Tina solved the equation using additive inverses. Copy her response and answer the questions:

Tina's response	Questions
$-7-(+3)=-11-\square$	
$-10=-11-\square$	i) How does Tina get -10?
$-10+\square=-11-\square+\square$	ii) What has Tina done on each side?
$-10+\square=-11+0$	iii) How did Tina get 0?
$\square=-11+10$	iv) How does Tina get $+10$ on the right of the equal sign?
$\square=-1$	v) This should be the same answer you got for Q2a. Is it?

Answers

2)
a) $\square=-1$
b)
   i) She "has" -7 and subtracts 3 more which gives -10.
   ii) Added box
   iii) $+\square$ is the additive inverse of $-\square$ so together they sum to zero.
   iv) She added the additive inverse of -10 to each side

Questions

3) Solve these equations using additive inverses. Refer to Kate and Tina's responses if you need help.
   a) $5-(-3)=9-\square$ also write what you did in each step in Q3a, and why you did it.
   b) $-7-(-5)=1-\square$
   c) $16-\square=-1-(-19)$
   d) $-2+\square=-7-3$ also write what you did in each step in Q3d, and why you did it.
   e) $9+(-3)=-8-\square$

Answers to Q3a and Q3d's steps

a) Simplify the right side: $5-(-3)=5+3=8$. Add box to each side to get $+\square$ on the left. Add the inverse of 8 to each side and get $9-8$ which is 1.
d) Simplify the left side: $-7-3=-10$. Add the inverse of -2 to each side to get $\square$ on its own.

Answers

3) **Final answers to Q3a to Q3e**
   a) $\square=1$
   b) $\square=3$
   c) $\square=-2$
   d) $\square=-8$
   e) $\square=-14$

See steps for Q3a and Q3d below the question.

Worksheet 2.7: Numerical equations with integers

This worksheet focuses on solving equations that contain a mixture of addition and subtraction There are two or more whole numbers on each side of the equal sign with ☐ in a variety of positions. As in Worksheets 2.1 to 2.6, our aim is to get ☐ on its own.

Questions
1) Look at this equation: $(-7) + (-8) = 3 - \square$. Note that it has $-\square$ at the end of the equation. a) Solve the equation by using additive inverses. b) Based on your answer to Q1a), answer the following questions: i) Could we rewrite $3 - \square$ as $\square - 3$? Explain. ii) How do you get $+\square$ on the left of the equal sign?
2) Given: $(-10) + (-13) + 4 = \square + (-11)$ a) Look for what is the same and different in this set of four equations: A. $(-10) + (-13) + 4 - 4 = \square + (-11)$ B. $(-10) + (-13) + 4 - 5 = \square + (-11)$ C. $(-10) + (-13) + 4 - 6 = \square + (-11)$ D. $(-10) + (-13) + 4 - 7 = \square + (-11)$ b) Do you expect the results for ☐ to be the same for each equation? Give a reason for your answer. c) Solve the equations. d) Was your prediction in Q2b correct?
3) Solve these equations using additive inverses. a) $10 - (-3) = 9 - \square$ b) $11 - \square = -12 - 4$ c) $-2 + \square = -5 + 3$ d) $9 + (-5) = 2 - \square$ e) $12 + (-3) = 18 - \square$
4) Here is a set of five statements: A. $\lozenge + (-4) + (-7) = \oslash + (-7)$ B. $\lozenge + (-4) + 7 = \oslash + 7$ C. $\lozenge = \oslash + (-4)$ D. $\lozenge + (-4) + 3 = \oslash$ E. $\lozenge - \oslash = -4$ You are now told that $\lozenge + (-4) = \oslash$ Use the equation $\lozenge + (-4) = \oslash$ to decide which statements (A to E) are **not** balanced. Say or show why they are not balanced and write them using a $\neq$ sign.

Worksheet 2.7: Numerical equations with integers

Answers

Questions	Answers
1) Look at this equation: $(-7)+(-8)=3-\square$. Note that it has $-\square$ at the end of the equation. a) Solve the equation by using additive inverses. b) Based on your answer to Q1a), answer the following questions: i) Could we rewrite $3-\square$ as $\square-3$? Explain. ii) How do you get $+\square$ on the left of the equal sign?	1) a) $(-7)+(-8)=3-\square$ $-15=3-\square$ $-15+\square=3-\square+\square$ $-15+15+\square=3+15$ $\square=18$ b) i) No. Subtraction is not commutative. $3-\square\neq\square-3$ ii) Add $\square$ to each side of the equation
2) Given: $(-10)+(-13)+4=\ \square\ +(-11)$ a) Look for what is the same and different in this set of four equations: A. $(-10)+(-13)+4-4=\ \square\ +(-11)$ B. $(-10)+(-13)+4-5=\ \square\ +(-11)$ C. $(-10)+(-13)+4-6=\ \square\ +(-11)$ D. $(-10)+(-13)+4-7=\ \square\ +(-11)$ b) Do you expect the results for $\square$ to be the same for each equation? Give a reason for your answer. c) Solve the equations. d) Was your prediction in Q2b correct?	2) a) $(-10)+(-13)+4$ and $\square\ +(-11)$ are the same. What is different is that in each equation we subtract one more on the left in the previous equation. b) No. The left side gets smaller by one each time so the value for $\square$ would also get smaller by 1 each time. c) A) $\square=-12$ B) $\square=-13$ C) $\square=-14$ D) $\square=-15$ d) Yes. Dependent on learners' response in Q2b.
3) Solve these equations using additive inverses. a) $10-(-3)=\ 9-\square$ b) $11-\square=-12-4$ c) $-2+\square=-5+3$ d) $9+(-5)=2-\square$ e) $12+(-3)=18-\square$	3) a) $\square=-4$ b) $\square=27$ c) $\square=0$ d) $\square=-2$ e) $\square=9$
4) Here is a set of five statements: A. $\lozenge+(-4)+(-7)=\oslash+(-7)$ B. $\lozenge+(-4)+7=\oslash+7$ C. $\lozenge=\oslash+(-4)$ D. $\lozenge+(-4)+3=\oslash$ E. $\lozenge-\oslash=-4$ You are now told that $\lozenge+(-4)=\oslash$ Use the equation $\lozenge+(-4)=\oslash$ to decide which statements (A to E) are **not** balanced. Say or show why they are not balanced and write them using a $\neq$ sign.	4) Given $\lozenge+(-4)=\oslash$ C $\lozenge=\oslash+(-4)$ but we were given $\lozenge+(-4)=\oslash$ if we subtract (-4) from each side we get $\lozenge-(-4)=\oslash$ and $\lozenge-(-4)\neq\oslash$ **or** substitute $\lozenge+(-4)$ for $\oslash$ into C: $\lozenge=[\lozenge+(-4)]+(-4)=\lozenge-8$ and $\lozenge\neq\lozenge-8$ <u>Conclusion</u>: the statement is not balanced $\lozenge\neq\oslash+(-4)$ D 3 is added to $\lozenge+(-4)$ and nothing is added to or subtracted from $\oslash$, changes on each side are different **or** by subtracting 3 from both sides of D we get $\lozenge+(-4)=\oslash-3$ but $\oslash\neq\oslash-3$ **or** substitute $\oslash$ for $\lozenge+(-4)$ on the left of D to get $\oslash+3$ but $\oslash+3\neq\oslash$ <u>Conclusion</u>: the statement is not balanced: $\lozenge+(-4)+3\neq\oslash$ E The given can be written as $\lozenge-4=\oslash$ by adding 4 to each side we get $\lozenge-\oslash=4$ comparing this to E, $4\neq-4$. <u>Conclusion</u>: the statement is not balanced: $\lozenge-\oslash\neq-4$

Worksheet 2.8: Numerical equations with integers

This worksheet focuses on solving equations involving multiplication. There are two integers on each side of the equal sign. Some answers are fractions.

Questions

1) Give the multiplicative inverse of each number:

a) 3 b) 2 c) -5 d) -7

When we multiply *multiplicative inverses*, we get a product of 1. e.g. $-4 \times -\frac{1}{4} = 1$

2) Work out the value of $\square$ to balance these statements:
Note: when we multiply a negative integer by another negative integer, we get a positive integer.

a) $4 \times \frac{1}{\square} = 4 \times 1$ b) $-8 \times -\frac{1}{\square} = 8 \times 1$ c) $5 \times \frac{1}{\square} = -5 \times (-1)$

3) Lina and Thabo solve the equation $10 \times (-2) = \square \times 4$ in different ways:

Lina uses *inspection:*	$10 \times (-2) = \square \times 4$
Lina first simplifies the equation	$-20 = \square \times 4$
She then thinks: "What multiplied by 4 gives -20?" and gets:	$\square = -5$

Thabo uses *multiplicative inverses:*	$10 \times (-2) = \square \times 4$
Thabo also simplifies the equation	$-20 = \square \times 4$
He then multiplies each side by $\frac{1}{4}$ to get $\square$ on its own.	$-20 \times \frac{1}{4} = \square \times 4 \times \frac{1}{4}$
Thabo then simplifies $-20 \times \frac{1}{4}$ like this: $-\frac{20}{1} \times \frac{1}{4} = -\frac{20}{4} = -5$	$-20 \times \frac{1}{4} = \square \times 1$
And gets:	$-5 = \square$
	So $\square = -5$

Solve these equations using *multiplicative inverses* in the way Thabo did. Then check your answers using *inspection*.

a) $(-4) \times 6 = \square \times 3$
b) $\square \times 5 = (-10) \times (-4)$

4) Lina made up this question to try Thabo's method. She found the answer was a fraction. Copy her response and answer the questions:

$$3 \times (-8) = (-7) \times \square$$
$$3 \times (-8) = \square \times (-7)$$
$$3 \times (-8) \times \left(-\frac{1}{7}\right) = \square \times (-7) \times \left(-\frac{1}{7}\right)$$
$$\frac{3}{1} \times \left(-\frac{8}{1}\right) \times \left(-\frac{1}{7}\right) = \square$$
$$\frac{24}{7} = \square$$

a) Lina rewrote $(-7) \times \square$ as $\square \times (-7)$. Explain why she can do this.
b) Why does Lina multiply each side by the multiplicative inverse of -7?
c) What is Lina doing in line 4?
d) Write Lina's answer as mixed number.

5) Solve the following equations using *multiplicative inverses*:

a) $\square \times (-6) = 5 \times 12$
b) $(-3) \times (-5) = \square \times 2$
c) $(-4) \times \square = (-7) \times (-5)$
d) $\square \times 6 = (-5) \times 5$
e) $9 \times 2 = (-3) \times \square$

Worksheet 2.8: Numerical equations with integers

Answers

Questions	**Answers**
1) Give the multiplicative inverse of each number: a) 3 b) 2 c) -5 d) -7	1) a) $\frac{1}{3}$ c) $-\frac{1}{5}$ b) $\frac{1}{2}$ d) $-\frac{1}{7}$
2) Work out the value of $\square$ to balance these statements: Note: when we multiply a negative integer by another negative integer, we get a positive integer. a) $4\times\frac{1}{\square}=4\times1$ b) $-8\times-\frac{1}{\square}=8\times1$ c) $5\times\frac{1}{\square}=-5\times(-1)$	2) a) $\square=1$ b) $\square=1$ c) $\square=1$
3) Lina and Thabo solve the equation $10\times(-2)=\square\times4$ in different ways: Lina uses *inspection:* $10\times(-2)=\square\times4$ Lina first simplifies the equation $-20=\square\times4$ She then thinks: "What multiplied by 4 gives -20?" and gets: $\square=-5$ Thabo uses *multiplicative inverses:* $10\times(-2)=\square\times4$ Thabo also simplifies the equation $-20=\square\times4$ He then multiplies each side by $\frac{1}{4}$ to get $\square$ on its own. $-20\times\frac{1}{4}=\square\times4\times\frac{1}{4}$ $-20\times\frac{1}{4}=\square\times1$ Thabo then simplifies $-20\times\frac{1}{4}$ like this: $-\frac{20}{1}\times\frac{1}{4}=-\frac{20}{4}=-5$ And gets: $-5=\square$ So $\square=-5$ Solve these equations using *multiplicative inverses* in the way Thabo did. Then check your answers using *inspection*. a) $(-4)\times6=\square\times3$ b) $\square\times5=(-10)\times(-4)$	3) a) $(-4)\times6=\square\times3$ $-24=\square\times3$ $-24\times\frac{1}{3}=\square\times3\times\frac{1}{3}$ $-8=\square$ b) $\square\times5=(-10)\times(-4)$ $\square\times5=40$ $\square\times5\times\frac{1}{5}=40\times\frac{1}{5}$ $\square=8$
4) Lina made up this question to try Thabo's method. She found the answer was a fraction. Copy her response and answer the questions: $3\times(-8)=(-7)\times\square$ $3\times(-8)=\square\times(-7)$ $3\times(-8)\times\left(-\frac{1}{7}\right)=\square\times(-7)\times\left(-\frac{1}{7}\right)$ $\frac{3}{1}\times\left(-\frac{8}{1}\right)\times\left(-\frac{1}{7}\right)=\square$ $\frac{24}{7}=\square$ a) Lina rewrote $(-7)\times\square$ as $\square\times(-7)$. Explain why she can do this. b) Why does Lina multiply each side by the multiplicative inverse of -7? c) What is Lina doing in line 4? d) Write Lina's answer as mixed number 4)	4) a) Multiplication is commutative $(-7)\times\square=\square\times(-7)$ b) To get box on its own c) She is writing all the values in fraction form. d) $\frac{24}{7}=3\frac{3}{7}$
5) Solve the following equations using *multiplicative inverses*: a) $\square\times(-6)=5\times12$ b) $(-3)\times(-5)=\square\times2$ c) $(-4)\times\square=(-7)\times(-5)$ d) $\square\times6=(-5)\times5$ e) $9\times2=(-3)\times\square$	5) Value of $\square$ a) -10 d) $-\frac{25}{6}$ b) $\frac{15}{2}$ e) -6 c) $-\frac{35}{4}$

Worksheet 2.9: Numerical equations with integers

This worksheet focuses on solving equations involving division where there are two integers on each side of the equal sign.

A statement that is balanced has the same result on each side of the equal sign.
So, a *balanced statement* is an equation.

Questions

1) If negative nine is divided by three, we can write it as $-9 \div 3$ or $-\frac{9}{3}$ or $-9 \times \frac{1}{3}$.
Write these divisions in two other ways:
a) $20 \div (-5)$ b) $-9 \div (-2)$ c) $-3 \div 7$ d) $4 \div (-4)$

a) Write these multiplications as divisions:
a) $7 \times \left(-\frac{1}{2}\right)$ b) $-9 \times \frac{1}{4}$ c) $-\frac{1}{5} \times (-8)$ d) $\frac{1}{6} \times (-6)$

3) Solve using i*nspection*. The first one has been done for you:
a) $18 \div (-3) = \square \div 6$
$-6 = \square \div 6$ Think: “What divided by 6 gives -6?”
$\square = -36$
b) Now try it this way: $-\frac{18}{3} = \frac{\square}{6}$
c) $\square \div (-8) = (-4) \div 2$ You can choose the method used in Q3a or in Q3b.

4) Give the multiplicative inverse of each number:
a) $\frac{1}{6}$ b) $-\frac{1}{2}$ c) $-\frac{1}{7}$ d) $\frac{3}{5}$

When we multiply *multiplicative inverses,* we get a product of 1. e.g. $-\frac{1}{6} \times (-6) = 1$

5)
a) Solve this equation using *inspection*: $(-6) \div 2 \times (-3) = \square \div (-4)$
b) Ravi solves the equation: $(-6) \div 2 \times (-3) = \square \div (-4)$ using *multiplicative inverses*. Copy his response and answer the questions:

$$(-6) \div 2 \times (-3) = \square \div (-4)$$
$$(-6) \times \frac{1}{2} \times (-3) = \square \times \left(-\frac{1}{4}\right)$$
$$9 = \square \times \left(-\frac{1}{4}\right)$$
$$9 \times (-4) = \square \times \left(-\frac{1}{4}\right) \times (-4)$$
$$-36 = \square$$

i) What has Ravi done in this step?
ii) Show how Ravi got 9 on the left side.
iii) Why did Ravi multiply by -4 on each side of the equal sign?

6) Solve the following equations using *multiplicative inverses* and then check using *inspection*:
a) $\frac{\square}{6} = -\frac{12}{3}$ b) $-\frac{\square}{5} = \frac{12}{4}$ c) $30 \div (-4) = \frac{\square}{2}$ d) $\frac{15}{3} = \square \div (-5)$ e) $-\frac{12}{4} = -\frac{\square}{3}$

7) Here is a set of three statements:
A. $15 \div 3 \times (-3) = 15 \times (-1)$
B. $15 \div (-3) = 5 \div 5 \times (-1)$
C. $-\frac{23}{3} \times 3 = 23 \times (-1)$

a) Which statements are **not** balanced? Re-write them
using a $\neq$ sign.
b) Change the right side of the unbalanced statements so that they are balanced.

Worksheet 2.9: Numerical equations with integers

Answers

Questions and answers

1) If negative nine is divided by three, we can write it as $-9 \div 3$ or $-\frac{9}{3}$ or $-9 \times \frac{1}{3}$.
Write these divisions in two other ways:
a) $20 \div (-5)$ b) $-9 \div (-2)$ c) $-3 \div 7$ d) $4 \div (-4)$

Answers
a) $\frac{20}{-5}$ or $20 \times \frac{1}{-5}$ b) $\frac{9}{2}$ or $9 \times \frac{1}{2}$ c) $-\frac{3}{7}$ or $-3 \times \frac{1}{7}$ d) $\frac{4}{-4}$ or $4 \times \frac{1}{-4}$

2) Write these multiplications as divisions:
a) $7 \times \left(-\frac{1}{2}\right)$ b) $-9 \times \frac{1}{4}$ c) $-\frac{1}{5} \times (-8)$ d) $\frac{1}{6} \times (-6)$

Answers
a) $7 \div (-2)$ b) $-9 \div 4$ c) $-\frac{1}{5} \div \frac{1}{-8}$ d) $\frac{1}{6} \div \frac{1}{-6}$

3) Solve using *inspection*. The first one has been done for you:
a) $18 \div (-3) = \square \div 6$
$-6 = \square \div 6$ Think: "What divided by 6 gives -6?"
$\square = -36$
b) Now try it this way: $-\frac{18}{3} = \frac{\square}{6}$
c) $\square \div (-8) = (-4) \div 2$ You can choose the method used in Q3a or in Q3b.

Answers
b) $\square = -36$
c) $\square \div -8 = -2$
$\square = 16$

4) Give the multiplicative inverse of each number:
a) $\frac{1}{6}$ b) $-\frac{1}{2}$ c) $-\frac{1}{7}$ d) $\frac{3}{5}$

Answers
a) 6 b) -2 c) -7 d) $\frac{5}{3}$

5)
a) Solve this equation using *inspection*: $(-6) \div 2 \times (-3) = \square \div (-4)$
b) Ravi solves the equation: $(-6) \div 2 \times (-3) = \square \div (-4)$ using *multiplicative inverses*. Copy his response and answer the questions:

$$(-6) \div 2 \times (-3) = \square \div (-4)$$
$$(-6) \times \frac{1}{2} \times (-3) = \square \times \left(-\frac{1}{4}\right)$$
$$9 = \square \times \left(-\frac{1}{4}\right)$$
$$9 \times (-4) = \square \times \left(-\frac{1}{4}\right) \times (-4)$$
$$-36 = \square$$

i) What has Ravi done in this step?
ii) Show how Ravi got 9 on the left side.
iii) Why did Ravi multiply by -4 on each side of the equal sign?

Answers
a) $\square = -36$ b) i) Changed the division to multiplication ii) $-3 \times (-3)$ or $18 \times \frac{1}{2}$ iii) -4 is the multiplicative inverse of $-\frac{1}{4}$

6) Solve the following equations using *multiplicative inverses* and then check using *inspection*:
a) $\frac{\square}{6} = -\frac{12}{3}$ b) $-\frac{\square}{5} = \frac{12}{4}$ c) $30 \div (-4) = \frac{\square}{2}$ d) $\frac{15}{3} = \square \div (-5)$ e) $-\frac{12}{4} = -\frac{\square}{3}$

Answers
a) $\square = -24$ b) $\square = -15$ c) $\square = -15$ d) $\square = -25$ e) $\square = 9$

7) Here is a set of three statements:
A. $15 \div 3 \times (-3) = 15 \times (-1)$
B. $15 \div (-3) = 5 \div 5 \times (-1)$
C. $-\frac{23}{3} \times 3 = 23 \times (-1)$

a) Which statements are **not** balanced? Re-write them using a $\neq$ sign.
b) Change the right side of the unbalanced statements so that they are balanced.

Answers
a) B. $15 \div (-3) \neq 5 \div 5 \times (-1)$
b) $15 \div (-3) = 5 \div 5 \times (-1)$ **$\times 5$**

Worksheet 2.10: Numerical equations with integers

This worksheet focuses on the equal sign as a balance for numerical equations involving a mixture of multiplication and division where there are two or more integers on each side of the equal sign.

Questions

1) Write the question and select the correct number for $\square$.
   a) $(-5) \times 2 \times (-3) = \square \times 6$
      A. -30 B. 30 C. -5 D. 5
   b) $6 \times \square \div (-2) = 4 \times (-3)$
      A. -12 B. 4 C. -1 D. 1
   c) $\square \times (-5) \times 5 = (-5) \times (-10) \times (-1)$
      A. -25 B. -50 C. 2 D. -2

2) Copy and solve by inspection:
   a) $-\frac{\square}{2} \times 5 \times (-2) = (-3) \times 5$
   b) $5 \times \frac{6}{4} \times (-4) = \square$

3)
   a) Use the equation $(-10) \times 4 = -40$ to complete the following:
      A. $(-10) \times 4 \times 2 = (-40) \times \square$
      B. $(-40) \times \frac{1}{5} = 4 \times (-10) \div \square$
      C. $(-10) \times 4 \div \square = (-40) \div 5$
   b) Which answers in Q3a are the same? Explain your answer.

4) Solve these equations by inspection. Rewrite the division as multiplication if you find it is easier.
   a) $3 \times \left(-\frac{6}{4}\right) \times (-4) = \square$
   b) $3 \times 6 \div (-4) \times 4 = \square$
   c) $\frac{\square}{2} \times (-4) \times 3 = (-3) \times 4$
   d) $\square \div 2 \times (-4) \times 2 = (-3) \times 4$

5) Solve these equations using inverses.
   a) $(-7) \times 3 \times 2 = \square \times (-6)$
   b) $\square \div (-2) \times 5 = 1 \times 10 \div 2$
   c) $\square \times (-6) \div 4 = (-10) \times 3 \div (-2)$
   d) $10 \div 5 \times (-2) = 16 \times \square \div (-2)$
   e) $(-9) \times \square \times 2 = (-3) \times (-5) \times (-2)$

6) Copy and give the value for $\square$ in each equation.
   A. $(-5) \times (-4) \times 3 = \square \times 4 \times (-3)$
   B. $(-5) \times (-4) \times 3 = \square \times 2 \times (-3)$
   C. $(-5) \times (-4) \times 3 = \square \times (-1) \times 6$
   D. $(-5) \times (-4) \times 3 = 5 \times 2 \times \square$
   E. $(-5) \times (-4) \times 3 = (-5) \times (-1) \times \square$
   F. $(-5) \times \square \times 3 = 3 \times 4 \times 5$
   G. $10 \times \square \times 3 = (-5) \times (-4) \times 3$
   H. $5 \times \square \times (-6) = (-4) \times (-5) \times 3$

7) Look at the set of eight equations in Q6 and answer the questions:
   a) Note that $\square$ is sometimes on the right as in A to E. What is the product of the numbers on the left in A to E?
   b) Sometimes $\square$ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H?
   c) So, what should the product be on the sides with a $\square$ in A to E? And in F to H?

8) Use the numbers $4, 5$ and -6 to make four multiplication equations:
   a) two equations with $\square$ on the right
      e.g. $4 \times 5 \times (-6) = \square \times (-6) \times 10$
   b) two equations with $\square$ on the left
      e.g. $\square \times (-6) \times 10 = -6 \times 5 \times 4$

Worksheet 2.10: Numerical equations with integers

Answers

Questions and answers

1) Write the question and select the correct number for ☐.

a) $(-5) \times 2 \times (-3) = \square \times 6$

A. -30 B. 30 C. -5 D. 5

b) $6 \times \square \div (-2) = 4 \times (-3)$

A. -12 B. 4 C. -1 D. 1

c) $\square \times (-5) \times 5 = (-5) \times (-10) \times (-1)$

A. -25 B. -50 C. 2 D. -2

Answers

a) D b) B c) C

6) Copy and give the value for ☐ in each equation.

A. $(-5) \times (-4) \times 3 = \square \times 4 \times (-3)$
B. $(-5) \times (-4) \times 3 = \square \times 2 \times (-3)$
C. $(-5) \times (-4) \times 3 = \square \times (-1) \times 6$
D. $(-5) \times (-4) \times 3 = 5 \times 2 \times \square$
E. $(-5) \times (-4) \times 3 = (-5) \times (-1) \times \square$
F. $(-5) \times \square \times 3 = 3 \times 4 \times 5$
G. $10 \times \square \times 3 = (-5) \times (-4) \times 3$
H. $5 \times \square \times (-6) = (-4) \times (-5) \times 3$

Answers for ☐

A) -5 C) -10 E) 12 G) 2
B) -10 D) 6 F) -4 H) -2

2) Copy and solve by inspection:

a) $-\frac{\square}{2} \times 5 \times (-2) = (-3) \times 5$

b) $5 \times \frac{6}{4} \times (-4) = \square$

Answers for ☐

a) -3 b) -30

3)

a) Use the equation $(-10) \times 4 = -40$ to complete the following:

A. $(-10) \times 4 \times 2 = (-40) \times \square$
B. $(-40) \times \frac{1}{5} = 4 \times (-10) \div \square$
C. $(-10) \times 4 \div \square = (-40) \div 5$

b) Which answers in Q3a are the same? Explain your answer.

Answers

a) A) 2 B) 5 C) 5

b) B and C. multiplying by $\frac{1}{5}$ is the same as dividing by 5.

7) Look at the set of eight equations in Q6 and answer the questions:

a) Note that ☐ is sometimes on the right as in A to E. What is the product of the numbers on the left in A to E?

b) Sometimes ☐ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H?

c) So, what should the product be on the sides with a ☐ in A to E? And in F to H?

Answers

a) 60
b) 60
c) 60

4) Solve these equations by inspection. Rewrite the division as multiplication if you find it is easier.

a) $3 \times \left(-\frac{6}{4}\right) \times (-4) = \square$
b) $3 \times 6 \div (-4) \times 4 = \square$
c) $\frac{\square}{2} \times (-4) \times 3 = (-3) \times 4$
d) $\square \div 2 \times (-4) \times 2 = (-3) \times 4$

Answers for ☐

a) 18 b) -18 c) 2 d) 3

5) Solve these equations using inverses.

a) $(-7) \times 3 \times 2 = \square \times (-6)$
b) $\square \div (-2) \times 5 = 1 \times 10 \div 2$
c) $\square \times (-6) \div 4 = (-10) \times 3 \div (-2)$
d) $10 \div 5 \times (-2) = 16 \times \square \div (-2)$
e) $(-9) \times \square \times 2 = (-3) \times (-5) \times (-2)$

Answers for ☐

a) 7 c) -10 e) $\frac{5}{3}$
b) -2 d) $\frac{1}{2}$

Note: $\times \frac{2}{1}$ then $\times \frac{1}{5}$ is the same as $\times \frac{2}{5}$

8) Use the numbers $4, 5$ and -6 to make four multiplication equations:

a) two equations with ☐ on the right
e.g. $4 \times 5 \times (-6) = \square \times (-6) \times 10$

b) two equations with ☐ on the left
e.g. $\square \times (-6) \times 10 = -6 \times 5 \times 4$

Answers will differ e.g.

a) $4 \times 5 \times (-6) = \square \times (-3) \times 2$
and $4 \times 5 \times (-6) = \square \times (-5) \times 2$

b) $\square \times (-2) \times 5 = -6 \times 5 \times 4$
and $\square \times (-3) \times 10 = -6 \times 5 \times 4$

Worksheet 2.11: Numerical equations with integers

This worksheet focuses on the equal sign as a balance for numerical equations involving a mixture of multiplication and division where there are two or more integers on each side of the equal sign.
Remember We can write multiplication in different ways using $\times$ or . or (…). For example, 3×2 or 3.2 or $3(2)$.

Questions

1) Write the question and select the correct number for $\square$.
   a) $(-2) \times 3 \times (-2) = \square \times (-6)$
      A. -12 B. 12 C. -2 D. 2
   b) $3(\square) \div 2 = 4 \times (-3)$
      A. -2 B. -8 C. 8 D. -12
   c) $\square.(-6)(-6) = (-6) \times 12 \times 1$
      A. -72 B. -6 C. -2 D. 2

2) Copy and solve by inspection:
   a) $\frac{\square}{3}(-7)(-3) = 3 \times 7$
   b) $(-8)\left(-\frac{6}{4}\right)(-5) = \square$

3) You know that $12(-3) = -36$.
   a) Use this fact to complete the following:
      i) $12(-3).2 = (-36).\square$
      ii) $(-36)\left(-\frac{1}{12}\right) = (-3).12 \div \square$
      iii) $12(3) \div \square = (-36) \div 12$
   b) Which answers are the same? Why?

4) Solve these equations by inspection. Rewrite the division as multiplication if it is easier for you.
   a) $9\left(-\frac{3}{2}\right) \times 2 = \square$
   b) $(-11) \times 6 \div (-3) \times (-3) = \square$
   c) $\frac{\square}{3}(-4)(3) = (-9)(4)$
   d) $\square \div 7 \times (-4)(-7) = 6 \times (-2)$

5) Solve these equations using inverses.
   a) $8 \times (-3) \times 2 = \square \times (-6)$
   b) $\square \div 2 \times (-7) = 1 \times (-14) \div 2$
   c) $\square \times (-8) \div 4 = 10 \times 4 \div (-2)$
   d) $12 \div 6 \times (-3) = (-24) \times \square \div (-2)$
   e) $(-10) \times \square \times (-2) = (-3) \times (-5) \times (-2)$

6) Copy and write down the value for $\square$ in each equation.
   A. $7 \times (-2) \times 3 = \square \times (-2) \times 3$
   B. $7 \times (-2) \times 3 = \square \times (-1) \times 3$
   C. $7 \times (-2) \times 3 = \square \times 1 \times (-6)$
   D. $7 \times (-2) \times 3 = 7 \times 2 \times \square$
   E. $7 \times (-2) \times 3 = (-7) \times 1 \times \square$
   F. $7 \times \square \times (-3) = 3 \times (-2) \times (-7)$
   G. $14 \times \square \times (-3) = 7 \times (-2) \times 3$
   H. $(-7) \times \square \times 6 = 2 \times 7 \times 3$

7) Look at the set of eight equations in Q6 and answer these questions:
   a) Note that $\square$ is sometimes on the right of the equal sign as in A to E. What is the product of the numbers on the left in A to E?
   b) Sometimes $\square$ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H?
   c) So, what should the product be on the sides with $\square$ in A to E? And in F to H?

8) Use the numbers -7, 8 and -5 to make multiplication equations:
   a) Two equations with $\square$ on the right
      e.g. $(-5)(8)(-7) = 8 \times \square \times 1$
   b) Two equations with $\square$ on the left
      e.g. $(8)(\square)(1) = (-7) \times 8 \times (-5)$

Worksheet 2.11: Numerical equations with integers

Answers

Questions and answers

1) Write the question and select the correct number for ☐.

a) $(-2) \times 3 \times (-2) = \square \times (-6)$
A. -12 B. 12 C. -2 D. 2

b) $3(\square) \div 2 = 4 \times (-3)$
A. -2 B. -8 C. 8 D. -12

c) $\square.(-6)(-6) = (-6) \times 12 \times 1$
A. -72 B. -6 C. -2 D. 2

Answers
a) C b) B c) C

2) Copy and solve by inspection:

a) $\frac{\square}{3}(-7)(-3) = 3 \times 7$

b) $(-8)\left(-\frac{6}{4}\right)(-5) = \square$

Answers
a) 3 b) -60

3) You know that $12(-3) = -36$.

a) Use this fact to complete the following:
i) $12(-3).2 = (-36).\square$
ii) $(-36)\left(-\frac{1}{12}\right) = (-3).12 \div \square$
iii) $12(3) \div \square = (-36) \div 12$

b) Which answers are the same? Why?

Answers
a) i) 2 ii) -12 iii) -12
b) ii) and iii) multiplying by $\frac{1}{12}$ is the same as dividing by 12

4) Solve these equations by inspection. Rewrite the division as multiplication if it is easier for you.

a) $9\left(-\frac{3}{2}\right) \times 2 = \square$
b) $(-11) \times 6 \div (-3) \times (-3) = \square$
c) $\frac{\square}{3}(-4)(3) = (-9)(4)$
d) $\square \div 7 \times (-4)(-7) = 6 \times (-2)$

Answers for ☐
a) -27 b) -66 c) 9 d) -3

Questions and answers

6) Copy and write down the value for ☐ in each equation.

A. $7 \times (-2) \times 3 = \square \times (-2) \times 3$
B. $7 \times (-2) \times 3 = \square \times (-1) \times 3$
C. $7 \times (-2) \times 3 = \square \times 1 \times (-6)$
D. $7 \times (-2) \times 3 = 7 \times 2 \times \square$
E. $7 \times (-2) \times 3 = (-7) \times 1 \times \square$
F. $7 \times \square \times (-3) = 3 \times (-2) \times (-7)$
G. $14 \times \square \times (-3) = -7 \times (-2) \times 3$
H. $(-7) \times \square \times 6 = 2 \times 7 \times 3$

Answers for ☐
A. 7 C. 7 E. 6 G. -1
B. 14 D. -3 F. -2 H. -1

7) Look at the set of eight equations in Q6 and answer these questions:

a) Note that ☐ is sometimes on the right of the equal sign as in A to E. What is the product of the numbers on the left in A to E?
b) Sometimes ☐ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H?
c) So, what should the product be on the sides with ☐ in A to E? And in F to H?

Answers
a) -42
b) 42
c) For A to E: -42
For F to H: 42

8) Use the numbers -7, 8 and -5 to make multiplication equations:

a) Two equations with ☐ on the right
e.g. $(-5)(8)(-7) = 8 \times \square \times 1$
b) Two equations with ☐ on the left
e.g. $(8)(\square)(1) = (-7) \times 8 \times (-5)$

Answers will differ e.g.
a) $(-5)(8)(-7) = 2 \times \square \times 2$ and
$(-5)(8)(-7) = 4 \times 2 \times \square$
b) $(4)(\square)(1) = (-7) \times 8 \times (-5)$ and
$(2)(\square)(-2) = (-7) \times 8 \times (-5)$

5) Solve these equations using inverses.

a) $8 \times (-3) \times 2 = \square \times (-6)$
b) $\square \div 2 \times (-7) = 1 \times (-14) \div 2$
c) $\square \times (-8) \div 4 = 10 \times 4 \div (-2)$
d) $12 \div 6 \times (-3) = (-24) \times \square \div (-2)$
e) $(-10) \times \square \times (-2) = (-3) \times (-5) \times (-2)$

Answers for ☐
a) 8 c) 10 e) $-\frac{3}{2}$
b) 2 d) $-\frac{1}{2}$

Note: Multiplying by $\left(-\frac{2}{1}\right)$ then multiplying by $\left(-\frac{1}{7}\right)$ is the same as multiplying by $\frac{2}{7}$

Worksheet 3.1: Algebraic equations

This worksheet provides a recap of numeric expressions involving integers and the commutative law for addition. We use a box or a letter to represent an unknown number.

> An *expression* consists of numbers, letters (variables) or placeholders that are linked by operations ($+, -, \times, \div$).

Questions

1) The tables contain verbal expressions and numeric expressions.

Verbal expressions	
A.	Add 2 and 13.
B.	Add 2 to negative 13.
C.	Add negative 2 and negative 13.
D.	Subtract 2 from 13
E.	Subtract 2 from negative 13.
F.	Subtract negative 2 from negative 13.

Numeric expressions	
1.	$-13+(-2)$
2.	$2-13$
3.	$13-(-2)$
4.	$2+(-13)$
5.	$13-2$
6.	$-2-13$
7.	$-2+(-13)$
8.	$2+13$

a) Match the columns. There may be more than one correct answer for some options!
b) If any items in the verbal expressions column do not have partners, provide the numeric expression.
c) If any items in the numeric expressions column do not have partners, provide the verbal expression.

2) Consider the expression $(-5)+3$. Match it with the expressions and results in A to F below. There is more than one match!

A. -8 B. 2 C. -2 D. -15 E. $3+(-5)$ F. $3-5$

3) Consider the expression $6-(-1)$.
a) Write the expressions and results from A to F that do **not** match $6-(-1)$.

A. 5 B. 7 C. 6 D. $6-1$ E. $6+1$ F. -7

b) Explain why they do not match.

4) Write numeric expressions to match the verbal expressions below. Use a box (☐) as a placeholder for the unknown number. There may be more than one way to write some expressions.
e.g. *Add* 5 *to a number*: $5+\square$ or $\square+5$

a) Add 13 to a number.
b) Subtract 2 from a number.
c) Subtract a number from negative 13.
d) Add a number to negative 5.
e) Subtract a number from 2.
f) Subtract negative 5 from number.

5) The letter n represents any number in the expressions below. State whether each statement is TRUE or FALSE. If the statement is false, give a numeric example to show why the statement is false.

a) $7+n$ is the same as $n+7$
b) $-7-n$ is the same as $-n-7$
c) $7-n$ is the same as $n-7$
d) $-7+(n)$ is the same as $n+(-7)$

Worksheet 3.1: Algebraic equations

Answers

Questions and Answers

1) The tables contain verbal expressions and numeric expressions.

Verbal expressions	
A.	Add 2 and 13.
B.	Add 2 to negative 13.
C.	Add negative 2 and negative 13.
D.	Subtract 2 from 13
E.	Subtract 2 from negative 13.
F.	Subtract negative 2 from negative 13.

Numeric expressions	
1.	$-13+(-2)$
2.	$2-13$
3.	$13-(-2)$
4.	$2+(-13)$
5.	$13-2$
6.	$-2-13$
7.	$-2+(-13)$
8.	$2+13$

Answers to a)

A. 8 see note
B. 4 see note
C. 1; 7 see note
D. 5
E. 1
F. None

a) Match the columns. There may be more than one correct answer for some options!
b) If any items in the verbal expressions column do not have partners, provide the numeric expression.
c) If any items in the numeric expressions column do not have partners, provide the verbal expression.

Note
We could include 4.: $13-(-2)$ for A. since it gives the same result as $2+13$.
For similar reasons we could include 2:.$2-13$ for B, and 6.: $-2-13$ for C.

Answers

b) F: $-13-(-2)$
c) 2: *13 subtracted from 2* or *subtract 13 from 2* (but see note)

2) Consider the expression $(-5)+3$. Match it with the expressions and results in A to F below. There is more than one match!

A. -8 B. 2 C. -2 D. -15 E. $3+(-5)$ F. $3-5$

Answers
C; E and F

4) Consider the expression $6-(-1)$.

a) Write the expressions and results from A to F that do **not** match $6-(-1)$.
A. 5 B. 7 C. 6 D. $6-1$ E. $6+1$ F. -7
b) Explain why they do not match.

Answers
a) A; C; D and F
b) Because $6-(-1)=6+1=7$

5) Write numeric expressions to match the verbal expressions below. Use a box (☐) as a placeholder for the unknown number. There may be more than one way to write some expressions.
e.g. *Add* 5 *to a number*: $5+\square$ or $\square+5$

a) Add 13 to a number.
b) Subtract 2 from a number.
c) Subtract a number from negative 13.
d) Add a number to negative 5.
e) Subtract a number from 2.
f) Subtract negative 5 from number.

Answers
a) $\square+13$
b) $\square-2$
c) $-13-\square$
d) $-5+\square$
e) $2-\square$
f) $\square-(-5)$

6) The letter n represents any number in the expressions below. State whether each statement is TRUE or FALSE. If the statement is false, give a numeric example to show why the statement is false.

a) $7+n$ is the same as $n+7$
b) $-7-n$ is the same as $-n-7$
c) $7-n$ is the same as $n-7$
d) $-7+(n)$ is the same as $n+(-7)$

Answers
a) True
b) True
c) False. Subtraction is not commutative.
d) True

Worksheet 3.2: Algebraic equations

This worksheet recaps algebraic simplification of like and unlike terms in preparation for algebraic equations. It also provides practice in linking verbal and algebraic expressions.

An *algebraic* expression consists of letters (variables), numbers and operations. An expression described in words is called a *verbal* expression.

Questions

1) In each cluster, identify the *unlike* term. If there is no unlike term, provide a term that would be unlike the rest of the cluster.
   a) $6x \quad 6x^2 \quad 3x$
   b) $-x \quad 3x \quad x$
   c) $6 \quad 6p \quad -6$
   d) $4ab \quad 7ba \quad ab$
   e) $8x \quad 8p \quad x$

Like terms have the same variables with the same exponents for the variables. Unlike terms have different variables or different exponents even if they have the same variables.

2) The tables below contain verbal and algebraic expressions. The letter h represents 'a number'. Match the columns. There may be more than one correct answer for some options!

	Verbal expressions
1.	Add 6 to a number
2.	A number multiplied by 6
3.	6 less than a number
4.	A number subtract 6
5.	A number subtracted from 6
6.	A number divided by 6

	Algebraic expressions
A.	$6 + h$
B.	$6h$
C.	$h + 6$
D.	$h - 6$
E.	$6(h)$
F.	$\frac{6}{h}$
G.	$6 - h$
H.	$g \div 6$

In algebra the multiplication sign ($\times$) is usually not written. So we write $5 \times h$ as $5.h$ or $5h$ or $5(h)$. The convention is that we write the number before the letter/s that are being multiplied. So, we write $5h$ rather than $h5$.

Did you identify both algebraic options for verbal expression 2?

3) Simplify the following expressions. If the expression cannot be simplified further, say so.
   a) $8p - 5p$
   b) $6 + 6y + 10 - 5y$
   c) $3 - 2c + c$
   d) $k - m + m - k + k$
   e) $9p - 9$
   f) $-7b + 4b + 6$
   g) $5d + 3e + 12f + 2d - e - 2f$
   h) $-5m - 4m + 3m - 2m + m$

4) The five examples show learners' answers and their explanations. There is an error in each explanation. Say what is wrong with the learners' reasoning and give the correct answer.

	Learners' answers	Learners' explanations
A.	$8p + 5p = 13p^2$	There are like terms so we can add them. 8 and 5 is 13. Then we add p and p, and we get p^2. The answer is $13p^2$.
B.	$8p - 5p = 3$	8 subtract 5 is 3. Then p subtract p is 0. So the answer is 3.
C.	$6 + 6y + 10 = 12y + 10$	6 add 6 is 12 then you write the y. So you get $12y$. But $12y$ and 10 are not like terms so you can't simplify further.
D.	$8a + b - 8ab = 0$	$8a$ add b gives me $8ab$. Then I have $8ab$ subtract $8ab$ which gives me zero.
E.	$5x + 3x - 11x + 4x = 7x$	I add 5 and 3 which gives me $8x$. Then I add 11 and 4 which is $15x$. Then it's $15x$ subtract $8x$ which is $7x$.

Worksheet 3.2: Algebraic equations

Answers

Questions	Answers
1) In each cluster, identify the *unlike* term. If there is no unlike term, provide a term that would be unlike the rest of the cluster. a) $6x \quad 6x^2 \quad 3x$ b) $-x \quad 3x \quad x$ c) $6 \quad 6p \quad -6$ d) $4ab \quad 7ba \quad ab$ e) $8x \quad 8p \quad x$	1) a) $6x^2$ b) No unlike term, $3y$ c) $6p$ d) No unlike term, $4a^2b$ e) $8p$
2) The tables below contain verbal and algebraic expressions. The letter h represents 'a number'. Match the columns. There may be more than one correct answer for some options!	2) 1. A and C 2. B and E 3. D 4. D 5. G 6. F and H

Verbal expressions	
1.	Add 6 to a number
2.	A number multiplied by 6
3.	6 less than a number
4.	A number subtract 6
5.	A number subtracted from 6
6.	A number divided by 6

Algebraic expressions	
A.	$6+h$
B.	$6h$
C.	$h+6$
D.	$h-6$
E.	$6(h)$
F.	$\frac{6}{h}$
G.	$6-h$
H.	$g \div 6$

Questions	Answers
3) Simplify the following expressions. If the expression cannot be simplified further, say so. a) $8p-5p$ b) $6+6y+10-5y$ c) $3-2c+c$ d) $k-m+m-k+k$ e) $9p-9$ f) $-7b+4b+6$ g) $5d+3e+12f+2d-e-2f$ h) $-5m-4m+3m-2m+m$	3) a) $3p$ b) $16+y$ c) $3-c$ d) k e) Can't f) $-3b+6$ g) $7d+2e+10f$ h) $-7m$

4) The five examples show learners' answers and their explanations. There is an error in each explanation. Say what is wrong with the learners' reasoning and give the correct answer.

	Learners' answers	Learners' explanations	Correct answer with explanation
A.	$8p+5p=13p^2$	There are like terms so we can add them. 8 and 5 is 13. Then we add p and p, and we get p^2. The answer is $13p^2$.	A. $8p$ and $5p$ are like terms. When we add like terms the letters do not get multiplied. The answer should be $13p$.
B.	$8p-5p=3$	8 subtract 5 is 3. Then p subtract p is 0. So the answer is 3.	B. We do not take out the letters from loke term and subtract them. The answer should be $3p$.
C.	$6+6y+10$ $=12y+10$	6 add 6 is 12 then you write the y. So you get $12y$. But $12y$ and 10 are not like terms so you can't simplify further.	C. 6 and $6y$ are not like terms so you can't add them. 6 and 10 are like terms. The answer should be $16+6y$
D.	$8a+b-8ab=0$	$8a$ add b gives me $8ab$. Then I have $8ab$ subtract $8ab$ which gives me zero.	D. $8a$, b and $8ab$ are unlike terms so we can't add them. The expression cannot be simplified.
E.	$5x+3x-11x+4x$ $=7x$	I add $5x$ and $3x$ which gives me $8x$. Then I add $11x$ and $4x$ which is $15x$. Then it's $15x$ subtract $8x$ which is $7x$.	E. We must subtract $11x$ from $8x$ then add $4x$. The correct answer is x

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Worksheet 3.3: Algebraic equations

This worksheet focuses on solving algebraic equations. It involves whole numbers and addition and/or subtraction.

An *equation* is a statement indicating that two *expressions* are equal.

Questions

1) Recall how we solved: $2 + \square = 5 + 3$, using inspection and inverses.

   a) Solve using *inspection:*
   Work out the value on the side with no $\square$ (the right side in this case) to see what the result of $2 + \square$ must be. Now think: "What must I add to 2 to get ___?" Then work out the value of $\square$.

   b) Solve using *inverses*:
   Work out the value on the side with no $\square$. Now think: To get $\square$ on its own, I must add the __________ of 2 to each side of the equation. Do this and give the value of $\square$.

A *solution* to an equation is the value that gives the same result on both sides of the equal sign, i.e. it **balances** the left and right sides of the equation.

2) Look at the equation: $2 + y = 5 + 3$.
It has an unknown, y, on the left side of the equation.
Amy and Pindi solve the equation in different ways.
Read their methods carefully and answer the questions.

Amy: *solving by inspection.*
Amy first works out the result on the side with no y.

$$2 + y = 5 + 3$$
$$2 + y = 8$$

Amy thinks 'What must I add to 2 to get 8?'
Amy gets: $y = 6$

Pindi: *solving using additive inverses.*
Pindi also first works out the result on the side with no y.

$$2 + y = 8$$

She rewrites the left side to make it easier for her to see what to do to both sides.

$$y + 2 = 8$$

Pindi then adds the additive inverse of 2 to each side.

$$y + 2 - 2 = 8 - 2$$

Pindi gets: $y = 6$

Questions:

a) Why do Amy and Pindi work out the results on the side with no y?
b) Why can Pindi rewrite $2 + y$ as $y + 2$?
c) Why does Pindi add the additive inverse of 2 to each side?
d) Complete this substitution to check that 6 is the correct answer: $2 + y = 2 + (\_\_) = 8$ and $5 + 3 =$ __ so $y =$ __.

3) Solve the following equations using additive inverses.

a) $n + 7 = 9$
b) $g + 5 = 9 + 5$
c) $k - 6 = 7 - 5$
d) $8 \times 2 = h - 5$
e) $7 - 3 = s - 3$
f) $7 - x = 5 \times 6$

4) Look at these five equations:

A. $r + 5 = 7 + 2$
B. $r + 5 = 7 - 2$
C. $5 + r = 7 + 2$
D. $r - 5 = 7 + 2$
E. $r - 5 = 7 - 2$

a) Which equations have the same left side?
b) Which equations have the same right side?
c) Work out the solutions to all equations using inverses.
d) Which equations have the same solution? Why does this happen?

Challenge

e) Use your answers to predict the answer to:
$$50 + r = 70 + 20$$
f) Now check by solving the equation using inverses.

Worksheet: 3.3: Algebraic equations

Answers

Questions and answers

1) Recall how we solved: $2 + \square = 5 + 3$, using inspection and inverses.

a) Solve using *inspection:*
Work out the value on the side with no $\square$ (the right side in this case) to see what the result of $2 + \square$ must be. Now think: "What must I add to 2 to get ___?" Then work out the value of $\square$.

b) Solve using *inverses*:
Work out the value on the side with no $\square$. Now think: To get $\square$ on its own, I must add the __________ of 2 to each side of the equation. Do this and give the value of $\square$.

Answers

a) 8 b) Additive inverse

2) Look at the equation: $2 + y = 5 + 3$.
It has an unknown, y, on the left side of the equation.
Amy and Pindi solve the equation in different ways.
Read their methods carefully and answer the questions.

Amy: <u>*solving by inspection*</u>.
Amy first works out the result on the side with no y.
$$2 + y = 5 + 3$$
$$2 + y = 8$$
Amy thinks 'What must I add to 2 to get 8?'
Amy gets: $y = 6$

Pindi: <u>*solving using additive inverses.*</u>
Pindi also first works out the result on the side with no y.
$$2 + y = 8$$
She rewrites the left side to make it easier for her to see what to do to both sides.
$$y + 2 = 8$$
Pindi then adds the additive inverse of 2 to each side.
$$y + 2 - 2 = 8 - 2$$
Pindi gets: $y = 6$

Questions:

a) Why do Amy and Pindi work out the results on the side with no y?
b) Why can Pindi rewrite $2 + y$ as $y + 2$?
c) Why does Pindi add the additive inverse of 2 to each side?
d) Use substution to check that 6 is the correct answer.

Answers

a) To make it easier to see what $y + 2$ should equal.
b) Addition is commutative
c) To get y on its own
d) $2 + y = 2 + (\mathbf{6}) = 8$ and $5 + 3 = \mathbf{8}$ so $y = \mathbf{6}$ is the correct answer

3) Solve the following equations using additive inverses.

Questions	Answers
a) $n + 7 = 9$	a) $n = 2$
b) $g + 5 = 9 + 5$	b) $g = 9$
c) $k - 6 = 7 - 5$	c) $k = 8$
d) $8 \times 2 = h - 5$	d) $h = 21$
e) $7 - 3 = s - 3$	e) $s = 7$
f) $7 - x = 5 \times 6$	f) $x = -23$

4) Look at these five equations:

A. $r + 5 = 7 + 2$
B. $r + 5 = 7 - 2$
C. $5 + r = 7 + 2$
D. $r - 5 = 7 + 2$
E. $r - 5 = 7 - 2$

Questions and answers

a) Which equations have the same left side?
A, B and C

b) Which equations have the same right side?
A, C and D
B and E

c) Work out the solutions to all equations using inverses.
A. $\boldsymbol{r = 4}$
B. $\boldsymbol{r = 0}$
C. $\boldsymbol{r = 4}$
D. $\boldsymbol{r = 14}$
E. $\boldsymbol{r = 10}$

d) Which equations have the same solution? Why does this happen?
A and C. This happens because addition is commutative i.e. $\boldsymbol{r + 5 = 5 + r}$ and both A and C have 7 + 2 on the right side.

e) Use your answers to predict the answer to: $50 + r = 70 + 20$.
This equation has the same structure as C but each *constant* has been multiplied by 10. So the solution to C should be multiplied by 10. $\boldsymbol{\therefore r = 40}$

f) Now check by solving the equation using inverses.
$$\boldsymbol{50 + r - 50 = 70 + 20 - 50}$$
$$\boldsymbol{r = 40}$$

Worksheet 3.4: Algebraic equations

This worksheet focuses on solving equations with one variable and one or more constants by applying additive and multiplicative inverses.

Questions

1) Use the information given in the table to fill the blank spaces. Q1a is done for you.

	Constant	Additive inverse	Multiplicative inverse
a)	7	-7 (because $7 + (-7) = 0$)	$\frac{1}{7}$ (because $7 \times \frac{1}{7} = 1$)
b)	-3		
c)	$\frac{1}{7}$		
d)		5	
e)			-5

2) Solve the following equations using multiplicative inverses. Q2a is done for you.

a) $5u = 20$

Multiply each side by $\frac{1}{5}$: $5u \times \frac{1}{5} = 20 \times \frac{1}{5}$

We can re-write these as: $\frac{5u}{5} = \frac{20}{5}$

$u = 4$

b) $3t = 15$

c) $99 = 11k$

d) $10x = 3$

e) $2d = 14$

3) Consider the equation $4x - 2 = 6$. We will solve it using additive and multiplicative inverses. Read the steps below and answer the questions from 3a to 3e.

Step 1: $4x - 2 = 6$

Step 2: $4x - 2 + 2 = 6 + 2$

Step 3: $4x = 8$

Step 4: Multiply each side by $\frac{1}{4}$.

$4x \times \frac{1}{4} = 8 \times \frac{1}{4}$

Step 5: $\frac{4x}{4} = \frac{8}{4}$

Step 6: $x = 2$

a) In Step 2, why do we add 2 on each side?

b) In Step 4, why did we multiply each side by $\frac{1}{4}$?

c) Is $8 \times \frac{1}{4}$ the same as $\frac{8}{4}$? Explain.

d) In which step (1 – 6) did we use additive inverses?

e) In which step did we use multiplicative inverses?

4) Solve these algebraic equations using additive and multiplicative inverses.

a) $3s + 9 = 15$

b) $2 + 5t = 11 - 4$

c) $10 + 4 = 7u + 7$

d) $9 - 3 = 4 + 5m$

5) Look at the following two equations and answer questions 5a to 5d.

A. $15 = 2t + 3$ B. $30 = 4t + 6$

a) What is the same and what is different in equations A and B?

b) Sibu predicts that both equations will have the same answers. Do you agree?

c) Solve equations A and B.

d) Was your prediction in Q5b correct? If not, what was wrong with your thinking?

Worksheet 3.4: Algebraic equations

Answers

Questions and answers

1) Use the information given in the table to fill the blank spaces. Q1a is done for you.

	Constant	Additive inverse	Multiplicative inverse
a)	7	-7 (because $7+(-7)=0$)	$\frac{1}{7}$ (because $7\times\frac{1}{7}=1$)
b)	-3	**3 (because $-3+3=0$)**	**$-\frac{1}{3}$ (because $-3\times-\frac{1}{3}=1$)**
c)	$\frac{1}{7}$	**$-\frac{1}{7}$ (because $\frac{1}{7}-\frac{1}{7}=0$)**	**7 (because $7\times\frac{1}{7}=1$)**
d)	$\mathbf{-5}$	5	$-\frac{1}{5}$
e)	$-\frac{1}{5}$	$\frac{1}{5}$	-5

2) Solve the following equations using multiplicative inverses. Q2a is done for you.

a) $5u = 20$

Multiply each side by $\frac{1}{5}$: $5u\times\frac{1}{5} = 20\times\frac{1}{5}$

We can re-write these as: $\frac{5u}{5} = \frac{20}{5}$

$u = 4$

b) $3t = 15$
c) $99 = 11k$
d) $10x = 3$
e) $2d = 14$

Answers

b) $t = 5$ d) $x = \frac{3}{10}$
c) $k = 9$ e) $d = 7$

3) Consider the equation $4x - 2 = 6$. We will solve it using additive and multiplicative inverses. Read the steps below and answer the questions from 3a to 3e.

Step 1: $4x - 2 = 6$
Step 2: $4x - 2 + 2 = 6 + 2$
Step 3: $4x = 8$
Step 4: Multiply each side by $\frac{1}{4}$.
$4x\times\frac{1}{4} = 8\times\frac{1}{4}$
Step 5: $\frac{4x}{4} = \frac{8}{4}$
Step 6: $x = 2$

Questions and answers

a) In Step 2, why do we add 2 on each side?
Because it is the additive inverse of -2.

b) In Step 4, why did we multiply each side by $\frac{1}{4}$?
Because it's the multiplicative inverse of 4.

c) Is $8\times\frac{1}{4}$ the same as $\frac{8}{4}$? Explain.
Yes. Multiplying by a number is the same as dividing by its reciprocal.

d) In which step (1 – 6) did we use additive inverses?
Step 2

e) In which step did we use s multiplicative inverses?
Step 4

4) Solve these algebraic equations using additive and multiplicative inverses.

a) $3s + 9 = 15$	c) $10 + 4 = 7u + 7$
b) $2 + 5t = 11 - 4$	d) $9 - 3 = 4 + 5m$

Answers

a) $s = 2$	c) $u = 1$
b) $t = 1$	d) $m = \frac{2}{5}$

5) Look at the following two equations and answer questions 5a to 5d.

A. $15 = 2t + 3$ B. $30 = 4t + 6$

a) What is the same and what is different in equations A and B?
Same: A and B have one number on left of the equal sign and a variable (t) and number added together on right of the equal sign. Different: Constants and coefficients are different.

b) Sibu predicts that both equations will have the same answers. Do you agree? Explain
Yes.If we multiply *each side* of equation A by 2 we get equation B.

c) Solve equations A and B. **$t = 6$ for both equations**

d) Was your prediction in Q5b correct? If not, what was wrong with your thinking? **Depends on your answer.**

Worksheet 3.5: Algebraic equations

This worksheet focuses on solving equations with the variable on one side of the equal sign by applying the additive inverse of the constant or the variable.

Questions

1) Give the additive inverse of the following:
   a) 5 b) -5 c) $3x$ d) $-8x$

2) Yvonne has simplified five expressions below. Check her responses. If she has simplified the expression incorrectly, write the correct expression.

	Expression	Simplification	Correct/ Incorrect	Correct simplification
a)	$f + 3f - 2$	$2f$		
b)	$n - 2 + 3$	$n - 5$		
c)	$7 + t - 3$	$5t$		
d)	$3a + 5 - 5a$	$-2a + 5$		

3) Mike solved the equation $14 = -x - 6$.
He applied the additive inverse *of the constant* to solve for x:

$$14 = -x - 6$$
$$14 + 6 = -x - 6 + 6$$
$$20 = -x$$
$$-20 = x$$

A constant is a number that has no variable (or letter) attached to it.

a) Copy Mike's solution and highlight where he applied the additive inverse.
b) Use substitution to check that his solution is correct.
c) Mike could have applied the additive inverse *of the letter* instead of the constant. Copy and complete to show what he could have done.

$$14 = -x - 6$$
$$14 + \_\_ = -x + \_\_ - 6$$
$$14 + x = -6$$

d) Now solve for x.
e) Do you get the same result for x as in Q3a?

4) Here are five equations:
   A. $24 = x + 6$ B. $-32 = a - 5$ C. $7 - b = 17$ D. $-18 + p = 11$ E. $30 = -m + 6$

   a) Solve the equations. Apply the additive inverse to the variable.
   b) Solve each equation again by applying the additive inverse to the constant that is on the same side as the variable.

These equations can be solved by inspection, but we want you to practise applying the additive inverse.

5) Consider the following two equations:
   A. $4x = 9$
   B. $4x + 1 = 9$

   a) What is the same and what is different about the equations?
   b) Which inverse must we apply to equation A? Why?
   c) We must apply two inverses to equation B. Which inverse must be applied first? Why?
   d) Solve the two equations using inverses.

6) Solve using inverses:
   a) $9x = 7$
   b) $7 = 2x + 3$
   c) $3x - 10 = -1$
   d) $6x = -4$

Worksheet 3.5: Algebraic equations

Answers

Questions | **Answers**

1) Give the additive inverse of the following:

a) 5 b) -5 c) $3x$ d) $-8x$

Answers 1)

a) -5 c) $-3x$
b) 5 d) $8x$

2) Yvonne has simplified five expressions below. Check her responses. If she has simplified the expression incorrectly, write the correct expression.

	Expression	Simplification	Answers: Correct/ Incorrect	Answers: Correct simplification
a)	$f+3f-2$	$2f$	Incorrect	$4f-2$
b)	$n-2+3$	$n-5$	Incorrect	$n+1$
c)	$7+t-3$	$5t$	Incorrect	$t+4$
d)	$3a+5-5a$	$-2a+5$	Correct	

3) Mike solved the equation

$$14=-x-6.$$

He applied the additive inverse *of the constant* to solve for x:

$$14=-x-6$$
$$14+6=-x-6+6$$
$$20=-x$$
$$-20=x$$

a) Copy Mike's solution and highlight where he applied the additive inverse.

b) Use substitution to check that his solution is correct.

c) Mike could have applied the additive inverse *of the letter* instead of the constant. Copy and complete to show what he could have done.

$$14=-x-6$$
$$14+\_\_=-x+\_\_-6$$
$$14+x=-6$$

d) Now solve for x.

e) Do you get the same result for x as in Q3a?

Answers 3)

a) $14+6=-x-6+6$

b) On the right side of the equation:
$-(-20)-6=20-6$
$=14$
And 14 is on the left side

c) $14=-x-6$
$14+\boldsymbol{x}=-x+\boldsymbol{x}-6$
$14+x-\mathbf{14}=-6-\mathbf{14}$

d) $x=-20$

e) Yes

4) Here are five equations:

A.	B.	C.	D.	E.
$24=x+6$	$-32=a-5$	$7-b=17$	$-18+p=11$	$30=-m+6$

a) Solve the equations. Apply the additive inverse to the variable.

b) Solve each equation again by applying the additive inverse to the constant that is on the same side as the variable.

Answers 4)

a)
A. $x=18$ D. $p=29$
B. $a=-27$ E. $m=-24$
C. $b=-10$

b) Same as above.

5) Consider the following two equations:

A. $4x=9$
B. $4x+1=9$

a) What is the same and what is different about the equations?

b) Which inverse must we apply to equation A? Why?

c) We must apply two inverses to equation B. Which inverse must be applied first? Why?

d) Solve the two equations using inverses

Answers 5)

a) Both equations have $4x$ on the left and 9 on the right side. Different: Equation B. has an additional $+1$ on the left side.

b) Multiplicative inverse. To get x on its own.

c) The additive inverse to collect the like terms

d) A. $x=\frac{9}{4}$ B. $x=2$

6) Solve using inverses:

a) $9x=7$ c) $3x-10=-1$

b) $7=2x+3$ d) $6x=-4$

Answers 6)

a) $x=\frac{7}{9}$ c) $x=3$

b) $x=2$ d) $x=-\frac{4}{6}$

Worksheet 3.6: Algebraic equations

This worksheet focuses on solving equations with the variable or the constant on both sides of the equal sign by applying the additive inverse of the constant or the letter.

Questions

1) Make 5 pairs of additive inverses from the list of terms below. If the additive inverse does not appear in the list, provide it.

   $-4 \quad \frac{1}{4} \quad -x \quad 6x \quad 6 \quad \frac{1}{6} \quad -\frac{1}{2} \quad 4 \quad x \quad 0{,}5$

2) In the table below, apply the additive inverse that is indicated. Then write down the new form of the equation after applying the inverse. The first one has been done for you.

Equation	Apply additive inverse of …	Equation after applying inverse
$x - 4 = 2x$	-4	$x = 2x + 4$
	x	
	$2x$	

3) This equation has k's on both sides of the equal sign: $4k = k + 6$
   a) If you apply the additive inverse of 6 to both sides, will there still be k's on both sides?
   b) Remember that the additive inverse of k is $-k$.
      i) Apply this additive inverse to both sides.
      ii) Are there still k's on both sides?
      iii) What remains on the right side?
   c) Continue to solve the equation and show that the solution is 2.
   d) What type of inverse did you use to continue solving the equation in Q3c?

4) Solve the following equations by applying the additive inverse of the variable

   a) $3p = p - 2$ b) $p = 3p - 2$ c) $7b = 5b + 4$ d) $20b = 50b - 10$

5) This equation has constants on both sides of the equal sign: $16 = 5m + 6$
   a) Apply the additive inverse of 16 to both sides. Are there constants on only one side?
   b) Apply the additive inverse of 6 to both sides. Are there constants on only one side?
   c) Choose the method that gives constants on one side to solve the equation.
   d) Why is it not a good idea to apply the additive inverse of $5m$ to both sides?

6) Solve the following equations using inverses.
   a) $5d = d - 20$ b) $s = 5s - 4$ c) $30x = 70x - 10$
   d) $5d + 8 = d - 20$ e) $s + 6 = 5s - 4$ f) $70x + 30 = 30x - 10$

Worksheet 3.6: Algebraic equations

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1) Make 5 pairs of additive inverses from the list of terms below. If the additive inverse does not appear in the list, provide it.
$-4 \quad \frac{1}{4} \quad -x \quad 6x \quad 6 \quad \frac{1}{6} \quad -\frac{1}{2} \quad 4 \quad x \quad 0{,}5$</td><td>1) Here are 7 pairs:
$-4; 4$
$-x; x$
$6x; -6x$
$\frac{1}{4}; -\frac{1}{4}$
$-\frac{1}{2}; \frac{1}{2}$
$6;\ -6$
$0{,}5;\ -0{,}5$</td></tr>
<tr><td colspan="2">2) In the table below, apply the additive inverse that is indicated. Then write down the new form of the equation after applying the inverse. The first one has been done for you.
<table>
<tr><th></th><th></th><th>Answers</th></tr>
<tr><th>Equation</th><th>Apply additive inverse of …</th><th>Equation after applying inverse</th></tr>
<tr><td rowspan="3">$x - 4 = 2x$</td><td>-4</td><td>$x = 2x + 4$</td></tr>
<tr><td>x</td><td>$\mathbf{-4 = 2x - x}$</td></tr>
<tr><td>$2x$</td><td>$\mathbf{x - 2x - 4 = 0}$</td></tr>
</table></td></tr>
<tr><td>3) This equation has k's on both sides of the equal sign: $4k = k + 6$
a) If you apply the additive inverse of 6 to both sides, will there still be k's on both sides?
b) Remember that the additive inverse of k is $-k$.
i) Apply this additive inverse to both sides.
ii) Are there still k's on both sides?
iii) What remains on the right side?
c) Continue to solve the equation and show that the solution is 2.
d) What type of inverse did you use to continue solving the equation in Q3c?</td><td>3)
a) Yes
b) i) $3k = 6$
ii) No, the terms with k are on left side of equal sign.
iii) 6
c) $3k = 6$
$k = 2$
d) Multiplicative inverse</td></tr>
<tr><td>4) Solve the following equations by applying the additive inverse of the variable
a) $3p = p - 2$ d) $7b = 5b + 4$
b) $p = 3p - 2$ d) $20b = 50b - 10$</td><td>4)
a) $p = -1$
b) $p = 1$
c) $b = 2$
d) $b = \frac{1}{3}$</td></tr>
<tr><td>5) This equation has constants on both sides of the equal sign: $16 = 5m + 6$
a) Apply the additive inverse of 16 to both sides. Are there constants on only one side?
b) Apply the additive inverse of 6 to both sides. Are there constants on only one side?
c) Choose the method that gives constants on one side to solve the equation.
d) Why is it not a good idea to apply the additive inverse of $5m$ to both sides?</td><td>5)
a) $0 = 5m + 6 - 16$; Yes
b) $10 = 5m$; Yes
c) $5m = 10$
$m = 2$
d) Because then we would have $-5m$ on the left.</td></tr>
<tr><td>6) Solve the following equations using inverses.
a) $5d = d - 20$ c) $s = 5s - 4$ e) $30x = 70x - 10$
b) $5d + 8 = d - 20$ d) $s + 6 = 5s - 4$ f) $70x + 30 = 30x - 10$</td><td>6)
a) $d = -5$ c) $s = 1$ e) $x = \frac{1}{4}$
b) $d = -7$ d) $s = \frac{5}{2}$ f) $x = 1$</td></tr>
</table>

Worksheet 3.7: Algebraic equations

This worksheet focuses on solving equations with variables and constants on both sides of the equal sign.

Questions

1) The equation: $x + 5 = 17 - 2x$ has x's and constants on both sides of the equal sign:
   a) Thandi solved the equation this way. Read her solution carefully and then answer the questions.

	$x + 5 = 17 - 2x$		
Step 1:	$x = 12 - 2x$	i)	How did Thandi get 12 in step 1?
Step 2:	$x + 2x = 12 - 2x + 2x$	ii)	Why did she add $2x$ in step 2?
Step 3:	$3x = 12$	iii)	How did she get $x = 4$ in step 4?
Step 4:	$x = 4$	iv)	Use substitution to check that her solution $x = 4$is correct.

   b) Solve the equation by first applying the additive inverse of 17. Do you get the same answer as Thandi?
   c) Now solve the equation by first applying the additive inverse of x. Do you get the same answer as Thandi?
   d) Lastly, solve the equation again by first applying the additive inverse of 5. Do you get the same answer as Thandi?
   e) What other type of inverse did you apply to solve each equation?

2) Focus on <u>how</u> we solve $4 - 3x = 12 + x$ by answering the following questions:
   a) We apply the additive inverse of x. Explain why.
   b) Then we apply the additive inverse of 4. Explain why.
   c) When we have applied both additive inverses above, we will be left with $-4x = 8$. Do you agree?
   d) Which inverse do we apply to isolate x?
   e) What is the solution to the equation?
   f) Now solve the equation again by gathering all the terms with x on the right side of the equal sign (and the constants on the left side).

3) We are going to solve the equation $3x - 10 = x + 6$ in using two different strategies.
   a) Apply inverses so that you collect the terms with variables on the left side, and the terms with constants on the right side.
   b) Now apply inverses so that you collect the terms with variables on the *right* side, and the terms with constants on the *left* side.

4) Solve the following using inverses.
   a) $2y = y + 9 - 3$ b) $2x + 3 = x + 9$ c) $r + 9 = 2 - 6r$ d) $r + 9 = 2r - 6$

5) Here is David's response to solving the equation: $2x + 3 = x - 4$

	$2x + 3 = x - 4$
Step 1:	$2x + 3 - 3 = x + 3 - 4$
Step 2:	$2x = x + 1$
Step 3:	$2x - x = x - x + 1$
Step 4:	$x = 1$

   a) Copy David's response and ring any errors he made.
   b) Correct his errors.
   c) Use substitution to confirm that your solution is correct.
   d) What happens on the right of the equal sign in Step 3 when he added $-x$ to both sides of the equation?

Worksheet 3.7: Algebraic equations

Answers

Questions and answers

1) The equation: $x + 5 = 17 - 2x$ has x's and constants on both sides of the equal sign:

a) Thandi solved the equation this way. Read her solution carefully and then answer the questions.

	$x + 5 = 17 - 2x$	
Step 1:	$x = 12 - 2x$	i) How did Thandi get 12 in step 1?
Step 2:	$x + 2x = 12 - 2x + 2x$	ii) Why did she add $2x$ in step 2?
Step 3:	$3x = 12$	iii) How did she get $x = 4$ in step 4?
Step 4:	$x = 4$	iv) Use substitution to check that her solution $x = 4$ is correct.

Answers

i) She applied the additive inverse of 5; $17 - 5 = 12$

ii) $2x$ is the additive inverse of $-2x$, adding it to both sides gets the terms with x on the left side of the equation.

iii) She used the multiplicative inverse of 3 on both sides of the equal sign.

iv) On the left: $(4) + 5 = 9$. On the right: $17 - 2(4) = 17 - 8 = 9$ So her solution $x = 4$ is correct

b) Solve the equation by first applying the additive inverse of 17. Do you get the same answer as Thandi?
Answer $x - 12 = -2x$; $3x = 12$; $x = 4$ Yes

c) Now solve the equation by first applying the additive inverse of x. Do you get the same answer as Thandi?
Answer $5 = 17 - 3x$; $-12 = -3x$; $4 = x$ Yes

d) Lastly, solve the equation again by first applying the additive inverse of 5. Do you get the same answer as Thandi?
Answer $x = 12 - 2x$; $3x = 12$; $x = 4$ Yes

e) What other type of inverse did you apply to solve each equation? **Answer** Multiplicative inverse

2) Focus on how we solve $4 - 3x = 12 + x$ by answering the following questions:

a) We apply the additive inverse of x. Explain why. **Answer** To collect the terms with x on the left side of the equation.

b) Then we apply the additive inverse of 4. Explain why. **Answer** To collect the constants on the right side of the equation.

c) When we have applied both additive inverses above, we will be left with $-4x = 8$. Do you agree? **Answer** Yes

d) Which inverse do we apply to isolate x? **Answer** Multiplicative inverse

e) What is the solution to the equation? **Answer** $x = -2$

e) Now solve the equation again by gathering all the terms with x on the right side of the equal sign (and the constants on the left side). **Answer** $x = -2$

3) We are going to solve the equation $3x - 10 = x + 6$ in using two different strategies.

a) Apply inverses so that you collect the terms with variables on the left side, and the terms with constants on the right side. **Answer** $2x = 16$; $x = 8$

b) Now apply inverses so that you collect the terms with variables on the *right* side, and the terms with constants on the *left* side. **Answer** $-16 = -2x$; $x = 8$

4) Solve the following using inverses.

a) $2y = y + 9 - 3$ b) $2x + 3 = x + 9$ c) $r + 9 = 2 - 6r$ d) $r + 9 = 2r - 6$

Answers

a) $y = 6$ b) $x = 6$ c) $r = -1$ d) $r = 15$

5) Here is David's response to solving the equation: $2x + 3 = x - 4$

	$2x + 3 = x - 4$
Step 1:	$2x + 3 - 3 = x + 3 - 4$
Step 2:	$2x = x + 1$
Step 3:	$2x - x = x - x + 1$
Step 4:	$x = 1$

a) Copy David's response and ring any errors he made.

b) Correct his errors.

c) Use substitution to confirm that your solution is correct.

d) What happens on the right of the equal sign in Step 3 when he added $-x$ to both sides of the equation?

Answers

a) $2x + 3 - 3 = x + 3 - 4$
$2x = x + 1$

b) $2x + 3 - 3 = x - 4 - \mathbf{3}$
$2x = x - 7$
$x = -7$

c) Left: $2(-7) + 3 = -11$
Right: $(-7) - 4 = -11$

d) The variables were on one side of the equation.

Worksheet 3.8: Algebraic equations

This worksheet focuses mostly on equations with variables on both sides of the equal sign.

Questions

1) Consider the following 2 equations

   A. $4 = 2n + 6$
   B. $4 + n = 2n + 6$

   a) What is the same and what is different about the two equations?
   b) Without solving the equations, try to decide whether they will have the same solution. Justify your answer.
   c) Sizwe says we apply only the additive inverse of 6 for equation A. Do you agree?
   d) Which inverse would you apply first for equation B? Why?
   e) Solve the two equations.
   f) Was your prediction in Q2b correct? If not, what was wrong with your thinking?

2)
   a) In the table below, apply the additive inverse and write down the equation after applying the inverse. The first one has been done for you.

Equation	Apply additive inverse of …	Equation after applying inverse
$2a - 3 = 5a - 4$	-3	$2a = 5a - 1$
	-4	
	$2a$	
	$5a$	

   b) Solve the equation in Q2a.

3) The six equations below are arranged in pairs.

   A. $5x - 3 = 2x + 6$
   B. $3 - 5x = 2x + 6$

   C. $3y = 6 - 2y$
   D. $3y - 5 = 6 - 2y - 5$

   E. $-5a = 6 - 2a$
   F. $0 = 5a - 2a + 6$

   a) Without solving the equations, try to predict whether each pair will have the same solution.
   b) Now solve all six equations using inverses.
   c) Were your predictions correct? If not, was there something you didn't pay attention to?

4) Solve the following equations using inverses.

   a) $5x - 3 = 4x + 6$
   b) $5a - 3 = 2a + 6$
   c) $5b = 2b + 6$
   d) $5c - 3 = 2c$

Worksheet 3.8: Algebraic equations

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1) Consider the following 2 equations
A. $4 = 2n + 6$
B. $4 + n = 2n + 6$
a) What is the same and what is different about the two equations?
b) Without solving the equations, try to decide whether they will have the same solution. Justify your answer.
c) Sizwe says we apply only the additive inverse of 6 for equation A. Do you agree?
d) Which inverse would you apply first for equation B? Why?
e) Solve the two equations.
f) Was your prediction in Q2b correct? If not, what was wrong with your thinking?</td><td>1)
a) Same: Both equations have 4 on the left side and $2n + 6$ on the right side.
Different: Equation B. has $+n$ on the left side.
b) No, because n was added to only one side, the left.
c) Yes, $-2 = 2n$ is easy to solve by inspection.
d) Additive inverse of n to collect the like terms.
e) A. $n = -1$ B. $n = -2$
f) May not have recognised that n is added only on left side in B, and right side is identical.</td></tr>
<tr><td colspan="2">2)
a) In the table below, apply the additive inverse and write down the equation after applying the inverse. The first one has been done for you.</td></tr>
</table>

Equation	Apply additive inverse of …	Answers What remains
$2a - 3 = 5a - 4$	-3	$2a = 5a - 1$
	-4	$\mathbf{2a - 7 = 5a}$
	$2a$	$\mathbf{-3 = 3a - 4}$
	$5a$	$\mathbf{-3a - 3 = -4}$

b) Solve the equation in Q2a **Answer**.

$$2a - 3 = 5a - 4$$
$$2a - 3 - 5a = 5a - 4 - 5a$$
$$-3a - 3 = -4$$
$$-3a - 3 + 3 = -4 + 3$$
$$-3a = -1$$
$$a = \frac{1}{3}$$

<table>
<tr><td>3) The six equations below are arranged in pairs.
A. $5x - 3 = 2x + 6$
B. $3 - 5x = 2x + 6$

C. $3y = 6 - 2y$
D. $3y - 5 = 6 - 2y - 5$

E. $-5a = 6 - 2a$
F. $0 = 5a - 2a + 6$
a) Without solving the equations, try to predict whether each pair will have the same solution.
b) Now solve all six equations using inverses.
c) Were your predictions correct? If not, was there something you didn't pay attention to?</td><td>3) Answers
a) Pair 1: No; Pair 2: Yes; Pair 3: Yes
b)
A. $x = 3$
B. $x = -\frac{3}{7}$
C. $y = \frac{6}{5}$
D. $y = \frac{6}{5}$
E. $a = -2$
F. $a = -2$
c) May have not recognised that:
A & B: $5x - 3 \neq 3 - 5x$;
D & E: Same number added to both sides;
E & F: additive inverse of $-5a$ has been applied in F, as well as commutative law.</td></tr>
<tr><td>4) Solve the following equations using inverses.
a) $5x - 3 = 4x + 6$ c) $5b = 2b + 6$
b) $5a - 3 = 2a + 6$ d) $5c - 3 = 2c$</td><td>4) Answers
a) $x = 9$ c) $b = 2$
b) $a = 3$ d) $c = 1$</td></tr>
</table>

Worksheet 3.9: Algebraic equations

This worksheet focuses on comparing solutions and on shorter methods of solving equations using inverses.

Questions

1) Consider the equation $4x + 1 = 7 + 2x$. Both Anu and Andrew solved it using additive and multiplicative inverses. Read the steps below carefully and answer the questions that follow.

Anu's response	Andrew's response
Step 1: $4x + 1 = 7 + 2x$	Step 1: $4x + 1 = 7 + 2x$
Step 2: $4x + 1 - 1 = 7 + 2x - 1$	Step 2: $4x + 1 - 4x = 7 + 2x - 4x$
Step 3: $4x = 7 - 1 + 2x$	Step 3: $1 = 7 - 2x$
Step 4: $4x = 6 + 2x$	Step 4: $1 - 7 = 7 - 2x - 7$
Step 5: $4x - 2x = 6 + 2x - 2x$	Step 5: $-6 = -2x$
Step 6: $2x = 6$	Step 6: …
Step 7: …	

a) Complete Step 7 for Anu and complete Step 6 for Andrew.
b) Explain why they get the same solution.
c) Now go back and look at the responses again.
   i) What is the difference in the two responses in Step 2?
   ii) In which step did Anu use the multiplicative inverse?
   iii) In which step did Andrew use the multiplicative inverse?
d) Whose response do you prefer? Why?
e) Try to go from Step 1 to step 6 in your head in Anu's solution and Step 1 to step 5 in your head in Andrew's solution. If you can, you will be able to work faster. If you can't, keep practicing the steps.

2) Consider the equation: $3x - 10 = x + 6$
a) If you want to collect like terms with the variable on the left side, what inverse must you apply, and which term must you apply it to?
b) If you want to collect constants on the left side, what inverse must you apply, and which term must you apply it to?
c) Solve the equation using inverses.
d) How many times did you apply the additive inverse? Which inverses did you apply?

3) Consider the equation: $1 - 3x = 5 - x$
Roger and Vas both solved it incorrectly.
a) Spot the errors in their responses.
b) Solve the equation correctly.

Roger's response	Vas's response
$1 - 3x = 5 - x$	$1 - 3x = 5 - x$
$-2x = 4x$	$1 - 3x + 3x = 5 - x + 3x$
$x = -2$	$1x = 7x$
	$x = 7$

4) You are given the following equation: $3x + 5 = x - 9$. Write TRUE or FALSE for each statement.
a) The solution is a fraction.
b) If you apply the additive inverse of 5, you will get -14 on the right side.
c) If you apply the additive inverse of x, you will get -14 on its own.
d) If you apply the additive inverse of -9, the equation won't be balanced.
e) If you apply the additive inverse of 5, and then add $-x$ to both sides, the result on the left side will be $-3x - x$.
f) The solution is $x = -7$.

Worksheet 3.9: Algebraic equations

Answers

Questions and answers

1) Consider the equation $4x + 1 = 7 + 2x$. Both Anu and Andrew solved it using additive and multiplicative inverses. Read the steps below carefully and answer the questions that follow.

Anu's response		Andrew's response	
Step 1:	$4x + 1 = 7 + 2x$	Step 1:	$4x + 1 = 7 + 2x$
Step 2:	$4x + 1 - 1 = 7 + 2x - 1$	Step 2:	$4x + 1 - 4x = 7 + 2x - 4x$
Step 3:	$4x = 7 - 1 + 2x$	Step 3:	$1 = 7 - 2x$
Step 4:	$4x = 6 + 2x$	Step 4:	$1 - 7 = 7 - 2x - 7$
Step 5:	$4x - 2x = 6 + 2x - 2x$	Step 5:	$-6 = -2x$
Step 6:	$2x = 6$	Step 6:	…
Step 7:	…		

a) Complete Step 7 for Anu and complete Step 6 for Andrew. **Answer** Step 7: $x = 3$ Step 6: $x = 3$

b) Explain why they get the same solution. **Answer** $-\frac{6}{-2} = \frac{6}{2}$

c) Now go back and look at the responses again.

i) What is the difference in the two responses in Step 2? **Answer** Anu applied the additive inverse of 1 and Andrew applied the additive inverse of $4x$.

ii) In which step did Anu use the multiplicative inverse? **Answer** Step 7

iii) In which step did Andrew use the multiplicative inverse? **Answer** Step 6

d) Whose response do you prefer? Why? **Answer** Anu's, he deals with positives

e) Try to go from Step 1 to step 6 in your head in Anu's solution and Step 1 to step 5 in your head in Andrew's solution. If you can, you will be able to work faster. If you can't, keep practicing the steps.

2) Consider the equation: $3x - 10 = x + 6$

a) If you want to collect like terms with the variable on the left side, what inverse must you apply to terms with variables?

b) If you want to collect constants on the left side, what inverse must you apply to constant terms?

c) Solve the equation using inverses.

d) How many times did you apply the additive inverse? Which inverses did you apply?

2) **Answers**

a) Additive inverse of x, i.e. add $-x$

b) Additive inverse of 6, i.e. add -6

c) Solution: $x = 8$

d) Will depend on approach. Will need to apply additive inverse to variable and to letter. Will then need to apply multiplicative inverse.

3) Consider the equation:
$1 - 3x = 5 - x$
Roger and Vas solved it incorrectly.

a) Spot the errors in their responses.

b) Solve the equation correctly.

Roger's response	Vas's response
$1 - 3x = 5 - x$	$1 - 3x = 5 - x$
$-2x = 4x$	$1 - 3x + 3x = 5 - x + 3x$
$x = -2$	$1x = 7x$
	$x = 7$

3) **Answers**

a) See circles

b) $1 - 3x = 5 - x$
$-2x = 4$
$x = -\frac{1}{2}$

4) You are given the following equation: $3x + 5 = x - 9$. Write TRUE or FALSE for each statement.

a) The solution is a fraction.

b) If you apply the additive inverse of 5, you will get -14 on the right side.

c) If you apply the additive inverse of x, you will get -14 on its own.

d) If you apply the additive inverse of -9, the equation won't be balanced.

e) If you apply the additive inverse of 5, and then add $-x$ to both sides, the result on the left side will be $-3x - x$.

f) The solution is $x = -7$.

4) **Answers**

a) False

b) True

c) True

d) False

e) False

f) True

Worksheet 3.10: Algebraic equations

This worksheet provides practice in solving equations using inverses. The equations have variables on one side or on both sides. Some equations have integer coefficients.

Questions

1) Solve the equations using additive and multiplicative inverses.

a) $5m - 6 = 3m + 2$
b) $6 - 5m = -2 - 3m$
c) $5m - 2 = 3m + 6$
d) $5m + 6 = 3m - 2$
e) $10m - 12 = 6m + 4$
f) $m + 1 - 2m + 2 = 3 + m + 4$

2) Consider the equation: $-9 + t = -3t + 3$

a) Read the statements A to E and decide if they are TRUE or FALSE.

A. The value of t (i.e. the solution) is positive.
B. The value of t will not change if we multiply each side of the equation by 3.
C. The value of t will not change if we divide each side of the equation by 3.
D. The value of t will not change if we multiply by 3 on the left side and divide by 3 on the right side of the original equation.
E. The value of t will change if we add 9 to each side.

b) Solve the equation.

c) Now test any of the statements you are unsure of, i.e. perform the required operations and then solve the new equation.

3) Solve these algebraic equations using additive and multiplicative inverses.

a) $d - 9 - 2d = 7$
b) $5p = -3 + p - 5$
c) $n - 3 = 7 - n$
d) $-s - 6 + 2s = 7 - s$
e) $3 - 2m = 2m + 3$
f) $k = 2k + k + 7$

4) There are five equations in the table.

a) Predict whether the solution to each equation will be: a positive integer, or a negative integer or a fraction.

b) Solve the equations to check your predictions.

	Equation	My prediction	Solution	Was I correct?
A.	$a + 5 = 2a - 3$			
B.	$5a + 7 = 2a - 3$			
C.	$5a - 7 = 2a - 3$			
D.	$6 - 3a = 2a - 3$			
E.	$5a + 5 - 7a = 2a - 3$			

5) Six equations are given below.

a) Without solving the equations, predict which equations will NOT have the solution $t = 7$.

b) Solve each equation to check your predictions.

A. $8 - t + 1$
B. $-21 = -3t$
C. $-21 + 4 = -3t + 4$
D. $-21 - 9 = -3t + 9$
E. $42 = 6t$
F. $21 = -3t + 3$

Worksheet 3.10: Algebraic equations

Answers

Questions	Answers
1) Solve the equations using inverses. a) $5m - 6 = 3m + 2$ d) $5m + 6 = 3m - 2$ b) $6 - 5m = -2 - 3m$ e) $10m - 12 = 6m + 4$ c) $5m - 2 = 3m + 6$ f) $m + 1 - 2m + 2 = 3 + m + 4$	1) a) $m = 4$ d) $m = -4$ b) $m = 4$ e) $m = 4$ c) $m = 4$ f) $m = -3$
2) Consider the equation: $-9 + t = -2t + 3$ a) Read the statements A to E and decide if they are TRUE or FALSE. A. The value of t (i.e. the solution) is positive. B. The value of t will not change if we multiply each side of the equation by 3. C. The value of t will not change if we divide each side of the equation by 3. D. The value of t will not change if we multiply by 3 on the left side and divide by 3 on the right side of the original equation. E. The value of t will change if we add 9 to each side. b) Solve the equation. c) Now test any of the statements you are unsure of, i.e. perform the required operations and then solve the new equation.	2) a) A. True B. True C. True D. False E. False b) $t = 3$ c) $-9 + t = -3t + 3$ $(-9 + t) \times \mathbf{3} = (-3t + 3) \times \mathbf{3}$ $-27 - 27t = -9t + 9$ $12t = 36$ $t = -3$ [Testing B.]
3) Solve these algebraic equations using additive and multiplicative inverses. a) $d - 9 - 2d = 7$ b) $5p = -3 + p - 5$ c) $n - 3 = 7 - n$ d) $-s - 6 + 2s = 7 - s$ e) $3 - 2m = 2m + 3$ f) $k = 2k + k + 7$	3) a) $d = 16$ b) $p = -2$ c) $n = 5$ d) $s = \frac{13}{2}$ e) $m = 0$ f) $k = -\frac{7}{2}$

4) There are five equations in the table.

a) Predict whether the solution to each equation will be: a positive integer, or a negative integer or a fraction.

b) Solve the equations to check your predictions.

Answers

	Equation	My prediction	Solution	Was I correct?
A.	$a + 5 = 2a - 3$	$\boldsymbol{a = 8}$	$\boldsymbol{a = 8}$	Depends on learners' predictions
B.	$5a + 7 = 2a - 3$	$\boldsymbol{a = -\frac{10}{3}}$	$\boldsymbol{a = -\frac{10}{3}}$	
C.	$5a - 7 = 2a - 3$	$\boldsymbol{a = \frac{4}{3}}$	$\boldsymbol{a = \frac{4}{3}}$	
D.	$6 - 3a = 2a - 3$	$\boldsymbol{a = \frac{9}{5}}$	$\boldsymbol{a = \frac{9}{5}}$	
E.	$5a + 5 - 7a = 2a - 3$	$\boldsymbol{a = 2}$	$\boldsymbol{a = 2}$	

Questions	Answers
5) Six equations are given below. a) Without solving the equations, predict which equations will NOT have the solution $t = 7$. b) Solve each equation to check your predictions.	5) a) D; F b) A. $t = 7$ D. $t = \frac{37}{3}$ B. $t = 7$ E. $t = 7$ C. $t = 7$ F. $t = -6$

A. $8 - t + 1$	D. $-21 - 9 = -3t + 9$
B. $-21 = -3t$	E. $42 = 6t$
C. $-21 + 4 = -3t + 4$	F. $21 = -3t + 3$

Worksheet 3.11: Algebraic equations

This worksheet provides practice in solving equations with fractions, using inverses.

Questions

1) Copy and complete the following products.

a) $\frac{1}{2} \times 9 = \_\_$
b) $6 \times \frac{2}{3} = \_\_$
c) $\frac{3}{5} \cdot \square = 6$
d) $\frac{2}{5} \cdot \square = -4$
e) $\frac{3x}{4} \times \frac{1}{3} = \_\_$
f) $\frac{3x}{4} \times 4 = \_\_$
g) $\frac{3x}{4} \times \frac{4}{3} = \_\_$
h) $\frac{-x}{2} \times \square = x$

2) The following equations have a fraction as a coefficient. Solve the equations. Q2a is done for you.

a)
$$\frac{1}{2}a + 7 = 5$$
$$\frac{1}{2}a + 7 - 7 = 5$$
$$\frac{1}{2}a = -2$$
$$\frac{1}{2}a \times 2 = -2 \times 2$$
$$a = -4$$

b) $4 - \frac{2}{3}c = 7$
c) $5 = \frac{1}{4}d - 2$
d) $9 = 6 + \frac{5}{4}d$

3) The following equation has a fraction as a constant: $7x - \frac{2}{3} = 1 + 5x$
Read Siya's response. Then, answer the questions from 3a to 3e.

Siya's response

	$7x - \frac{2}{3} = 1 + 5x$
Step 1:	$7x - \frac{2}{3} - 5x = 1 + 5x - 5x$
Step 2:	$7x - 5x - \frac{2}{3} = 1$
Step 3:	$2x - \frac{2}{3} = 1$
Step 4:	$2x - \frac{2}{3} + \frac{2}{3} = 1 + \frac{2}{3}$
Step 5:	$2x = 1 + \frac{2}{3}$
Step 6:	$2x = \frac{1 \times 3 + 2}{3}$
Step 7:	$2x = \frac{5}{3}$
Step 8:	$2x \times 3 = \frac{5}{3} \times 3$
Step 9:	$6x = 5$
Step 10:	$x = \frac{5}{6}$

a) In Step 1, Siya used an additive inverse of ____.
b) In which step did Siya use a multiplicative inverse?
c) Explain what Siya did in Step 6.
d) Siya used the multiplicative inverse of ____ in Step 8.
e) Try to use a different way to solve this equation.
f) Compare the steps of your approach with Siya's steps.

4) Find the solutions to the following equations.

a) $5 - 3t + \frac{1}{2} = 7t$
b) $-9 = -\frac{3}{5}h$
c) $-27 + 4 = -3t + 14t$
d) $\frac{3}{4}m - 9 = -m + 9$
e) $42 + \frac{1}{5}g = 3g$
f) $4 = -9t + \frac{3}{6}$

Worksheet 3.11: Algebraic equations

Answers

Questions	Answers
1) Copy and complete the following products. a) $\frac{1}{2} \times 9 =$ ___ b) $6 \times \frac{2}{3} =$ ___ c) $\frac{3}{5} \cdot \square = 6$ d) $\frac{2}{5} \cdot \square = -4$ e) $\frac{3x}{4} \times \frac{1}{3} =$ ___ f) $\frac{3x}{4} \times 4 =$ ___ g) $\frac{3x}{4} \times \frac{4}{3} =$ ___ h) $\frac{-x}{2} \times \square = x$	1) a) $\frac{9}{2}$ b) 4 c) 10 d) -10 e) $\frac{x}{4}$ f) $3x$ g) x h) -2
2) The following equations have a fraction as a coefficient. Solve the equations. Q2a is done for you. a) $\frac{1}{2}a + 7 = 5$ $\frac{1}{2}a + 7 - 7 = 5$ $\frac{1}{2}a = -2$ $\frac{1}{2}a \times 2 = -2 \times 2$ $a = -4$ b) $4 - \frac{2}{3}c = 7$ c) $5 = \frac{1}{4}d - 2$ d) $9 = 6 + \frac{5}{4}d$	2) b) $-\frac{9}{2}$ c) $d = 28$ d) $d = \frac{12}{5}$
3) The following equation has a fraction as a constant: $7x - \frac{2}{3} = 1 + 5x$ Read Siya's response. Then, answer the questions from 3a to 3e. **Siya's response** $7x - \frac{2}{3} = 1 + 5x$ Step 1: $7x - \frac{2}{3} - 5x = 1 + 5x - 5x$ Step 2: $7x - 5x - \frac{2}{3} = 1$ Step 3: $2x - \frac{2}{3} = 1$ Step 4: $2x - \frac{2}{3} + \frac{2}{3} = 1 + \frac{2}{3}$ Step 5: $2x = 1 + \frac{2}{3}$ Step 6: $2x = \frac{1\times3+2}{3}$ Step 7: $2x = \frac{5}{3}$ Step 8: $2x \times 3 = \frac{5}{3} \times 3$ Step 9: $6x = 5$ Step 10: $x = \frac{5}{6}$ a) In Step 1, Siya used an additive inverse of ____. b) In which step did Siya use a multiplicative inverse? c) Explain what Siya did in Step 6. d) Siya used the multiplicative inverse of ____ in Step 8. e) Try to use a different way to solve this equation. f) Compare the steps of your approach with Siya's steps.	3) a) $5x$ b) Step 8 c) created a single fraction, changing1 to be $\frac{3}{3}$ d) $\frac{1}{3}$ e) $7x - \frac{2}{3} - 5x = 1 + 5x - 5x$ $7x - 5x - \frac{2}{3} = 1$ $2x - \frac{2}{3} = 1$ $2x - \frac{2}{3} + \frac{2}{3} = 1 + \frac{2}{3}$ $2x = 1 + \frac{2}{3}$ $2x = \frac{1\times3+2}{3}$ $2x = \frac{5}{3}$ $x = \frac{5}{3} \times \frac{1}{2}$ $x = \frac{5}{6}$ f) I used the multiplicative inverse of 2 instead of × 3 on each side (Step 9]
4) Find the solutions to the following equations. a) $5 - 3t + \frac{1}{2} = 7t$ b) $-9 = -\frac{3}{5}h$ c) $-27 + 4 = -3t + 14t$ d) $\frac{3}{4}m - 9 = -m + 9$ e) $42 + \frac{1}{5}g = 3g$ f) $4 = -9t + \frac{3}{6}$	4) a) $t = \frac{11}{20}$ b) $h = 15$ c) $t = -\frac{23}{11}$ d) $t = \frac{72}{7}$ e) $g = 15$ f) $t = -\frac{17}{18}$

Worksheet 3.12: Algebraic equations

This worksheet provides practice in solving equation using inverses. This will require dealing with the four arithmetic operations and brackets.

Questions

1) Solve following equations using inverses.

a) $\frac{9}{2}h + 5 = 27$

b) $5t - 7 = -27 + 7t$

c) $1 - \frac{3}{2}m = m - 9$

d) $y + \frac{7}{6} = -y - \frac{7}{6}$

e) $b - 11 = 13$

f) $4 - 3n + 9 + 5n = 21$

2) This question focuses on the distributive law. Complete the table. Q2a is done for you.

	Expression	Using distributive law	Simplification
a)	$2(a + 7)$	$2 \times a + 2 \times 7$	$2a + 14$
b)	$2(a - 7)$		
c)		$2 \times (-a) + 2 \times 7$	
d)			$-2a - 14$

3) Seven equations are given below.

a) Without solving the equations, predict which equations will have the same solution as $3(s - 2) = 9$.

b) Solve each equation to check your predictions.

A. $3s - 2 = 9$

B. $3s - 6 = 9$

C. $3s - 6 = 27$

D. $9 = 3(s - 2)$

E. $9 = 2(s - 3)$

F. $(s - 2)3 = 9$

G. $s - 3 = \frac{9}{6}$

4) Six equations are given below.

a) Without solving the equations, predict which equations DO NOT have the same solution as $-3(s - 2) = 9$.

b) Solve each equation to check your predictions.

A. $-3s - 2 = 9$

B. $-3s + 2 = 9$

C. $-3s + 6 = 9$

D. $-3s - 6 = 9$

E. $-9 = 3(s - 2)$

F. $3(s - 2) = -27$

G. $s - 2 = -3$

5) Solve the following equations using inverses. Q5a has been done for you.

a) $13 = 2(k - 4) + 5$

$13 = 2 \times k - 2 \times 4 + 5$

$13 = 2k - 8 + 5$

$13 = 2k - 3$

$13 + 3 = 2k$

$16 = 2k$

$8 = k$ or $k = 8$

b) $5(-6 - j) = 2$

c) $2z - 3(2) = 5$

d) $2(n - 3) = 5(4)$

e) $-4(f - 3) = 7$

6) Lee solved the equation: $-3(2 - n) = \frac{3}{4}$

Her response is given alongside.

Spot the error(s) and write the correct solution.

$-3(2 - n) = \frac{3}{4}$

Step 1: $-3 \times 2 - 3 \times n = \frac{3}{4}$

Step 2: $-6 - 3n = \frac{3}{4}$

Step 3: $-3n = \frac{3}{4} + 6$

Step 4: $-3n = \frac{3+6}{4}$

Step 5: $-3n = \frac{9}{4}$

Step 6: $n = \frac{9}{4} \times \frac{1}{3}$

Step 7: $n = \frac{3}{4}$

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Worksheet 3.12: Algebraic equations

Answers

Questions

1) Solve following equations using inverses.

a) $\frac{9}{2}h + 5 = 27$
b) $5t - 7 = -27 + 7t$
c) $1 - \frac{3}{2}m = m - 9$
d) $y + \frac{7}{6} = -y - \frac{7}{6}$
e) $b - 11 = 13$
f) $4 - 3n + 9 + 5n = 21$

Answers

1)
a) $h = \frac{44}{9}$
b) $t = 10$
c) $m = 4$
d) $y = -\frac{7}{6}$
e) $b = 24$
f) $n = 4$

2) This question focuses on the distributive law. Complete the table. Q2a is done for you.

Answers

	Expression	Using distributive law	Simplification
a)	$2(a + 7)$	$2 \times a + 2 \times 7$	$2a + 14$
b)	$2(a - 7)$	$\boldsymbol{2 \times a - 2 \times 7}$	$\boldsymbol{2a - 14}$
c)	$\boldsymbol{2(-a + 7)}$	$2 \times (-a) + 2 \times 7$	$\boldsymbol{-2a + 14}$
d)	$\boldsymbol{-2(a + 7)}$	$\boldsymbol{-2 \times a - 2 \times 7}$	$-2a - 14$

3) Seven equations are given below.

a) Without solving the equations, predict which equations will have the same solution as $3(s - 2) = 9$.
b) Solve each equation to check your predictions.

A. $3s - 2 = 9$
B. $3s - 6 = 9$
C. $3s - 6 = 27$
D. $9 = 3(s - 2)$
E. $9 = 2(s - 3)$
F. $(s - 2)3 = 9$
G. $s - 3 = \frac{9}{6}$

3)
a) B; D; F
b)
$s = 5$ for B; D and F
A. $s = \frac{11}{3}$
C. $s = 11$
E. $s = \frac{15}{2}$
G. $s = \frac{9}{2}$

4) Six equations are given below.

a) Without solving the equations, predict which equations DO NOT have the same solution as
$-3(s - 2) = 9$.
b) Solve each equation to check your predictions.

A. $-3s - 2 = 9$
B. $-3s + 2 = 9$
C. $-3s + 6 = 9$
D. $-3s - 6 = 9$
E. $-9 = 3(s - 2)$
F. $3(s - 2) = -27$
G. $s - 2 = -3$

4)
a) A; B; D; F
b) $s = -1$ for C; E; G
A. $-\frac{11}{3}$
B. $-\frac{7}{3}$
D. $s = -5$
F. $s = -7$

5) Solve the following equations using inverses. Q5a has been done for you.

a) $13 = 2(k - 4) + 5$
$13 = 2 \times k - 2 \times 4 + 5$
$13 = 2k - 8 + 5$
$13 = 2k - 3$
$13 + 3 = 2k$
$16 = 2k$
$8 = k$ or $k = 8$

b) $5(-6 - j) = 2$
c) $2z - 3(2) = 5$
d) $2(n - 3) = 5(4)$
e) $-4(f - 3) = 7$

6) Lee solved the equation: $-3(2 - n) = \frac{3}{4}$

Her response is given alongside.
Spot the error(s) and write the correct solution.

$-3(2 - n) = \frac{3}{4}$
Step 1: $-3 \times 2 - 3 \times n = \frac{3}{4}$
Step 2: $-6 - 3n = \frac{3}{4}$
Step 3: $-3n = \frac{3}{4} + 6$
Step 4: $-3n = \frac{3+6}{4}$
Step 5: $-3n = \frac{9}{4}$
Step 6: $n = \frac{9}{4} \times \frac{1}{3}$
Step 7: $n = \frac{3}{4}$

5)
b) $j = -\frac{32}{5}$
c) $z = \frac{11}{2}$
d) $n = 13$
e) $f = \frac{5}{4}$

6) See circled errors
Correction:
Step 4: $-3n = \frac{3+24}{4}$
Step 5: $-12n = 27$
Step 6: $n = -\frac{27}{12}$
Lee's Step 6 from $-3n = \frac{9}{4}$ should have been: $\times -\frac{1}{3}$

Worksheet 3.13: Algebraic equations

This worksheet provides practice in solving equations using inverses. In the last question, you will need to create the equations from verbal expressions.

Questions

1) Solve following equations using inverses.
   a) $h + 8 = 15$
   b) $h + 15 = 8$
   c) $h + 15 = 8h$
   d) $8m - 11 = 10 + m$
   e) $2b - 8b = 3(5 - 3)$
   f) $2b - 8 = 3(b - 3)$
   g) $6s + \frac{2}{3} = 3 - s$
   h) $y + \frac{7}{6} - 3y = 1 + \frac{7}{6}$
   i) $\frac{1}{2}(2x - 4) = 7$
   j) $\frac{1}{2}(2x + 4) = 7x$

2) Check whether each of the given values is the solution of the equation.

	Equation	Value	Is it the solution: Yes/ No?	Evidence
a)	$v - 7 = 0$	$v = -7$		
b)	$2(a - 5) = -5$	$a = -\frac{5}{2}$		
c)	$9m - 7 = 2m - 7$	$m = \frac{2}{9}$		
d)	$\frac{b}{3} - 1 = 1$	$b = 6$		

3) Look at the pairs of equations.
   a) Predict whether the solution to the two equations will be the same for each pair. Give reasons for your answers.
   b) Solve the equations and check your predictions.

	Equation 1	Equation 2	Same solution?	Reason
A.	$3f + 7 = 11$	$3f + 10 = 14$		
B.	$g - 8 = 15 - g$	$g - 8 = -15 - g$		
C.	$g + 8 = 15 - g$	$2g + 16 = 30 - 2g$		
D.	$2p - 3 = 3(p - 7)$	$\frac{1}{3}(2p - 3) = p - 7$		
E.	$\frac{20b}{3} - 7 = 49$	$\frac{20b}{3} - 7 - b = 49 - b$		

4) Write equations for the following statements. Solve the equations after forming them.
   a) The sum of 5 and a number is -1.
   b) The product of a number and -7 is 140.
   c) A number decreased by 8 is 11.
   d) A number divided by 5 gives 10 times 7.
   e) Three times a number added to 7 gives 19.
   f) Seven times a number added to 4 gives -15.
   g) The sum of one-third of a number and 6 gives 10.
   h) Three-quarters of a number gives 5.

Worksheet 3.13: Algebraic equations

Answers

Questions and answers

1) Solve following equations using inverses.

a) $h + 8 = 15$
b) $h + 15 = 8$
c) $h + 15 = 8h$
d) $8m - 11 = 10 + m$
e) $2b - 8b = 3(5 - 3)$
f) $2b - 8 = 3(b - 3)$
g) $6s + \frac{2}{3} = 3 - s$
h) $y + \frac{7}{6} - 3y = 1 + \frac{7}{6}$
i) $\frac{1}{2}(2x - 4) = 7$
j) $\frac{1}{2}(2x + 4) = 7$

1) **Answers**

a) $h = 7$
b) $h = -7$
c) $h = \frac{15}{7}$
d) $m = 3$
e) $b = -1$
f) $b = 1$
g) $s = \frac{1}{3}$
h) $y = -\frac{1}{2}$
i) $x = 9$
j) $x = 5$

2) Check whether each of the given values is the solution of the equation.

Answers

	Equation	Value	Is it the solution: Yes/ No?	Evidence
a)	$v - 7 = 0$	$v = -7$	**No**	$\boldsymbol{v = 7}$
b)	$2(a - 5) = -5$	$a = -\frac{5}{2}$	**No**	$\boldsymbol{a = \frac{5}{2}}$
c)	$9m - 7 = 2m - 7$	$m = \frac{2}{9}$	**No**	$\boldsymbol{m = 0}$
d)	$\frac{b}{3} - 1 = 1$	$b = 6$	**Yes**	$\boldsymbol{b - 3 = 3}$ $\boldsymbol{b = 6}$

3) Look at the pairs of equations.

a) Predict whether the solution to the two equations will be the same for each pair. Give reasons for your answers.
b) Solve the equations and check your predictions.

Answers

	Equation 1	Equation 2	Same solution?	Reason
A.	$3f + 7 = 11$	$3f + 10 = 14$	Yes	$11 - 7$ and $14 - 10$ are both 4
B.	$g - 8 = 15 - g$	$g - 8 = -15 - g$	No	15 is negative in equation 2
C.	$g + 8 = 15 - g$	$2g + 16 = 30 - 2g$	Yes	Each term in equation 1 is multiplied by 2 to get corresponding term in equation 2
D.	$2p - 3 = 3(p - 7)$	$\frac{1}{3}(2p - 3) = p - 7$	Yes	If you multiply both sides of equation 2 by 3, then you get equation 1.
E.	$\frac{20b}{3} - 7 = 49$	$\frac{20b}{3} - 7 - b = 49 - b$	Yes	Subtracted b on each side of equation 2

4) Write equations for the following statements. Solve the equations after forming them.

a) The sum of 5 and a number is -1.
b) The product of a number and -7 is 140.
c) A number decreased by 8 is 11.
d) A number divided by 5 gives 10 times 7.
e) Three times a number added to 7 gives 19.
f) Seven times a number added to 4 gives -15.
g) The sum of one-third of a number and 6 gives 10.
h) Three-quarters of a number gives 5.

4) **Answers**

	Equation	Solution
a)	$5 + x = -1$	$x = -6$
b)	$-7y = 140$	$y = -20$
c)	$p - 8 = 11$	$p = 19$
d)	$\frac{a}{5} = 10 \times 7$	$a = 350$
e)	$3x + 7 = 19$	$x = 4$
f)	$7y + 4 = -15$	$y = -\frac{19}{7}$
g)	$\frac{b}{3} + 6 = 10$	$b = 4$
h)	$\frac{3x}{4} = 5$	$x = \frac{120}{3}$

Worksheet 3.14: Algebraic equations

This worksheet provides practice in solving equations using inverses. The equations have variables on one side or on both sides. There are examples with brackets and fractions too.

Questions

1) Solve following equations using inverses.
   a) $5h + 7 = 33$
   b) $-2(z - 11) = 7z - 41$
   c) $\frac{1}{3}(b + \frac{9}{2}) = \frac{18}{2}$
   d) $x + 2(1 - 3x) = 3 \times 9$
   e) $k(7 - 3) = 24$
   f) $\frac{1}{2}(6x - 8) = 7x - 25$

2) Check whether the given values are the solution of the equation.

	Equation	Value	Is it the solution: Yes/ No?	Reasons
a)	$6 - 2a = 0$	$a = -3$		
b)	$\frac{1}{2}\left(m - \frac{1}{2}\right) = -5$	$a = \frac{21}{2}$		
c)	$m - 7m = -\frac{66}{11}$	$m = -3$		
d)	$\frac{2f}{3} - 9 = 11$	$f = 19$		

3) Write equations for the following statements. Solve the equations after forming them.
   a) Two times a number subtract 9 gives -1.
   b) Two times a number subtracted from 9 gives -1.
   c) The sum of a number and one-quarter gives 2.
   d) The product of a number and one-quarter gives -2.
   e) The difference between 2 and a number gives -21.
   f) The sum of a number and 5 is the same as the product of the number and 6.
   g) A number subtract 5 is the same as the product of the number and 6.
   h) Ten subtract a number is the same as the number decreased by 10.

4) Which of the equations have the same solution?
   Try to do this without solving the equations. Then check your predictions by solving the equations.
   a) $a - (5 - a) = 5$
   b) $-a + (a - 5) = -5$
   c) $2a - (5 - a) = 10$
   d) $2a - 2(5 - a) = 10$
   e) $a - 5 + a - 5 = 2(5 - a)$

Worksheet 3.14: Algebraic equations

Answers

Questions	Answers
1) Solve following equations using inverses. a) $5h + 7 = 33$ b) $-2(z - 11) = 7z - 41$ c) $\frac{1}{3}(b + \frac{9}{2}) = \frac{18}{2}$ d) $x + 2(1 - 3x) = 3 \times 9$ e) $k(7 - 3) = 24$ f) $\frac{1}{2}(6x - 8) = 7x - 25$	1) a) $h = \frac{26}{5}$ b) $z = 7$ c) $b = \frac{45}{2}$ d) $x = -\frac{16}{5}$ e) $k = 6$ f) $x = \frac{21}{4}$

2) Check whether the given values are the solution of the equation.

	Equation	Value	Is it the solution: Yes/ No?	Reasons
a)	$6 - 2a = 0$	$a = -3$	No	$a = 3$
b)	$\frac{1}{2}\left(m - \frac{1}{2}\right) = -5$	$a = \frac{21}{2}$	No	$m = -\frac{19}{2}$
c)	$m - 7m = -\frac{66}{11}$	$m = -3$	No	$m = 1$
d)	$\frac{2f}{3} - 9 = 11$	$f = 19$	Yes	$2f - 27 = 11$ $2f = 38$ $f = 19$

3) Write equations for the following statements. Solve the equations after forming them.

a) Two times a number subtract 9 gives -1.
b) Two times a number subtracted from 9 gives -1.
c) The sum of a number and one-quarter gives 2.
d) The product of a number and one-quarter gives -2.
e) The difference between 2 and a number gives -21.
f) The sum of a number and 5 is the same as the product of the number and 6.
g) A number subtract 5 is the same as the product of the number and 6.
h) Ten subtract a number is the same as the number decreased by 10.

3)

	Equation	Solution
a)	$2x - 9 = -1$	$x = 4$
b)	$9 - 2x = -1$	$x = 5$
c)	$y + \frac{1}{4} = 2$	$y = \frac{7}{4}$
d)	$a \times \frac{1}{4} = -2$	$a = -8$
e)	$2 - x = -21$ or $x - 2 = -21$	$x = 23$ or $x = -19$
f)	$x + 5 = 6x$	$x = 1$
g)	$x - 5 = 6x$	$x = -1$
h)	$10 - x = x - 10$	$x = 10$

Questions	Answers
4) Which of the equations have the same solution? Try to do this without solving the equations. Then check your predictions by solving the equations. a) $a - (5 - a) = 5$ b) $-a + 2(a - 5) = -5$ c) $2a + (5 - a) = 20$ d) $2a - 2(5 - a) = 10$ e) $a - 5 + a - 5 = 2(5 - a)$	4) a; b; d; e have the same solution a) $a = 5$ b) $a = 5$ c) $a = 15$ d) $a = 5$ e) $a = 5$