

VERSION 1.0







#equali = ymatters: Practice in working with linear equations

These materials were produced by the Wits Maths Connect Secondary (WMCS) project at the University of the Witwatersrand.

Visit us at www.witsmathsconnectsecondary.co.za

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#equali practice in working with linear equations version 1.0

About this booklet

This booklet contains 38 worksheets on linear equations, together with answers. These materials differ from our other materials in that we provide a pedagogical approach for the first-time teaching of equations. We do not assume that learners have already been taught equations. However, there are still many practice examples particularly in section 3.

We begin with numerical equations involving whole numbers only, and focus on the ideas of equality and balance to give learners a sense of solving equations without the distractions of algebraic notation. In these worksheets we use a place holder (\Box) and a space (____) to represent the unknown quantity. Many of the worksheets are suitable for learners in Grade 7, maybe even Grade 6 learners.

We then move on to integer equations where we continue to use \Box and $__$ to represent the unknown. Here we reinforce the ideas introduced with whole number equations and extend the

number range to integers. This helps learners to get more comfortable in working with negatives and subtraction without the complications of algebraic notation. We also push learner to pay attention to the *structure* of equations by posing questions about sets of numerical and algebraic equations that have minor variations.

If $9 - (-3) + (-2) = \Box + 2 + 4$, does \Box have the same value in these 3 equations?
$9 - (-3) + (-2) - 12 = \Box + 2 + 4 - 12$
$9 - (-3) + (-2) + 40 - 40 = \Box + 2 + 4$
$32 + 9 - (-3) + (-2) = \Box + 2 + 4 - 32$

There are 14 worksheets that deal with algebraic equations and provide practice for learners. In these worksheets we emphasise the shift from equations with the variable on one side only, e.g. 2x - 5 = 7, to equations with the variable on both sides, e.g. 2x - 5 = 7 - x. Local and international research shows that this shift is a substantial obstacle for learners to overcome, partly because it requires them to make use of inverses and inverse operations to solve equations. In addition, our research shows that learners' difficulties with linear equations frequently stem from their difficulties in simplifying algebraic expressions.

In addition to the sets of worksheets, we provide 12 pages of notes on the key ideas for solving equations as well as a section on what makes equations difficult for learners. For those who wish to jump straight to algebraic equations, the first two worksheets in Section 3 provide a recap of integers and algebraic simplification respectively.

Section	Content	No. of worksheets
	Notes and key ideas for solving equations	
1	Numerical equations with whole numbers	13
2	Numerical equations with integers	11
3	Algebraic equations	14

NOTES ON LINEAR EQUATIONS

In these notes we explain important concepts, terminology and procedures for solving linear equations. We also provide lots of practice in the form of worksheets. We have written these notes in simple language for Grade 8 and 9 learners.

The topic of *Equations* is a very important section of mathematics. We use equations in all topics of mathematics. For example, we may need to solve an equation to work out the sizes of angles in a triangle or the lengths of sides of a triangle. Linear equations lay the foundation for other kinds of equations in Grade 9 and beyond. We need to solve a linear equation to find the simple interest rate in financial maths, and we can use an equation to find the number of terms in a number pattern that were added to get a particular total. At the end of the notes we discuss some reasons why learners may have difficulty with linear equations.

1. What is an equation

An equation tells us about the numeric relationship between quantities. For example, say we have two boxes of balls and we know that there are 10 more balls in the one box than in the other. Let's use x to indicate the number of balls in the first box. Then there are x + 10balls in the second box. To find the total number of balls we can make an equation: Total = x + (x + 10), which gives: Total = 2x + 10. If we know that there are 46 balls altogether, then we can say 2x + 10 = 46. When we solve this equation, we get x = 18. So we know there are 18 balls in one box and 28 balls in the other.

Before we define an equation in more detail, we need to introduce two other terms: *expression* and *statement*.

- An **expression** consists of terms that are combined using addition, subtraction, multiplication and division. Numeric expressions: 7-5; $100 \div 5+1$; $6 \times 10-2+5$ Algebraic expressions: 7x-5; $\frac{100p}{3}+1$; $7x^2+3x-x-2$
- A statement compares two expressions using relationship signs such as equal to (=), not equal to (≠), greater than (>), less than (<), less than or equal to (≤), etc. When we deal with equations, we use the equal sign.

Numeric statements:7 = 5 + 2; $100 \div 5 < 25;$ $-7 < 0; 6 \times 10 - 2 + 5 = 63$ Algebraic statements:7x - 5 = 9; $\frac{100p}{3} + 1 \ge 0;$ $7x^2 + 3x - x - 2 \le 6$

- A statement is **balanced** if the result on each side of the equal sign is the same. We use *result* in these materials to indicate the answer we get when we evaluate the expression on each side of the equal sign.
- A **balanced statement** is called an **equation**. So an equation consists of two expressions that produce the same result.

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For numeric equations we will use a place holder (\Box)
or "space" (___) for the unknown value.
Examples:
3 + \_ = 7 - 2 We refer to the
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 $\begin{array}{l} 3 + _ = 7 - 2 \\ \Box - 4 = 9 - 2 \\ 13 - 9 = _ + 4 \times 2 \\ 7 - \Box = 6 + 8 \end{array}$ We refer to the place holder, \Box , as "box".

For algebraic equations, we will use letters for the unknown value. Examples:

$$3 + x = 7 - 2a - 4 = 9 - 213p - 9 = p + 4 \times 27 - 2k = 6 + 8k2(n - 7) = 10$$

• When there are no unknowns (such as letters or place holders) in a statement, it is easy to see if the statement is balanced. For example, 4 + 3 - 1 = 4 + 2 is balanced because the result on the left of the equal sign is 6 and the result on the right of the equal sign is also 6.







But the statement $5 \times 2 - 7 = 5 \times 7 - 2$ is not balanced because the result on the left is 3 and the result on the right is 33. So we write $5 \times 2 - 7 \neq 5 \times 7 - 2$.

• When there are unknowns in a statement, we need to find the value/s that will make the statement balanced. This is called *solving an equation*. The value that makes the statement balanced is called the *solution*.

Consider the example, 4x + 3 = 11. This statement will be balanced if we can find a value for x that will give a result of 11 on the left side. It should be easy to see that x must equal 2. So the solution to the equation 4x + 3 = 11 is x = 2.

A statement that is balanced has the same result on both sides of the equal sign. So, a *balanced statement* is an equation.

Consider another example, 2x - 1 = x + 3. This is more difficult to solve without a special procedure. The solution is x = 4. If we substitute x = 4 on the left, we get 2(4) - 1 = 7. If we substitute x = 4 on the right, we get 4 + 3 = 7. So the statement is balanced when x = 4. It is not balanced when x = 0 or x = 5 or x = -2 or any other number you choose. You can check by substituting these values in both sides.

We have chosen to use *balanced* and *not balanced* rather than *true* or *false* when dealing with equations. This helps us to remember that working with equations is always about maintaining balance. So we won't speak about a *true statement* or a *false statement* but rather a statement that is balanced or not balanced.

	Word problem 1	Word problem 2
Verbal statement	I am thinking of a number. If you add 3 to my number, you get 10.	I am thinking of a number. If you subtract 3 from my number and then double it, you get 20.
Convert verbal statement into a numeric statement or an algebraic statement:	$\Box + 3 = 10$ $n + 3 = 10$	$2(\Box - 3) = 20$ 2(n - 3) = 20
Solution	We can work out in our heads that $n = 7$. So the number I am thinking of is 7.	You may be able to see that $n = 13$. You can check this by substituting 13 into the left side to confirm that the result is 20.

Equations are essential for solving many problems. Here are two simple word problems:

2. How do we work with equations?

The most typical action to perform on an equation is to *solve* it, i.e. to find the solution. This applies to both numeric equations and algebraic equations. In this section we explain briefly the basics of solving equations using inspection and inverses. In section 3.5 we give more detailed explanations of solving equations using inverses.

2.1 Solving equations using inspection

When we solve equations using inspection, we do most of the work in our heads. Here are three examples:

$4 \times \Box = 11 - 3$	$2(\Box - 3) = 20$	4x + 3 = 11
The result on the right side of	We can reason that $2 \times 10 = 20$.	We can reason: $11 - 3 = 8$. Then $8 \div 4 = 2$.
the equal sign is 8. This	This means that the bracket must	So $x = 2$.
means we must get 8 on the	have a value of 10 . Which means	The general approach is that you do the
left side too. So "4 times	something subtract 3 equals 10. This	"opposite operation". On the left of the equal
something must equal 8". So	means $\Box = 13$. Note this is the same	sign, we were adding 3 so we must subtract 3.
$\Box = 2.$	as word problem 2 above.	We were multiplying by 4 so we must divide
		by 4.



Although inspection is a powerful method, it is only useful when the unknown is on one side of the equal sign. It is essential that we know how to solve equations using inverse operations.

2.2 Solving equations using inverses

The goal in solving equations using inverses is to collect the terms with variables on one side of the equal sign and the terms with constants on the other side. It does not matter which side you choose. We illustrate the method with brief explanations. We explain these processes in more detail in section 3.5.

Example 1: Solve for m : $m + 5 = 7$	
We choose to collect all the constants on the right side	
Add -5 to both sides so that the constants on the left of the equal sign sum	m + 5 + (-5) = 7 + (-5)
to zero	m + 5 - 5 = 7 - 5
The solution is 2.	m = 2
We can see that this is the correct solution by using inspection	
Example 2 : Solve for y : $3y - 5 = y + 8$	
We choose to collect terms with letters on the left and constants on the right.	
Add $-y$ to both sides so that terms with variables sum to zero on right side	3y - 5 + (-y) = y + 8 + (-y)
	2y - 5 = 8
Add 5 to both sides so that constant terms sum to zero on the left side	2y - 5 + 5 = 8 + 5
We get $2y = 13$	2y = 13
	$2v \times \frac{1}{2} = 13 \times \frac{1}{2}$
We want to determine the value of y , not $2y$, therefore we multiply both	
sides by the multiplicative inverse of 2. The solution is $\frac{13}{2}$	$y = \frac{1}{2}$ or $6\frac{1}{2}$
2	

In example 1 we added negative 5. We could also have said "subtract 5" because "adding a negative number" is the same as "subtracting a positive number". In example 2, we multiplied by the multiplicative inverse of 2 which is $\frac{1}{2}$. We could also have said "divide by 2" because "dividing by 2" is the same as "multiplying by $\frac{1}{2}$ ".

3. Important knowledge for understanding equations

There are seven important ideas that need to be understood well in order to solve linear equations successfully. There are: 1) seeing the equal sign as an equivalence, i.e. "is the same as"; 2) the idea of *balancing* an equation; 3) paying attention to structure; 4) commutative and associative properties; 5) inverses and inverse operations; 6) the meaning of a solution; and 7) the distributive law. We discuss each of these in more detail below. At the end of this section we provide examples of different ways to use inverse operations to solve one equation.

3.1 Seeing the equal sign as equivalence, i.e. "is the same as"

When we first encounter the equal sign in primary school, we treat it as "gives me", e.g. $4 + 5 = \Box$. Here we say "4 add 5 *gives me* 9". But when we have a statement with at least two terms on each side, e.g. $4 + 5 = 3 + \Box$, we need to reason as follows: "4 add 5 *is the same as* 3 add something". The left side of the equal sign adds to 9 so the right side must also add to 9. This means the place holder (\Box) must have a value of 6. So we have 4 + 5 = 3 + 6 and we say "4 add 5 *is the same as* 3 add 6". The statement is balanced because we get the same result on both sides of the equal sign. We say the left side is equivalent to the right side which means that the sides have the same result but they don't look the same.



Here is another example: $4 + 5 = \Box - 2$. Once again, we must treat the equal sign as "is the same as". So we say "4 add 5 is the same as a number subtract 2". If the left side adds to 9, then the right side must also give a result of 9. This means the place holder must have a value of 11. So we have 4 + 5 = 11 - 2 and we say "4 add 5 *is the same as* 11 subtract 2". We can also write this as an equation using a letter such as p: 4 + 5 = p - 2. If we solve the equation, we will get p = 11. Here are three more numeric examples where we need treat the equal sign as "is the same as". Can you work out the value of \Box ?

 $3 + 4 = \Box - 1$ $2 + 7 \times 3 = \Box \times 2 - 1$ $80 \div 4 + 15 \times 2 = \Box \times 100$

3.2 The idea of balance in working with equations

The idea of balance is very important when we work with equations. Some teachers use the idea of balancing a scale and they might say things like "what you do on the left, you must do on the right". This can be a helpful reminder. Consider the statement, 4 + 5 = 3 + 6. This is an equation because the result on the left is 9 and the result on the right is 9. If we subtract 2 from both sides we get: 4 + 5 - 2 = 3 + 6 - 2. This will give a result of 7 on both sides. If we add 8 to both sides of the original equation, we will have 4 + 5 + 8 = 3 + 6 + 8. The result will now be 17 on both sides. But if we add 8 to left side and subtract 8 from the right side we get 4 + 5 + 8 = 3 + 6 - 8. Now the result on the left will be 17 but the result on the right will be 1. So the equation is no longer balanced, and we must write $4 + 5 + 8 \neq 3 + 6 - 8$ to show that the left side is not equal to the right side.

There is another way to maintain balance in an equation. Consider the example: 4 + 5 + 8 - 8 = 3 + 6. Notice that we started with a balanced statement, then we added 8 and then subtracted 8 which means we have "added zero". This maintains the balance of the equation.

Now consider a numeric equation with a box: $4 + 6 = \Box - 2$. The left side gives a result of 10 so we must put 12 in the box so that the right side to give a result of 10. If we add the same number to both sides, or subtract the same number from both sides, the numerical equation will still be balanced. But if we perform different operations on each side (or use different numbers with the same operation), the numeric equation will no longer be balanced. Here are three examples to illustrate these cases (LHS = left hand side, RHS = right hand side):

Original equation:	$4+6=\Box-2$	
Add 5 to both sides:	Subtract 3 from both sides:	Add 10 on left, subtract 10 on right:
$4 + 6 = \Box - 2$ $4 + 6 + 5 = \Box - 2 + 5$	$4 + 6 = \Box - 2$ $4 + 6 - 3 = \Box - 2 - 3$	$4 + 6 = \Box - 2$ $4 + 6 + 10 = \Box - 2 - 10$
LHS: $4 + 6 + 5 = 15$ RHS: $12 - 2 + 5 = 15$	LHS: $4 + 6 - 3 = 7$ RHS: $12 - 2 - 3 = 7$	LHS: $4 + 6 + 10 = 20$ RHS: $12 - 2 - 12 = 0$ The equation is no longer balanced.

Note that we don't need to work out the value of \Box before we make changes to the original equation. If we know that the shaded parts are the same $4+6+5 = \Box - 2 + 5$, when we add 5 to both sides, we maintain the balance. The same applies if we subtract 3 from both sides.



It works in the same way when we work with algebraic equations. Consider the example: x - 4 = 7 + 5. It is easy to work out that x must have a value of 16 to balance the equation. Here are four examples where we change the equation but still keep it balanced. Note that the parts of the original equation are shaded.

Add 1 to both sides:	x - 4 + 1 = 7 + 5 + 1	We call these equivalent equations because
Subtract 6 from both sides:	x - 4 - 6 = 7 + 5 - 6	they all come from the same original equation
Multiply by 2 on both sides:	2(x-4) = 2(7+5)	equation would be $x - 4 = 12$ or even $x = 16$.
Add 6, then subtract 6 from the same side:	x - 4 = 7 + 5 + 6 - 6	•

When we solve equations, each line must be an *equivalent equation* that maintains balance. The phrase *equivalent equation* means that the equations don't look the same but the expressions on each side have the same value. This will happen if we have performed the same arithmetic operation on each side, or if we have manipulated the terms in the same way. We say the left side of the equal sign is equivalent to the right side which means that the sides have the same result if we substitute a value for the variable. See section 3.8.

Assume you are given the equation: x - 3 = 12. The five equations in column A are equivalent to this equation, but the five equations in column B are not. Can you see why? There are some hints to help you.

A: 5 equations that are equations that are equations that are equations that are equated as the second seco	uivalent to $x - 3 = 12$	12 B: 5 equations that are NOT equivalent to $x - 3 = 12$	
x - 4 = 11	subtract from both sides	x - 4 = 10	subtract on LHS, subtract on RHS
x + 10 = 25		x + 10 = 19	
2(x-3) = 24	multiply both sides by	2x - 3 = 24	didn't multiply by 2
x + 9 - 9 = 12 + 3		x + 9 = 12 + 3	3 – 9
-x + 3 = -12	multiply both sides by -1	3 - x = 12	

3.3 Paying attention to the structure of equations

In these materials you will also find questions with *sets of equations* (and sets of statements) that focus on the idea of balance. In such questions you are expected to pay attention to the *structure* of the equations, i.e. look at the expressions on each side of the equal sign and focus on what is the same and what is different. Here is a set of statements that all have 12 - 3 on the left side and 4 + 5 on the right of the equal sign. Now we make some changes to each side and then ask you to respond, depending on the question:

- A. What value must we put in the box to keep the statement balanced: $12 3 + 7 = 4 + 5 + \Box$
- B. What value must we put in the box to keep the statement balanced: $12 3 + \Box = 4 + 5 5$
- C. Is this statement balanced: 12 3 6 = 4 + 5 6
- D. Is this statement balanced: 12 3 6 + 6 = 4 + 5

In A and B above, you should have noticed that we have added/subtracted a number on one side of the equal sign. To maintain balance, we must perform the same operation with the same number on the other side. In D, notice that we have "added zero" on the left side of the equal sign and we have not made any changes on the right side.

There are also some more challenging examples like the ones below. Here too you need to focus on the structure. For example, look at the numbers, symbols and operations on each side of the equal sign, and look for repetition of these groups of number/operations, etc.



You are told that $\Box + \odot = 13$,

d) Is this statement balanced:

e) Is this statement balanced:

a) What is the value of:

c)

 $\Box + \odot + 1 = \cdots$ (Hint: what did we add on the left side?)

- b) Is this statement TRUE or FALSE: $\Box + \odot - 5 = 13 - 5$
 - $\Box + \odot 5 = 8$
 - Is this statement TRUE or FALSE: $\Box + \odot + 25 - 20 = 13$
 - $\Box + \odot + m = 13 + m$

Note: We don't know the individual values of \Box or \odot . Of course, we can make up some combinations like $\Box = 10$ and $\odot = 3$.

3.4 Commutative and associative properties

The commutative property and associative property are helpful in solving equations because they allow us to change the appearance of the equation, without changing the relationships between the components. We will first revisit these properties for whole numbers and integers. Then we will use them to manipulate expressions in algebraic equations.

Adding and subtracting whole numbers 3.4.1

The commutative property for whole numbers states that when we add two numbers, the order does not matter, e.g. 4 + 5 = 5 + 4. When we add more than two whole numbers, we apply the associative property.

This means we can change their order, for example, 4 + 5 + 6 = 4 + 6 + 5 = 5 + 4 + 6. We often do this when we want to combine some terms, like we know that 4 + 6 = 10, so we want to add them first.

But these properties do not apply to subtraction. For example, $5-4 \neq 4-5$ (the result on the left is 1, while the result on the right is -1).

3.4.2 Adding and subtracting integers

When we add and subtract integers, it is easy to get confused with the extra symbols in the expressions. If we add 5 and -2, we write 5 + (-2). Using the commutative property, we can write: 5 + (-2) = (-2) + 5. If we see this as an equation, then both sides give the same result, i.e. 3. If we have -7 + 3, this can be rewritten as 3 + (-7) using the commutative property.

When we subtract 5 and -2, the commutative property will not hold. For example, compare 5 - (-2) and (-2) - 5. The left side gives a result of 7 but the result on the right side is -7.

Although the commutative property doesn't hold for subtraction, we can convert a subtraction operation into the addition of a negative number. Then the commutative property will hold. For example, 4-6 can be rewritten as 4 + (-6). Now this sum can be rewritten as -6 + 4 because addition is commutative. It is important to note that we will still get the same result (of -2) whether we use 4 - 6 or 4 + (-6) or -6 + 4.

Here is an example using the associative property for adding integers: (-6) + 5 + (-3) and 5 + (-6) + (-3):

(-6) + 5 + (-3) can be written as: -6 + 5 - 3 = -4

5 + (-6) + (-3) can be written as: 5 - 6 - 3 = -4

This shows that the order of adding does not matter. Notice that we started by adding the three numbers. This is important because this is where the associative property applies. It allows us to change the order of the numbers. However, when we simplified each expression and so that it has no brackets, some addition operations changed to subtraction. But we still got the same result.



3.4.3 Adding and subtracting terms with variables in equations

We use the commutative and associative properties when solving algebraic equations. For example, consider 4 + 5k + 7 = 9. We know that addition is commutative so we can rewrite 5k + 7 as 7 + 5k, and rewrite the equation as 4 + 7 + 5k = 9. This helps us in adding the like terms, 4 and 7.

Similarly, the commutative property can be used in terms with a negative constant or coefficient. For example, the equation 4 - 5k - 7 = 9 can be rewritten as 4 + (-5k) + (-7) = 9 and therefore we can write 4 + (-7) + (-5k) = 9.

This means we can rewrite the original equation directly because 4 - 5k - 7 = 9 is the same as 4 - 7 - 5k = 9. In these materials, we have separate worksheets for numeric equations with whole numbers and numeric equations with integers. Both set of worksheets will provide practice in applying the commutative and associative properties.

3.5 Inverses and inverse operations

In section 2.2 we described briefly how inverses and inverse operations are used to solve equations. In this section we give more detail.

We use two inverses when working with equations: the additive inverse and the multiplicative inverse. There are two pairs of inverse operations: 1) addition and subtraction; and 2) multiplication and division. Inverse operations "undo" each other. For example, if we start with 12 and then add 5, we get 17. If we then subtract 5 from 17, we get back to 12. So the subtraction "reversed" the effect of adding.

Similarly, if we start with 12 and multiply by 3, we get 36. If we then divide 36 by 3, we get back to 12. So dividing by 3 reversed the effect of multiplying by 3. If we start with 12 and divide by 2, we get 6. Now multiply 2 by 6 and we get 12 again. So multiplying by 2 has reversed the effect of dividing by 2.

3.5.1 Additive inverse

The *additive identity* when working with numbers and algebraic expressions is zero. We use the idea of additive identity to find the *additive inverse* of a number. The additive inverse of a number is another number that must be added to make the sum (or the result of addition) to be zero. For example, the additive inverse of 5, is -5. We know this because 5 + (-5) = 0. The additive inverse of -7 is 7 because -7 + 7 = 0. We ask ourselves: "what must I add to this number to get zero?"

The same applies with variables. The additive inverse of x is -x because x + (-x) = 0. The additive inverse of -3p is 3p because -3p + 3p = 0. We can see that the sum is zero by simplifying the like terms but we can also check by substituting values for x and p.



Here are three examples using substitution:

Let $x = 4$, then $-x = -4$	Let $p = 2$,	Let $p = -4$,
So $x + (-x) = 4 + (-4)$	then $-3p = -3(2) = -6$	then $-3p = -3(-4) = 12$
= 4 - 4 = 0	and $3p = 3(2) = 6$	and $3p = 3(-4) = -12$
	So - 3p + 3p = -6 + 6 = 0	So - 3p + 3p = 12 + (-12) = 12 - 12 = 0

3.5.2 Multiplicative inverse

We also use multiplicative inverses in solving equations. Remember that 1 is the multiplicative identity of a number because when you multiply any number by 1, the product is the original number, e.g. $5 \times 1 = 5$ and $-7 \times 1 = -7$. We use the multiplicative identity to find the *multiplicative inverse* of a number. To determine the multiplicative inverse of a number we way "what number must I multiply by to get a product of 1?". For example, the multiplicative inverse of 5 is $\frac{1}{5}$ because $5 \times \frac{1}{5} = 1$. Also, the multiplicative inverse of $\frac{1}{5}$ is 5. So we can say that 5 and $\frac{1}{5}$ are multiplicative inverses of each other. The multiplicative inverse of -2 is $-\frac{1}{2}$ because $(-2)\left(-\frac{1}{2}\right) = 1$. In section 3.7 we give further examples to illustrate the use of the multiplicative inverse.

3.6 The meaning of "solution"

In section 1, we defined the *solution* of an equation as the value of the variable that makes the left side of the equation equal to the right side of the equation. When we want to check the solution, it is important to separate the left side from the right to test whether they are equal. There are three examples below, labelled A - C.

Sometimes we can easily find the solution by inspection (e.g. A). This usually means we can find the answer by using mental arithmetic. When there are variables on both sides of the equal sign, it is more difficult to find the solution and so we use inverse operations (e.g. B). Solutions can be integers or fractions. It is also possible to have a solution of zero (e.g. C).

	Equation	Solution	Checking the solution
А	2x - 3 = 15	x = 9	LHS: $2(9) - 3 = 15$
			RHS: 15
В	2x + 15 = 3 + x	x = -12	LHS: $2(-12) + 15 = -9$
	2x + 15 - 15 = 3 + x - 15		RHS: $3 + (-12) = -9$
	2x = -12 + x		
	2x - x = -12 + x - x		
	x = -12		
С	4x + 6 = 2(x + 1) + 4	x = 0	LHS: $4(0) + 6 = 6$
			RHS: $2(0+1) + 4 = 6$

It is possible to get linear equations where there is no solution, e.g. 4x + 6 = 4(x + 1) + 4. It is also possible to get linear equations that are true for all values of the variable, e.g. 1 + 2x + 3 = 2(x + 2). However, we have not included such examples in our materials.

In the maths class, we sometimes refer to the answer (of any maths problem) as the "solution". In these materials we talk about someone's *response* to a question, and to their *answers*. When we use the word *solution*, we specifically mean the value that makes the equation true, i.e. the value that makes the result on the left to be the same as the result on the right.



3.7 Distributive law

We apply the distributive law when we multiply a monomial (a single term) by an expression containing two or more unlike terms. For example, 2(x + 5) means the 2 must be multiplied by each term in the bracket. Note that the expression could also be written as: (x + 5)2. In both cases the 2 is multiplied by the binomial to give 2x + 10. A typical example of an equation with brackets is: 2(x + 5) = -3(4 + x) - 2. To solve this equation, we must apply the distributive law.

We must apply the distributive law on the left side and the right side.	2(x+5) = -3(4+x) - 2
On the left, 2 is multiplied by x and 5.	
On the right, -3 is multiplied by 4 and x. Note that -2 is not multiplied by the	2x + 10 = -12 - 3x - 2
terms in the brackets, i.e. we read -2 as "subtract 2".	2x + 10 = -14 - 3x
Add additive inverse of $-3x$ to both sides.	2x + 10 + 3x = -14 - 3x + 3x
	5x + 10 = -14
Add additive inverse of 10 to both sides.	5x + 10 + (-10) = -14 + (-10)
	5x = -24
Divide both sides by 5 (or apply multiplicative inverse of 5).	$\frac{5x}{24} = -\frac{24}{24}$
	5 5
	$x = -\frac{24}{5}$ or $-4\frac{4}{5}$

3.8 Examples of different strategies for using inverse operations to solve equations

We have already explained the procedures for solving linear equations and how to use additive and multiplicative inverses. In this section, we focus on one example that has letters on both sides, 3x - 4 = 5x + 6. We show four different ways to solve this equation by applying the inverse operations in different orders and by collecting the variable on the left side in three cases, and on the right side in one case. In response 1 we explicitly show *adding* the additive inverse when it is a negative number. In responses 2, 3 and 4, we show it as subtraction. Each "new" equation is an equivalent equation to the equation in the previous line. It looks different but it doesn't change the value of the solution. Note that response 4 involves factorising. This is not a typical response, but it shows that the multiplicative inverse can be used at different stages of the process to find the solution.

Response 1		Response 2		
Collect variable on right	3r - 4 = 5r + 6	Collect variable on left	3r - 4 = 5r + 6	
Additive inverse of $3x$	3x - 4 + (-3x) = 5x + 6 + (-3x)	Additive inverse of $5x$	3x - 4 - 5x = 5x + 6 - 5x	
	-4 = 2x + 6		-2x - 4 = 6	
Additive inverse of 6	-4 + (-6) = 2x + 6 + (-6)	Additive inverse of -4	-2x - 4 + 4 = 6 + 4	
	-10 = 2x		-2x = 10	
Multiplicative inverse of 2	$\frac{-10}{2} = \frac{2x}{2}$	Multiplicative inverse of -2	$\frac{-2x}{-2} = \frac{10}{-2}$	
	-5 = x		x = -5	
Response 3		Response 4		
Collect variable on left	3x - 4 = 5x + 6	Collect variable on left	3r - 4 = 5r + 6	
Additive inverse of -4	3x - 4 + 4 = 5x + 6 + 4	Additive inverse of $5x$	3x - 4 - 5x = 5x + 6 - 5x	
	3x = 5x + 10		-2x - 4 = 6	
Additive inverse of $5x$	3x - 5x = 5x + 10 - 5x	Factorise on left, taking out common $-2(x+2) = 6$		
	-2x = 10	factor of -2, then multiplicative $\frac{-2(x+2)}{-2} = \frac{6}{-2}$		
Multiplicative inverse of -2	$\frac{-2x}{-2} = \frac{10}{-2}$	inverse of -2	x + 2 = -3	
	x = -5	Additive inverse of 2	x + 2 - 2 = -3 - 2	
			x = -5	



4. What makes equations difficult

Many learners have difficulty in solving equations because they are still getting used to other new aspects of high school maths – like working with integers and algebraic symbols. In this section we discuss six of the typical difficulties that learners have with these sections and we provide a summary of the essential aspects that need to be mastered to succeed in solving linear equations. The subsections are: 1) the view of the equal sign; 2) limitations of solving by inspection; 3) negatives and subtraction; 4) fractions and division; 5) algebraic notation; and 6) working with brackets.

4.1 Viewing the equal sign as "do-something" instead of having an equivalence view

If learners can only view the equal sign as a "do something" signal, they will have difficulty in making sense of equations. For example, if asked to fill the blank in a numerical equation like $7 + 2 = __+3$, some learners give the answer of 9 because they focus on $7 + 2 = __$, but they ignore the other part of the equation. When learners do this, it tells us that they are interpreting the equal sign as something "to do" or "gives me". They need to view the equal sign as indicating equivalence between the left and the right side of the statement. In the example, they need to work out what must be put in place of the blank so that the result on the right side *is the same as* the result on the left side, i.e. 7 + 2 = 6 + 3. See section 3.1 where this is discussed in more detail.

4.2 Recognising the limitations of solving by inspection

Solving equations by inspection is a very important skill but it only works well for equations with the variable on one side, such as 2x - 5 = 1 and 4 - x = 20. When the variable appears on both sides of the equal sign, we need to use inverses to solve the equation, e.g. 2x - 5 = 3 - x. Many learners still try to solve this kind of equation by inspection and then they get stuck.

So this difficulty that learners experience is partly self-inflicted. It stems from a resistance to learn to solve equations using inverses because they can successfully solve easy equations using inspection.

4.3 Difficulties with negatives and subtraction

Learners encounter negative numbers for the first time in Grade 7. Until then, the minus symbol (-) has only one meaning: the *operation* of *subtraction*. When learners encounter negative numbers, the minus symbol takes on another meaning: it can represent a *sign* (negative) or an *operation* (subtraction). This can be very confusing. Consider the following example:



Reading from left to right, the first minus symbol represents a *sign* (*negative* five). The second minus symbol represents the operation *subtraction* (three subtract five). The third minus symbol represents a *sign* (*negative* two).



For this reason, we recommend the use of precise language when working with integers:

For operations, we say:add and subtractFor signs, we say:positive and negative

We avoid using *plus* and *minus* because these words don't tell us whether we are dealing with a sign or an operation. So we rather say:

Negative six add (positive) nine:	-6 + 9 = 3	
Five subtract positive seven:	5 - (+7) = -2	(equals negative two)
Negative five add negative seven:	-5 + (-7) = -12	(equals negative twelve)
Negative five subtract negative seven:	-5 - (-7) = 2	(equals positive two)

4.4 Working with fractions and division

In Grades 8 and 9, we mainly use fractions as coefficients of the variable. For example, $\frac{3}{2}x + 4 = 8$. We also encounter fractions as the multiplicative inverses of integers, for example $\frac{1}{4}$ is the multiplicative inverse of 4.

In the examples in section 3.5, we multiplied both sides of the equation by the multiplicative inverse, for example:

$$2y = 13$$
$$2y \times \frac{1}{2} = 13 \times \frac{1}{2}$$
$$y = \frac{13}{2}$$

Most times when we need to apply the multiplicative inverse, we actually focus on division. For example, instead of multiplying by

```
\frac{1}{2}, we can divide by 2:

2y = 13

\frac{2y}{2} = \frac{13}{2}

y = \frac{13}{2}
```

We can do this because multiplying by $\frac{1}{2}$ is the same as dividing by 2.

Remember: when we divide by the coefficient of the variable, we are actually applying the multiplicative inverse to both sides of the equation.

4.5 Difficulties with algebraic symbols

Our research has shown that many difficulties that learners have with equations stem from their difficulties in working with algebraic expressions. Here we discuss three ways in which algebraic notation and terminology can be confusing.

4.5.1 Sometimes a symbol represents a sign, sometimes it represents an operation

We have already noted that the minus symbol can represent a sign or an operation. Similarly, the plus sign can represent a sign or an operation. Consider the expression: 4 - 3x. We say "4 subtract 3x". This sounds as if the minus symbol does not belong to 3x. We say the expression has two terms that are separated by the operation of subtraction. This also suggests that the minus symbol does not belong to the 3x. But then we say the terms are 4 and -3x (four and negative three x) which means the minus symbol *is* connected to the 3x. We also say "the coefficient of x is negative 3". Once again, this indicates that the minus symbol belongs to the 3.

This is confusing because sometimes we are separating the minus symbol from the 3 and sometimes we are attaching it to the 3. Part of learning algebra involves learning when to combine the minus (or plus) symbol with the letter or number and when to separate it from the letter or number.



4.5.2 Errors in operating on like and unlike terms

We can add and subtract *like* terms. We cannot add and subtract *unlike* terms. Many learners combine unlike terms. This is often called the *conjoining* error. Typical examples of this error are: 7 + x = 7x; 2g + h = 2gh; 3x - 3 = x. We also see errors with *like* terms such as 3x - x = 3 (instead of 2x)

When working with equations, we aim to collect terms with variables on one side of the equal sign, and terms with constants on the other side of the equal sign. To do this we must apply additive inverses. See section 3.5.1 for more details on additive inverses.

4.6 Difficulties with brackets

Brackets have many different uses in mathematics. We describe some of these in detail in the x.act materials. When working with linear equations, we generally use brackets in four ways:

- i. To substitute numbers into expressions: we usually do this when checking the solution of an equation. See example A below.
- ii. To separate signs and operations: we usually do this when adding an additive inverse that is a negative number. e.g. given the equation x + 3 = 2x + 5x, we write: +3 + (-3) = 2x + 5 + (-3).
- iii. When applying the distributive law to a given equation, as shown in section 3.7.
- iv. To multiply an entire side of an equation by a number, e.g. C.

A. Checking the solution	B. Applying the additive inverse	C. Multiplying the entire side of an equation by a number
Given: $2x + 5 = x - 3$ LHS: $2(-8) + 5 = -16 + 5 = -11$ RHS: $(-8) - 3 = -11$	2x + 5 = x - 3 2x + 5 + (-5) = x - 3 + (-5) 2x + 5 - 5 = x - 3 - 5 2x = x - 8 etc.	$\frac{1}{2}x = x - 3$ $2 \times \frac{1}{2}x = (x - 3) \times 2$ $x = 2x - 6 \text{ etc.}$
We have substituted -8 in place of x	We add -5 so we use a bracket to separate the <i>addition operation</i> from the <i>negative sign.</i>	We multiplied the entire right side by 2. We need to use brackets to ensure that all terms on the right side are multiplied by 2 (distributive law).

Note that some of the ways of working with brackets were also dealt with in sections 4.1, 4.2 and 4.3.



Worksheet 1.1: Numerical equations with whole numbers

This worksheet focuses on solving numerical equations involving addition. There are two whole numbers on each side of the equal sign.

Qu	estions				
1) Thabo solved these equations:					
	A. $5 + 3 = 4 + \Box$	Answer:	$\Box = 4$	a)	Correct any mistakes Thabo has made.
	B. $5 + 3 = \Box + 6$	Answer:	$\Box = 8$	b)	Why do you think he made the mistakes?
	C. $7 + \Box = 4 + 4$	Answer:	$\Box = 4$		When solving a linear equation, we find
	D. $8 + \Box = 5 + 3$	Answer:	$\Box = 0$		a value to balance the result on each side of the equal sign.
	Some of his answers are wi	ong.			
2)	a) What is the additive inv b) What is the additive inv	verse of —4? verse of 6? H	How do you kno ow do you knov)w? /?	
3)	Look at the equation $5 + 7$	= 🗆 + 4. H	lelena and Nisha	solve	e the equation in different ways. Both girls
	find that the solution is 8.				
	Read their methods careful	ly. Make sur	e you can link th	e wo	rds and the statements.
	Helena uses solving by inspe	ection.			
	Helena first works out the v	alue on the si	de without the \Box		$5+7 = \Box + 4$
	she adds the 5 and 7 to get	12.			
	She now knows \Box + 4 must	st be 12.			$12 = \Box + 4$
	Helena thinks 'What must I	add to 4 to ge	t 12?' to get a val	ue for	
	Helena gets:				
	Nisha uses solving using add	litive inverses	<u>-</u>		$5+7=\Box+4$
	Nisha's first step is to simpli	fy 5 + 7 just l	ike Helena did:		$12 = \bigsqcup + 4$
	Nisha then subtracts 4 from	both sides of	the equation to g	et the	$12 - 4 = \Box + 4 - 4$
	own. (She uses the <i>additive</i>	inverse of 4)			$8 = \Box + 0$
	Nisha then simplifies both s	ides of the eq	uation to get a val	ue for	· ⊔. 8 = □
	She gets: Which is the same as:				$\Box = 8$
	Look at the equations belo	w:			
	A. $6+3 = \Box + 5$ B. $3+3 = \Box + 1$ C. $\Box + 7 = 9 + 2$				
	a) Quickly work out the anb) Now try to solve the ecc) Go back to your response	nswer using Juations usin Ise for equat	Helena's methoo g Nisha's metho ion A. Write wha	l. Wri d whi at you	te down only the answers. ich uses <i>additive inverses.</i> I did in each step and say why you did it.
4)	If we know that $7 + \Delta = 0$,	then which	of the following	state	ments are TRUE? Give reasons.
	A. The value of Δ is 7.			D. T	The value of Δ is -7 .
	B. Δ can be any whole null	mber.		E. Δ	and 7 are additive inverses of each other.
	C. The value of Δ is 0.			F. Δ	is the additive identity for addition.



Worksheet 1.1: Numerical equations with whole numbers

Qu	estions	Answers
1)	Thabo solved these equations:A. $5 + 3 = 4 + \Box$ Answer: $\Box = 4$ B. $5 + 3 = \Box + 6$ Answer: $\Box = 8$ C. $7 + \Box = 4 + 4$ Answer: $\Box = 4$ D. $8 + \Box = 5 + 3$ Answer: $\Box = 0$ Some of his answers are wrong.a)a) Correct any mistakes Thabo has made.b) Why do you think he made the mistakes?	 a) b) B. □ = 2 The left side was 8 so he made □ = 8 C. □ = 1 4 was added on the right in the same position as □ on the left side so he replaced □ with 4.
2)	 a) What is the additive inverse of -4? How do you know? b) What is the additive inverse of 6? How do you know? 	 2) a) 4 because -4 + 4 = 0 b) -6 because 6 + (-6) = 0
3)	 Look at the equation 5 + 7 = □ + 4. Helena and Nisha solve the equation in different ways. Both girls find that the solution is 8. Read their methods carefully. Make sure you can link the words and the statements. Look at the equations below: A. 6 + 3 = □ + 5 B. 3 + 3 = □ + 1 C. □ + 7 = 9 + 2 a) Quickly work out the answer using Helena's method. Write down only the answers. b) Now try to solve the equations using Nisha's method which uses <i>additive inverses</i>. c) Go back to your response for equation A. Write what you did in each step and say why you did it. 	3) a) A. $\Box = 4$ B. $\Box = 5$ C. $\Box = 4$ b) A. $6+3 = \Box + 5$ C. $\Box + 7 = 9 + 2$ $9 = \Box + 5$ $\Box + 7 = 11$ $9-5 = \Box + 5 - 5$ $\Box + 7 - 7 = 11 - 7$ $4 = \Box + 0$ $\Box + 0 = 4$ $4 = \Box$ $\Box = 4$ B. $3+3 = \Box + 1$ $6 = \Box + 1$ $6-1 = \Box + 1 - 1$ $5 = \Box + 0$ $5 = \Box$ c) Simplify $6+3$ to see what $\Box + 5$ must equal. Subtract 5 from both sides to get \Box on own. Simplify both sides to get a result for \Box .
4)	If we know that $7 + \Delta = 0$, then which of the following statements are TRUE? Give reasons. A. The value of Δ is 7. B. Δ can be any whole number. C. The value of Δ is 0. D. The value of Δ is 0. D. The value of Δ is -7 . E. Δ and 7 are additive inverses of each other. F. Δ is the additive identity for addition.	 4) Statements which are TRUE: D: The value of Δ is -7 because 7 - 7 = 0 E: Δ and 7 are additive inverses of each other. Additive inverses add up to 0.



Worksheet 1.2: Numerical equations with whole numbers

This worksheet focuses on solving numerical equations. Most equations involve subtraction and there are two whole numbers on each side of the equal sign.

	A statement that is balanced has the same
Questions	a <i>balanced statement</i> is an equation.
 1) Solve the equations by inspection: a) 5 - 0 = 9 - □ b) □ - 2 = 9 - 6 c) 7 - 5 = □ - 4 d) 9 + 3 = □ + 4 	 3) If you know that 8 - 4 = 7 - 3, use it to solve these equations: a) 8 - 4 - 7 = □ b) 8 - 4 + 3 = □ c) 7 - 3 + 4 = □ d) 8 - 4 + 3 - 7 = □
 2) Helena and Nisha solve the equation 6 - 3 = □ - 4 in different ways: Read them carefully and make sure you can see the links between the words and the statements. 	4) If $\Box - \Delta = 29 - 5$, give <i>four</i> sets of values for \Box and Δ that will balance the statement.
Helena uses inspection, as you did in Q1.Helena subtracts 3 from 6 to get: $6-3 = \Box -4$ Helena then thinks,'What subtract 4 gives 3?' $3 = \Box -4$ She gets: $\Box = 7$ Nisha uses additive inverses. $6-3 = \Box -4$ Nisha subtracts 3 from 6 to get: $3 = \Box -4$ Nisha adds 4 to both sides to get $3+4 = \Box -4+4$ the \Box on its own. $3+4 = \Box -0$ Nisha then simplifies further to get: $7 = \Box$ which is the same as: $\Box = 7$	 5) Look at these four statements and answer the questions that follow: A. 2 + 3 = 4 + 1 B. 2 + 3 + 5 = 4 + 1 C. 2 + 3 - 1 + 1 = 4 + 1 D. 2 + 3 - 3 = 4 + 1 a) Highlight 2 + 3 and 4 + 1 in each statement. This will help you see the structure of the statements. b) Which statements are equations? c) Rewrite the statements that are not equations with the ≠ sign.
 Look at the equations below: A. 6-3 = □ - 2 B. □ - 4 = 10 - 3 C. □ - 7 = 13 - 11 a) Solve them using Helena's method. Just write the answers. b) Now try to solve the equations using Nisha's method which uses additive inverses. c) Write down what you did in each step of A in Q2b and why you did it. 	6) Here is a list of six statements: A. $\diamond - 2 = \diamond$ B. $\diamond + 2 = \diamond$ C. $\diamond + 6 = \diamond + 2 + 6$ D. $\diamond + 5 = \diamond + 2$ E. $\diamond + 5 = \diamond + 7$ F. $\diamond - 2 + 3 = \diamond + 3$ You are now told that $\diamond = \diamond + 2$. Use this information to decide which statements are balanced. Write 'EQUATION' for those statements that are balanced.

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Worksheet 1.2: Numerical equations with whole numbers

Qu	estions	Answers
1)	Solve the equations by inspection:	1)
	a) $5 - 0 = 9 - \Box$ c) $7 - 5 = \Box - 4$	a) $\Box = 4$ c) $\Box = 6$
	b) $\Box - 2 = 9 - 6$ d) $9 + 3 = \Box + 4$	b) $\Box = 5$ d) $\Box = 8$
2)	Helena and Nisha	$2) \qquad \qquad 2) \qquad A \square = 5 B \square = 11 C \square = 0$
	$\Delta 6-3 = \Box - 2$	h) $A = -3$ $B = -11$ $C = -3$
	B. $\Box - 4 = 10 - 3$	
	C. $\Box - 7 = 13 - 11$	A. $6-3 = \Box - 2$ C. $\Box - 7 = 13 - 11$
	a) Solve them using Helena's method. Just write the	$3 = \Box - Z$ $\Box - 7 = Z$ $3 + 2 - \Box + 2 - 2$ $\Box - 7 + 7 - 2 + 7$
	answers.	$5 + 2 = \Box + 2 = 2$ $\Box + 0 = 9$
	b) Now try to solve the equations using Nisha's	$5 = \Box$ $\Box = 9$
	method which uses additive inverses.	B. $\Box - 4 = 10 - 3$ c) Simplify $6 - 3$ to see what
	C) Write down what you did in each step of A in	$\Box - 4 = 7$ the right side must equal.
	Q2b and why you did it.	$\Box - 4 + 4 = 7 + 4$ Subtract 2 from both sides to
		$\Box - 0 = 11$ get \Box on own. Simplify both
		$\Box = \Pi$ sides to get the result for \Box
3)	If you know that $8 - 4 = 7 - 3$, use it to	3)
	solve these equations:	a) $\Box = -3$ Subtracted 7 on both sides
	a) $8 - 4 - 7 = \Box$ b) $8 - 4 + 3 = \Box$	b) $\Box = 7$ Added 3 to both sides
	c) $7 - 3 + 4 = \Box$	d) $\Box = 0$ Added 3 and subtracted 7 on both sides
	d) $8-4+3-7 = \Box$	-,
4)	If $\Box = \Delta = 29 - 5$, give <i>four</i> sets of values for \Box and Δ that will belance the statement	4) Many possible answers will result in a difference of 24 e.g. 25 and 1: 26 and 2: 39 and 15
5)	Look at these four statements and answer the	5)
	questions that follow:	a) Highlighting of $2 + 3$ and $4 + 1$
	A. $2 + 3 = 4 + 1$ B. $2 + 3 + 5 = 4 + 1$	b) A. $2 + 3 = 4 + 1$ C $2 + 3 = 1 + 1 = 4 + 1$
	C. $2+3-1+1=4+1$	(2+3-1+1-4+1)
	D. $2+3-3=4+1$	B. $2 + 3 + 5 \neq 4 + 1$
	will help you see the structure of the statements	D. $2 + 3 - 3 \neq 4 + 1$
	b) Which statements are equations?	
	c) Rewrite the statements that are not equations	
	with the \neq sign.	
6)	Here is a list of six statements: A = A = A	b) Given: $\Diamond = \heartsuit + 2$
1	A. $\sqrt{-2} = 0$ B. $0 + 2 = 0$ F. $0 + 5 = 0 + 2$ F. $0 + 5 = 0 + 7$	$A \Diamond -2 = \Diamond \qquad \text{FOULATION}$
	C. $\diamond + 6 = \diamond + 2 + 6$ F. $\diamond - 2 + 3 = \diamond + 7$	3 B. $\bigotimes + 2 = \diamondsuit$ EQUATION
		C. $\diamond + 6 = \diamond + 2 + 6$ EQUATION
1	You are now told that $\diamond = \odot + 2$. Use this information t	To D. $\diamond + 5 = \diamond + 2$ Not an equation
	decide which statements are balanced. Write 'EQUATION	N' for E. $\diamond + 5 = \diamond + 7$ Not an equation
	those statements that are balanced.	F. $\diamond - 2 + 3 = \diamond + 3$ EQUATION



Worksheet 1.3: Numerical equations with whole numbers

This worksheet focuses on solving whole number numerical equations that have addition on both sides, or subtraction on both sides of the equal sign.

Qu	estions			
1)	1) Solve these equations by inspection:			
	a) $5 + 3 = 7 + \Box$			
	b) $\Box - 2 = 8 - 5$			
	c) $10 - \Box = 12 - 3$			
	d) $4 + \Box = 5 + 7$			
2)	Select the correct option for \Box :			
	a) $10 - 2 = \Box - 4$			
	i) 8 ii) 12 iii) 14 iv) 16			
	b) $7 + \Box = 9 + 2$			
	i) 0 ii) 2 iii) 3 iv) 4			
3)	Solve the equations using <i>inspection</i> and then using <i>additive inverses</i> .			
	a) $10 - 3 = \Box - 1$			
	b) $\Box + 3 = 8 + 9$			
	c) $\Box + 6 = 7 + 3$			
	$dj 12 - 4 = \Box - 6$			
4)	A balanced statement has the same result one each side of the equal sign.			
	e.g. $9 + 3 = 10 + 2$. The result on each side of the equal sign is 12.			
	If the result on each side of the equal sign is not the same, the statement is not balanced.			
	e.g. $9 + 3 = 10 + 2 - 3$			
	The result on the left of the equal sign is 12 but the result on the right is 9. So, the statement is not balanced. We write $9 + 3 \neq 10 + 2 - 3$. We say that "the left side <i>is not equal to</i> the right side".			
	Look at these three statements:			
	A. $12 - 3 = 4 + 5$			
	B. $12 - 3 - 5 = 4 + 5$			
	C. $12 - 3 + 6 = 4 + 5 - 6$			
	a) Decide which statements are not balanced. Re-write them using the \neq sign.			
	b) Make one change to the right side so that the statement becomes balanced.			
5)	The equation $8 - 4 = \Box - 3$ has two whole numbers on each side. Use this information to decide			
	whether the following equations are balanced or NOT balanced. If they are not balanced, re-write them			
	with the \neq sign.			
	a) $8 - 4 + 3 = \Box + 3$			
	$\begin{array}{c} D \\ \hline \end{array} \\ \begin{array}{c} -3 + 4 = 8 \\ \hline \end{array} \\ \begin{array}{c} 2 \\ \end{array} \\ \end{array}$			
	$ (1 \ 0 - 4 + 1) - 3 $ $ (1 \ 0 - 4 + 1) - 3 $ $ (2 \ 0 - 4 + 1) - 3 $			

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Worksheet 1.3: Numerical equations

Questions	Answers
 1) Solve these equations by inspection: a) 5 + 3 = 7 + □ b) □ - 2 = 8 - 5 c) 10 - □ = 12 - 3 d) 4 + □ = 5 + 7 	1) a) $\Box = 1$ b) $\Box = 5$ c) $\Box = 1$ d) $\Box = 8$
2) Select the correct option for \Box : a) $10 - 2 = \Box - 4$ i) 8 ii) 12 iii) 14 iv) 16 b) $7 + \Box = 9 + 2$ i) 0 ii) 2 iii) 3 iv) 4	 2) a) □ = 12 option ii) b) □ = 4 option iv)
 3) Solve the equations using <i>inspection</i> and then using <i>additive inverses</i>. a) 10 - 3 = □ - 1 b) □ + 3 = 8 + 9 c) □ + 6 = 7 + 3 d) 12 - 4 = □ - 6 	3) Inspection Additive inverses hint a) $\Box = 7$ Add 1 to both sides b) $\Box = 14$ Subtract 3 from both sides c) $\Box = 4$ Subtract 6 from both sides d) $\Box = 14$ Add 6 to both sides
 4) A balanced statement has the same result on both sides of the equal sign. e.g. 9 + 3 = 10 + 2 The result on each side of the equal sign is 12 If the result on both sides of the equal sign is not the same, the statement is not balanced e.g. 9 + 3 = 10 + 2 - 3 The result on the left of the equal sign is 12 but the result on the right is 9 Look at these three statements: A. 12 - 3 = 4 + 5 B. 12 - 3 - 5 = 4 + 5 C. 12 - 3 + 6 = 4 + 5 - 6 a) Decide which statements are not balanced. Re-write them using the ≠ sign. b) Make one change to the right side so that the statement becomes balanced. 	a) B. $12 - 3 - 5 \neq 4 + 5$ c. $12 - 3 + 6 \neq 4 + 5 - 6$ b) B. Subtract 5 from the right side: 12 - 3 - 5 = 4 + 5 - 5 C. Add 6 to the right side: 12 - 3 - 6 = 4 + 5 + 6
 5) The equation 8 - 4 = □ - 3 has two whole numbers on each side. Use this information to decide whether the following equations are balanced or NOT balanced. If they are not balanced, re-write them with the ≠ sign. a) 8 - 4 + 3 = □ + 3 b) □ - 3 + 4 = 8 c) 8 = 4 + □ - 3 d) □ = 8 - 4 + 3 	5) Given: $8 - 4 = \Box - 3$ a) $8 - 4 + 3 \neq \Box + 3$ NOT balanced b) $\Box - 3 + 4 = 8$ balanced c) $8 = 4 + \Box - 3$ balanced d) $\Box \neq 8 - 4 + 3$ NOT balanced



Worksheet 1.4: Numerical equations with whole numbers

This worksheet focuses on solving equations involving: 1) addition only; and 2) subtraction only. There are two or more whole numbers on each side of the equal sign.

Qu	Questions				
1)	1) Three equations are given below. Shane's answers are given next to each equation. Copy the equations				
	and answers. If Shane's answer is correct, give it a tick 🗸. If Shane's answer is incorrect, give it a cross 🗙				
	and give the correct answer.	and give the correct answer.			
	a) $1 + 3 + 2 = \Box + 2 + 1$ Ar	iswer: $\Box = 6$			
	b) $8-5-\Box = 14-7-6$ Ar	nswer: $\Box = 2$			
	c) $9 - 4 - \Box = 10 - 4 - 4$ Ar	swer: $\Box = 2$			
2)	Solve the following equations:				
	a) $7 + 3 = \Box + 7$	If you are not told which method to use			
	b) $\Box + 4 = 6 + 3$	inspection or additive inverses.			
	c) $12 - 2 = \Box - 9$				
	d) $\Box - 4 = 10 - 5$				
2)	Pelow is a set of four equations:	a) Highlight another part which is the same in all			
3)	$12 - 2 - 5 - 0 - \Box - 1$	a) The aquations			
	A. $12 - 3 - 5 - 0 - 1 - 1$ P. $12 - 2 - 5 - 1 - 1 - 1$	the equations.			
	B. $12 - 3 - 5 - 1 - \Box - 1$	b) Solve all the equations.			
	C. $12 - 3 - 5 - 2 - \Box - 1$	c) Compare your answers to the four equations.			
	D . 12 3 3 5 5 $ \Box$ 1	What is the relation between the values of \Box in			
	that these numbers appear in all the equation	each case? What causes this relation?			
	that these humbers appear in an the equation.	3.			
4)	Below is a set of four equations:	a) Highlight the parts which are the same for each			
	A. $5 + 3 + 2 = \Box + 4$	equation.			
	B. $5 + 3 + 2 + 7 = \Box + 4 + 7$	b) Predict TRUE or FALSE: The value of the box will			
	C. $40 + 5 + 3 + 2 = \Box + 4 + 40$	NOT be the same for all the equations.			
	D. $3 + 2 + 5 + 12 = 12 + \Box + 4$	Give reasons for your response.			
l r		c) Solve all the equations.			
	To predict the answers: Look for what is the	d) Was your prediction in Q4b correct? Why/why			
	same and different in the equations and write what you expect the value of \Box to be	not?			
5)	You are given the equation: $\diamond + 4 = \diamond - 2$ T	his equation has two unknown values \Diamond and \heartsuit When			
,	you balance the statements below both unknow	own values must appear in the equation. They could be on			
	different sides of the equal sign, or they could	be on the same side as in Q5b			
	a) 4 ± 0				

b) $\diamond + 4 - \diamond =$

- c) $\otimes -2 + 0 =$
- d) $\diamond + 4 + 2 =$
- e) $\bigcirc -2 4 =$



Worksheet 1.4: Numerical equations with whole numbers

Qı	lestions	Ans	swer	S
1)	Three equations are given below. Shane's answers are given next to each equation. Copy the equations and answers. If Shane's answer is correct, give it a tick \checkmark . If Shane's answer is incorrect, give it a cross X and give the correct answer. a) $1 + 3 + 2 = \Box + 2 + 1$ Answer: $\Box = 6$ b) $8 - 5 - \Box = 14 - 7 - 6$ Answer: $\Box = 2$ c) $9 - 4 - \Box = 10 - 4 - 4$ Answer: $\Box = 2$	1)	a) b) c)	Answer: $\Box = 6 \times \Box = 3$ Shane used the left side as his answer Answer: $\Box = 2 \checkmark$ Answer: $\Box = 2 \times \Box = 3$ Shane used the right side as his answer
2)	Solve the following equations: a) $7 + 3 = \Box + 7$ b) $\Box + 4 = 6 + 3$ c) $12 - 2 = \Box - 9$ d) $\Box - 4 = 10 - 5$	2)	lnsp a) b) c) d)	DectionAdditive inverse hint $\Box = 3$ Subtract 7 from each side $\Box = 5$ Subtract 4 from each side $\Box = 19$ Add 9 to each side $\Box = 9$ Add 4 to each side
3)	 Below is a set of four equations: A. 12-3-5-0= □ -1 B. 12-3-5-1= □ -1 C. 12-3-5-2= □ -1 D. 12-3-5-3= □ -1 Look at the highlighted part in equation A. Note that these numbers appear in all the equations. a) Highlight another part which is the same in all the equations. b) Solve all the equations. c) Compare your answers to the four equations. What is the relation between the values of □ in each case? What causes this relation? 	3)	a) b) c)	$\begin{vmatrix} -1 \\ A. \end{vmatrix} = 5 \\ B. \end{vmatrix} = 4 \\ C. \end{vmatrix} = 3 \\ D. \end{vmatrix} = 2 \\ \end{vmatrix}$ is 1 less each time. This is because on the left of the equal sign we subtract one extra each time, e.g. 0 then 1 etc.
4)	 Below is a set of four equations: A. 5 + 3 + 2 = □ + 4 B. 5 + 3 + 2 + 7 = □ + 4 + 7 C. 40 + 5 + 3 + 2 = □ + 4 + 40 D. 3 + 2 + 5 + 12 = 12 + □ + 4 a) Highlight the parts which are the same for each equation. b) Predict TRUE or FALSE: The value of the box will NOT be the same for all the equations. Give reasons for your response. c) Solve all the equations. d) Was your prediction in Q4b correct? Why/why not? 	4)	a) b) c) d)	$5 + 3 + 2$ and $\Box + 4$ FALSE: The value of the box will be the same becausethe same number is added on each side of eachequation. $\Box = 6$ for all equationsPrediction was correct. All the equations have thesame value for the box. [If we subtract 7 from eachside of B. It is the same as adding 0, The same appliesto subtracting 40 in C. and 4 in D. This makes eachequation go back to A's equation]
5)	You are given the equation: $\diamond + 4 = \heartsuit - 2$. This equation has two unknown values, \diamond and \heartsuit . When you balance the statements below, both unknown values must appear in the equation. They could be on different sides of the equal sign, or they could be on the same side as in Q5b. a) $4 + \diamondsuit =$ b) $\diamond + 4 - \heartsuit =$ c) $\heartsuit - 2 + 0 =$ d) $\diamondsuit + 4 + 2 =$ e) $\heartsuit - 2 - 4 =$ f) $\heartsuit =$	5)	Give a) b) c) d) e) f)	n: $\diamond + 4 = \heartsuit - 2$ $4 + \diamond = \heartsuit - 2$ $\diamond + 4 - \heartsuit = -2$ $\circlearrowright -2 + 0 = \diamondsuit + 4 + 0$ $\diamond + 4 + 2 = \heartsuit$ $\circlearrowright -2 - 4 = \diamondsuit + 4 - 4$ $\circlearrowright = \diamondsuit + 4 + 2$



Worksheet 1.5 Numerical equations with whole numbers

This worksheet focuses on solving equations that contain a mixture of addition and subtraction. There are two or more whole numbers on each side of the equal sign.

Questions	
1) Match the columns:	
COLUMN A CO	OLUMN B
a) $9 + 4 - \Box = 10 + 4 - 7$	I. $\Box = 0$
b) $7 - 4 + 6 = 8 + 6 - \Box$	$\Box = 5$
c) $3 + 9 - 5 = 11 - 4 - \Box$	I. $\Box = 6$
	$I. \Box = 7$
2) Solve these four equations by inspection:	3) Solve these equations using additive inverses:
a) $5 + \Box = 10 - 3$	a) $8 + 3 - 5 = \Box + 2$
b) $9-2 = \Box + 2$	b) $\Box + 4 - 2 = 6 + 7 - 3$
c) $8 - \Box = 7 - 1$	c) $11 - 1 - 3 = \Box + 4 + 2$
d) $\Box - 4 = 9 - 8$	
	To predict the answers: Look for what is the
4) Given: $4 + 3 + 6 = \Box + 5$	same and different in the equations and write what you expect the value of \Box to be.
a) Look for what is the same and different in	n
this set of four equations:	b) Do you expect the values for \Box to be the same
A. $4 + 3 + 6 - 4 = \Box + 5$	for each equation? Give a reason for your
B. $4 + 3 + 6 - 5 = \Box + 5$	answer.
C. $4 + 3 + 6 - 6 = \Box + 5$	c) Solve the equations.
D. $4 + 3 + 6 - 7 = \Box + 5$	d) Was your prediction in Q4b correct?
5) Given: $9 + 3 + 2 = \Box + 2 + 4$	
a) Solve each equation:	c) Predict the answers to the following:
A. $9 + 3 + 2 - 1 = \Box + 2 + 4 - 1$	C. $9 + 3 + 2 - 12 = \Box + 2 + 4 - 12$
B. $9 + 3 + 2 + 2 = \Box + 2 + 4 + 2$	D. $9 + 3 + 2 + 40 - 40 = \Box + 2 + 4$
b) You should have got the same answers to	E. $32 - 32 + 9 + 3 + 2 = \Box + 2 + 4$
equations A and B. Why does this happen	$F. 3 + 2 + 9 - 14 = 2 + 4 + \Box - 14$
	d) Use any method to check your predictions.
6) Here is a set of five statements:	
A. $\diamond -2 + 5 = \diamond -2$	You are now told that $\Diamond -2 = \bigcirc$
$B. \diamondsuit -2 + 5 = \circlearrowright + 5$	Use the equation $\Diamond - 2 = \odot$ to decide which of the
C. $\diamond = \odot + 2$	statements (A to E) are balanced and why.
D. $\diamond -2 + 3 = \diamond$	· · · ·
E. $\diamond - \odot - 1 = 1$	



Worksheet 1.5: Numerical equations whole numbers

Qu	lestions and answers	Questions and answers
1)	Match the columns:	5) Given: $9 + 3 + 2 = \Box + 2 + 4$
Í	COLUMN A COLUMN B	a) Solve each equation:
	a) $9 + 4 - \Box = 10 + 4 - 7$	A $9+3+2-1 = \Box + 2 + 4 - 1$
	b) $7-4+6=8+6-\Box$	$B_{1} = 9 + 3 + 2 + 2 = \Box + 2 + 4 + 2$
	$\begin{array}{c} 5 & 7 & 1 + 6 = 6 + 6 & \Box \\ \hline 1 & 1 & - 5 \\ \hline 1 & 2 & - 5 \\ \hline 1 & - 5 & - 11 - 4 - \Box \\ \hline 1 & 1 & - 5 \\ $	b) You should have got the same answers to
		equations A and B. Why does this happen?
	\mathbb{IV} . $\square = 7$	c) Predict the answers to the following:
	Answers	$C = 9 \pm 3 \pm 2 \pm 12 = \Box \pm 2 \pm 4 \pm 12$
	a) III $\Box = 6$ b) II $\Box = 5$ c) I $\Box = 0$	D. $9+3+2+40-40 = \Box + 2 + 4$
2)	Solve these four equations by inspection:	E. $32 - 32 + 9 + 3 + 2 = \Box + 2 + 4$
	a) $5 + \Box = 10 - 3$ Answer $\Box = 2$	F. $3 + 2 + 9 - 14 = 2 + 4 + \Box - 14$
	b) $9-2=\Box+2$ Answer $\Box=5$	d) Use any method to check your predictions.
	c) $8 - \Box = 7 - 1$ Answer $\Box = 2$	
	d) $\Box - 4 = 9 - 8$ Answer $\Box = 5$	Answers
	·	a) A. $\Box = 8$ B. $\Box = 8$
3)	Solve these equations using additive inverses:	b) Same answers because the $9 + 3 + 2$ on the
	a) $8+3-5 = \Box + 2$	left and the \Box + 2 + 4 on the right stay the
	b) $\Box + 4 - 2 = 6 + 7 - 3$	same and we subtract the same value from
	c) $11 - 1 - 3 = \Box + 4 + 2$	(or add the same value to) the left and right.
		c) $\Box = 8$ for all equations
	Answers	d) Check A: using inverses
	a) $\Box = 4$ subtract 2 from each side	$2 = \Box - 6$
	b) $\Box = 8$ subtract 2 from each side	$2+6 = \Box - 6 + 6$
	c) $\Box = 1$ subtract 6 from each side	8 = 🗆
		or inspection $2 = \Box - 6$ so $8 = \Box$
4)	Given: $4 + 3 + 6 = \Box + 5$	6) Here is a set of five statements:
	a) Look for what is the same and different in this set of four	A. $\diamond -2 + 5 = \diamond -2$
	equations:	$B. \diamondsuit-2+5=\heartsuit+5$
	A. $4 + 3 + 6 - 4 = \Box + 5$	$C. \diamondsuit = \heartsuit + 2$
	B. $4+3+6-5 = \Box + 5$	D. $\Diamond - 2 + 3 = \bigcirc$
	C. $4+3+6-6 = \Box + 5$	$E. \diamondsuit - \circlearrowright - 1 = 1$
	D. $4+3+6-7 = \Box + 5$	You are now told that $\Diamond -2 = \oslash$
	b) Do you expect the values for \Box to be the same for each	Use the equation $\Diamond -2 = \odot$ to decide which of the
	equation? Give a reason for your answer.	statements (A to E) are balanced and why.
	c) Solve the equations.	
	d) Was your prediction in Q4b correct?	Answers Given $\Diamond - 2 = \bigcirc$
		B: The same number is added to each side so when it is
	Answers	subtracted from each side we get $\Diamond -2 = \heartsuit$ which we
	a) $4+3+6$ and $\Box + 5$ is the same in each equation.	were given, so the statement is balanced or substitute \heartsuit
	What is different is that a different number is subtracted	for $\Diamond - 2$ into $\Diamond - 2 + 5 = \bigcirc + 5$ and get
	from the left of each equation each time.	\otimes + 5 = \otimes + 5 so the statement is balanced.
	b) 🗌 will not to be the same. We subtract a different	C: Subtract 2 from each side to get $\Diamond - 2 = \bigcirc$ or substitute
	number each time. This means the value in \Box will have	$\Diamond - 2$ for \odot into $\Diamond = \odot + 2$ and get $\Diamond = \Diamond - 2 + 2$
	to change to ensure the equations remain balanced	which simplifies to $\Diamond = \Diamond$ hence the statement is balanced.
1	c) $A \Box = 4 B \Box = 3 C \Box - 2 D \Box - 1$	E: Subtract 1 from each side to get $\Diamond - \heartsuit - 2 = 0$ then add
	d) Vos The answers were different	\otimes to each side to get $\Diamond - 2 = \Diamond$
	ען ובא. וווב מואשבוא שבוב מוופופות	Note: substitution for \bigcirc gets complicated (substitute
1		\wedge 2 - \wedge for \wedge into Γ and $\rightarrow \uparrow \wedge$ (\wedge 2) 1 1
1		$\vee - 2 = \odot$ for \odot into E. and get $\vee - (\vee - 2) - 1 = 1$
1		which involves integers and a solution of $1 = 1$.
		Inis type of solution is discussed in algebraic equations]
1		

#equali = y matters

PRACTICE IN WORKING WITH LINEAR EQUATIONS



Worksheet 1.6: Numerical equations with whole numbers

This worksheet focuses on the \Box in different positions in equations with whole numbers. The position of the \Box makes some equations more difficult than others. As in Worksheets 1.1 to 1.5, our aim is to get \Box on its own.

Qı	Questions					
1)	Loc	ok at t	this equation: $5 + 6 = 9 + 6$	\Box . Note that it has \Box at the end of the equation.		
	a)	Solv	e the equation by inspection	on.		
	b)	Jack	solved $5 + 6 = 9 + \Box$ using	ng additive inverses.		
		Сору	his response and answer	the questions:		
		[Jack's response Questions			
			$5 + 6 = 9 + \Box$	i) Is Jack correct to re-write $9 + \Box$ as $\Box + 9$? Why?		
			$11 = \Box + 9$	ii) How does Jack get 11?		
			$11 - 9 = \Box + 9 - 9$	iii) Why does Jack subtract 9 from each side?		
			$2 = \Box + 0$	iv) How does Jack get 2?		
				v) And how does he get 0?		
			2 = 🗆	vi) Does Jack get the correct answer?		
		L				
2)	2) Here is another equation: $7 - 3 = 9 - \Box$. Note that it has "subtract \Box " at the end of the equation.					
	a) Solve the equation by inspection.					
	b) Mbali solved the equation using additive inverses.					
		Сору	y her response and answer	the questions:		
			Mbali's response	Questions		
			$7-3 = 9 - \square$			
			$4 = 9 - \Box$	i) How does Mbali get 4?		
	$4 + \Box = 9 - \Box + \Box$ ii) What has Mbali done on each side?		ii) What has Mbali done on each side?			
	$4 + \Box = 9 + 0$ iii) Why is there a 0?		iii) Why is there a 0?			
	$\Box + 4 = 9$ iv) Why does Mbali change $\Box + 4$ to $4 + \Box$?		iv) Why does Mbali change $\Box + 4$ to $4 + \Box$?			
	$\Box = 9 - 4$ v) How does Mbali get -4 on the right of the equal sign?		v) How does Mbali get -4 on the right of the equal sign?			
			$\Box = 5$	vi) This should be the same answer you got for Q2a. Is it?		

vii) Go back to the first line of Mbali's response. Could she have/ changed $9 - \Box$ to $\Box - 9$? Explain.

3) Solve these equations using additive inverses. Refer to Jack and Mbali's responses if you need help.

a) $5-3 = 9 - \Box$ also write what you did in each step in Q3a, and why you did it.

- b) $7-5 = 1 + \Box$
- c) $16 \Box = 12 4$
- d) $2 + \Box = 7 3$ also write what you did in each step in Q3d, and why you did it.
- e) $9 + 3 = 18 \Box$



Worksheet 1.6: Numerical equations whole numbers

Questions	Answers	
1) Look at this equation: 5 + a) Solve the equation b) Jack solved 5 + 6 = Copy his response Jack's response 5 + 6 = 9 + \Box 11 = \Box + 9 11 - 9 = \Box + 9 - 9 2 = \Box + 0 2 = \Box	 6 = 9 + □. Note that it has □ at the end of the equation. by inspection. = 9 + □ using additive inverses. and answer the questions: Questions i) Is Jack correct to re-write 9 + □ as □ + 9? Why? ii) How does Jack get 11? iii) Why does Jack subtract 9 from each side? iv) How does Jack get 2? v) And how does he get 0? vi) Does Jack get the correct answer? 	1) a) $\Box = 2$ b) i) Yes, because addition is commutative ii) Adds 5 and 6 iii) To get \Box on its own iv) By subtracting 9 from 11 v) 9 - 9 = 0 vi) Yes
2) Here is another equation the equation. a) Solve the equation b) Mbali solved the equation copy her response $7-3 = 9 - \Box$ $4 = 9 - \Box$ $4 + \Box = 9 - \Box + \Box$ $4 + \Box = 9 + 0$ $\Box + 4 = 9$ $\Box = 9 - 4$ $\Box = 5$	 n: 7 - 3 = 9 - □. Note that it has "subtract □" at the end of by inspection. quation using additive inverses. and answer the questions: Questions i) How does Mbali get 4? ii) What has Mbali done on each side? iii) Why there a 0? iv) Why does Mbali change □ + 4 to 4 + □? v) How does Mbali get -4 on the right of the equal sign? vi) This should be the same answer you got for Q2a. Is it? vii) Go back to the first line of Mbali's response. Could she have changed 9 - □ to □ - 9? Explain. 	2) a) $\Box = 5$ b) i) Subtracted 3 from 7 ii) Added box iii) $-\Box + \Box = 0$ iv) Addition is commutative. i.e. $4 + \Box = \Box + 4$ and it is easier to see what to do to each side v) Added the additive inverse of 4 to each side vi) Yes vii) No because subtraction is not commutative i.e. $9 - \Box \neq \Box - 9$
3) Solve these equations u you need help. a) $5-3 = 9 - \Box$ a b) $7-5 = 1 + \Box$ c) $16 - \Box = 12 - 4$ d) $2 + \Box = 7 - 3$ a e) $9 + 3 = 18 - \Box$ Answers to Q3a and Q3d 's s a) Subtract 3 from 5 to sin \Box Subtract 2 from eac d) Subtract 3 from 7 to sin before the equal sign. S	3) Final answers to Q3a to Q3e a) □ = 7 b) □ = 1 c) □ = 8 d) □ = 2 e) □ = 6 See steps for Q3a and Q3d below the question.	



Worksheet 1.7: Numerical equations with whole numbers

This worksheet focuses on solving equations that contain a mixture of addition and subtraction. There are two or more whole numbers on each side of the equal sign with \Box in a variety of positions. As in Worksheets 1.1 to 1.6, our aim is to get \Box on its own.

Qu	Questions			
1)	Look at this equation: $7 + 8 = 20 - \Box$ Note that it has \Box at the end of the equation			
-,	a) Solve the equation by using additive inverses			
	b) Pased on your answer to 01a answer the following questions:			
	b) based on your answer to Q1a answer the following questions. i) Could we rewrite $20 \square cc \square = 202$ Eveloin			
	i) How do you get $ \Box $ on the left of the equal sign?			
	ii) How do you get $+\Box$ on the left of the equal sign:			
2)	Given: $10 + 13 + 4 = \Box + 11$			
	a) Look for what is the same and different in this set of four equations:			
	A. $10 + 13 + 4 - 4 = \Box + 11$			
	B. $10 + 13 + 4 - 5 = \Box + 11$			
	C. $10 + 13 + 4 - 6 = \Box + 11$			
	D. $10 + 13 + 4 - 7 = \Box + 11$			
	b) Do you expect the results for \Box to be the same for each equation? Give a reason for your answer.			
	c) Solve the equations.			
	d) Was your prediction in Q2b correct?			
3)	Solve these equations using additive inverses.			
	a) $10 - 3 = 9 - \Box$			
	b) $11 - \Box = 12 - 4$			
	c) $2 + \Box = 5 + 3$			
	d) $9-5=2+\Box$			
	e) $12 + 3 = 18 - \Box$			
4)	Here is a set of five statements:			
.,	$A \diamondsuit -4 + 7 = \bigotimes -4$			
	B. $\Diamond -4 + 7 = \heartsuit + 7$			
	C. $\Diamond = \bigotimes + 4$			
	D. $\diamond -4 + 3 = \diamond$			
	E. $\Diamond - \heartsuit = 4 + 7$			
	You are now told that $\Diamond -4 = \oslash$			
	Use the equation $\Diamond-4=\odot$ to decide which statements (A to E) are not balanced. Say or show why			
	they are not balanced and write them using a \neq sign.			



Worksheet 1.7: Numerical equations with whole numbers

Questions				we	rs
1)	 Look at this equation: 7 + 8 = 20 - □. Not end of the equation. a) Solve the equation by using additive involution by Based on your answer to Q1a answer tiin in Could we rewrite 20 - □ as □ - iiin How do you get +□ on the left of iin the left of the equation of the equation of the equation. 	e that it has □ at the verses. he following questions: 20? Explain. the equal sign?	1)	a) b)	$7 + 8 = 20 - \square$ $15 = 20 - \square$ $15 + \square = 20 - \square + \square$ $\square + 15 - 15 = 20 - 15$ $\square = 5$ i) No. Subtraction is not commutative i.e. $20 - \square \neq \square - 20$ ii) add \square to each side of the equation
2)	 Given: 10 + 13 + 4 = □ + 11 a) Look for what is the same and different equations: A. 10 + 13 + 4 - 4 = □ + 11 B. 10 + 13 + 4 - 5 = □ + 11 C. 10 + 13 + 4 - 6 = □ + 11 D. 10 + 13 + 4 - 7 = □ + 11 b) Do you expect the results for □ to be t equation? Give a reason for your answer c) Solve the equations. d) Was your prediction in Q4b correct? 	in this set of four he same for each er.	2)	a) b) c)	$10 + 13 + 4$ and $\Box + 11$ is the same in each equation. What is subtracted on the left of the equation is one more each time. So -4 in the first, -5 in the second, etc. No. The left side gets smaller by one each time so the value for \Box would also get smaller each time.A. $\Box = 12$ B. $\Box = 11$ C. $\Box = 10$ D. $\Box = 9$ Yes
3)	Solve these equations using additive inverse a) $10 - 3 = 9 - \square$ b) $11 - \square = 12 - 4$ c) $2 + \square = 5 + 3$ d) $9 - 5 = 2 + \square$ e) $12 + 3 = 18 - \square$ Here is a set of five statements: A. $\Diamond -4 + 7 = \bigcirc -4$ B. $\Diamond -4 + 7 = \bigcirc +7$ C. $\Diamond = \bigcirc +4$ D. $\Diamond -4 = \bigcirc -4$ E. $\Diamond - 0 = 0 + 4 = 7$	s. 4) Given $\Diamond - 4 = \bigcirc$ A On the left of the first subtracted changed in the or subtracting for the first subtracting for the first subtracting for subtr	3) he equ from same 7 from	a) b) c) d) e) ual si \odot . T way	$\Box = 2$ $\Box = 3$ $\Box = 6$ $\Box = 2$ $\Box = 3$ ign: 7 is added to $\Diamond - 4$. On the right he left and the right have not been $\bullet \Diamond - 4 + 7 \neq \Diamond - 4$ and $\heartsuit \neq \heartsuit + 7$ th side we get $\Diamond - 4$ on the left and $\heartsuit - 4$
E. $\Diamond - \heartsuit = 4 + 7$ You are now told that $\Diamond - 4 = \heartsuit$ Use the equation $\Diamond - 4 = \heartsuit$ to decide which statements (A to E) are not balanced. Say or show why they are not balanced and write them using a \neq sign.		$ \begin{array}{c} 11 \neq \odot \text{ or Sut} \\ \odot + 7 \text{ and } \odot \\ \text{balanced: } \diamond - \\ \text{D} \\ \text{Left: 0 is addect} \\ \text{on each side ar} \\ \text{on the left and} \\ 4 \text{ on the left and} \\ 4 \text{ on the left to} \\ \text{is not balanced} \\ \text{E} \\ \text{Simplifying the} \\ \diamond - 4 = \odot \text{ wh} \\ \underline{\text{Conclusion: the}} \end{array} $	+ 7 ≠ 4 + 7 d to \Diamond re diffe \odot on get \bigcirc l: \Diamond – right s hich is e state	$e \odot - 4$ $\Rightarrow \odot - 4$. erent the \Rightarrow \Rightarrow and $4 \neq =$ side, the \Rightarrow men	A conclusion: the statement is not 2 - 4 Right:4 is subtracted from \bigcirc , changes t or by adding 4 to both sides we get \diamondsuit right and $\diamondsuit \neq \bigcirc$ or substitute \bigcirc for $\diamondsuit -$ d $\bigcirc \neq \oslash - 4$ <u>Conclusion</u> : the statement $\oslash - 4$ we get $\diamondsuit - \oslash = 11$ but we were given same as $\diamondsuit - \oslash = 4$ and $11 \neq 4$ at is not balanced: $\diamondsuit - \heartsuit \neq 4 + 7$



Worksheet 1.8: Numerical equations with whole numbers

This worksheet focuses on solving equations involving multiplication where there are two whole numbers on each side of the equal sign. Some answers are not whole numbers.

Questions					
1) Give the multiplicative inverse of each number:					
a) 3 b) 2 c) 5 d) 7	When we multiply multiplicative inverses,				
	we get a product of 1. e.g. $4 \times \frac{1}{4} = 1$				
2) Work out the value of \Box to balance these statements					
a) $4 \times \frac{1}{\Box} = 4 \times 1$ b) $8 \times \frac{3}{\Box} = 8 \times 1$	1 c) $5 \times \frac{6}{\Box} = 5 \times 1$				
3) Marti and Musa solve the equation $6 \times 2 = \Box \times 4$ in	different ways:				
Marti uses inspection:	$6 \times 2 = \Box \times 4$				
Marti first simplifies the equation	$12 = \Box \times 4$				
She then thinks: "What multiplied by 4 gives 12?"					
And she gets:	$\Box = 3$				
Musa uses multiplicative inverses:	$6 \times 2 = \Box \times 4$				
Musa also simplifies the equation	$12 = \Box \times 4$				
Us then multiplies both sides by $\frac{1}{2}$ to get \Box on its own	$12 \times 1 - \Box \times 4 \times 1$				
He then multiplies both sides by $\frac{1}{4}$ to get \Box of its own.	$12 \times \frac{1}{4} - \Box \times 4 \times \frac{1}{4}$				
Musa then simplifies $12 \times \frac{1}{4}$ like this: $\frac{1}{1} \times \frac{1}{4} = \frac{1}{4} = 3$	$12 \times \frac{1}{4} = \Box \times 1$				
And gets:	3 = 🗆				
	So $\Box = 3$				
Solve these equations using multiplicative inverses in the inspection. a) $4 \times 6 = \Box \times 3$ b) $\Box \times 5 = 10 \times 4$	the way Musa did. Then check your answers using				
4) Marti made up this question to try Musa's method. She found the answer was a fraction. Copy her					
response and answer the questions:					
$4 \times 8 = 5 \times \square$					
$4 \times 8 = \Box \times 5$ a) Marti rewrote	$5 \times \Box$ as $\Box \times 5$. Explain why she can do this.				
$4 \times 8 \times \frac{1}{r} = \Box \times 5 \times \frac{1}{r}$ b) Why does Mar	ti multiply both sides by the multiplicative inverse of 5?				
$\overline{1} \times \overline{1} \times \overline{5} = \Box$ c) What is Marti of	doing here?				
32	ver a common fraction or an improper fraction?				
$\frac{1}{5} = \Box$					
5) Solve the following equations using multiplicative involutions $\nabla = \frac{1}{2} = \frac{1}{2}$	erses:				
a) $\Box \times 6 = 5 \times 12$ b) $\Box \times 6 = 5 \times 5$					
c) $9 \times 2 = 3 \times \square$ d) $4 \times \square = 7 \times 5$					
e) $3 \times 5 = \Box \times 2$					



Worksheet 1.8: Numerical equations whole numbers

Qu	estions	Answers
1)	Give the multiplicative inverse of each number:	1)
	a) 3 b) 2 c) 5 d) 7	a) $\frac{1}{3}$ c) $\frac{1}{5}$
		b) $\frac{1}{2}$ d) $\frac{1}{7}$
2)	Work out the value of \Box to balance these statements:	2)
	a) $4 \times \frac{1}{\Box} = 4 \times 1$ b) $8 \times \frac{3}{\Box} = 8 \times 1$ c) $5 \times \frac{3}{\Box} = 5 \times 1$	a) $\Box = 1$ b) $\Box = 3$
		c) $\Box = 6$
3)	Marti and Musa solve the equation $6 \times 2 = \Box \times 4$ in different ways:	3)
	Marti uses inspection: $6 \times 2 = \Box \times 4$	a) $4 \times 6 = \Box \times 3$
	Marti first simplifies the equation $12 = \Box \times 4$	$24 = \Box \times 3$
	She then thinks: "What multiplied by 4 gives 12?"	$24 \times \frac{1}{3} = \Box \times 3 \times \frac{1}{3}$
	And she gets: $\Box = 3$	8 = 🗆
	Musa uses multiplicative inverses: $6 \times 2 = \Box \times 4$	b) $\Box \times 5 = 10 \times 4$
	Musa also simplifies the equation $12 = \Box \times 4$	$\Box \times 5 = 40$
	He then multiplies both sides by $\frac{1}{7}$ to get \Box on its $12 \times \frac{1}{7} = \Box \times 4 \times \frac{1}{7}$	$\Box \times 5 \times \frac{1}{5} = 40 \times \frac{1}{5}$
	own. $12 \times \frac{1}{2} - \Box \times 1$	$\Box = 8$
	Musa then simplifies $12 \times \frac{1}{4}$ like this:	
	$\frac{12}{1} \times \frac{1}{4} = \frac{12}{4} = 3$	
	And gets: $3 = \Box$	
	So, □ = 3	
	Solve these equations using multiplicative inverses in the way Musa did. Then shock	
	your answers using inspection.	
	a) $4 \times 6 = \Box \times 3$	
	b) $\Box \times 5 = 10 \times 4$	
4)	Marti made up this question to try Musa's method. She found the answer was a	4)
	fraction. Copy her response and answer the questions:	a) Multiplication is
	$4 \times 8 = 5 \times \square$	commutative,
	$4 \times 6 = 1 \times 5$ a) Marti rewrote 5×1 as 1×5 . Explain why she can do this.	b) To isolate the box.
	$4 \times 9 \times 1$ b) Why does Marti multiply both sides by	$5 \times \frac{1}{5} = 1$
	$4 \times 8 \times \frac{5}{5} = 1 \times 3 \times \frac{5}{5}$ the multiplicative inverse of 5?	c) Converting 4 and 8 to
	$\frac{4}{4} \times \frac{8}{4} \times \frac{1}{4} = \Box$ c) What is Marti doing here?	fractions
	1 1 5 32 d) Is Marti's answer a common fraction or	d) Improper fraction
	$\frac{1}{5} = \Box$ an improper fraction?	
5)	Solve the following equations using <i>multiplicative inverses</i> : 5) Answe	l rs
	a) $\Box \times 6 = 5 \times 12$ d) $\Box \times 6 = 5 \times 5$ a)	$\Box = 10$ d) $\Box = \frac{25}{5}$
	b) $9 \times 2 = 3 \times \square$ e) $4 \times \square = 7 \times 5$ b)	$\Box = 6$ e) $\Box = \frac{35}{3}$
	c) $3 \times 5 = \Box \times 2$ c)	$\Box = \frac{15}{2}$
		2



Worksheet 1.9: Numerical equations with whole numbers

This worksheet focuses on solving equations involving division. There are two whole numbers on each side of the equal sign.

Qu	estions	A statement that is balanced has the same result on each side of the equal sign. So, a <i>balanced statement</i> is an equation.	
1)	If twelve is divided by three, we can write it as $12 \div 3$ or	$r \frac{12}{2}$ or $12 \times \frac{1}{2}$.	
,	Write these divisions in two other ways:	3 3	
	a) $20 \div 5$ b) $9 \div 2$ c) $3 \div 3$	$\div 7$ d) $4 \div 4$	
2)	Write these multiplications as divisions:		
	a) $7 \times \frac{1}{2}$ b) $9 \times \frac{1}{4}$ c) $\frac{1}{5} \times \frac{1}{5}$	< 8 d) $\frac{1}{6} \times 6$	
3)	Solve using inspection. The first one has been done for y a) $18 \div 3 = \Box \div 6$ $6 = \Box \div 6$ Think: "What divided by 6 gives 6 $\Box = 36$ b) Now try it this way: $\frac{18}{3} = \frac{\Box}{6}$	ou: ?"	
	c) $\Box \div 8 = 4 \div 2$ You can choose the method used	l in Q3a or in Q3b.	
4)	Give the multiplicative inverse of each number: a) $\frac{1}{6}$ b) $\frac{1}{2}$ c) $\frac{1}{7}$ d) $\frac{2}{3}$	When we multiply <i>multiplicative inverses,</i> we get a product of 1. e.g. $\frac{1}{4} \times 4 = 1$	
5)	 a) Solve this equation using <i>inspection</i>: 16 ÷ 2 × 3 = □ b) Andrew solves the equation: 16 ÷ 2 × 3 = □ ÷ 4 us and answer the questions: 	☐ ÷ 4 sing <i>multiplicative inverses</i> . Copy his response	
	$16 \div 2 \times 3 = \Box \div 4$		
	$16 \times \frac{1}{2} \times 3 = \Box \times \frac{1}{4}$ a) What has Andrew dor	ne in this step?	
	$24 = \Box \times \frac{1}{4}$ b) Show how Andrew go	ot 24 on the left side.	
	$24 \times 4 = \Box \times \frac{1}{4} \times 4$ c) Why did Andrew mult $96 = \Box$	tiply by 4 on each side of the equal sign?	
6)	Solve the following equations using <i>multiplicative invers</i>	es and then check using inspection:	
•,	a) $\frac{\Box}{6} = \frac{12}{3}$ b) $\frac{\Box}{5} = \frac{12}{4}$ c) $30 \div 4 = \frac{\Box}{2}$	$\frac{1}{2}$ d) $\frac{15}{3} = \Box \div 5$ e) $\frac{12}{4} = \frac{\Box}{3}$	
7)	Here is a set of three statements: a) Which st	atements are not balanced? Re-write them using	
	A. $15 \div 3 \times 3 = 15 \times 1$ a \neq sign.		
	B. $15 \div 3 = 5 \div 5 \times 1$ b) Say why C. $\frac{23}{3} \times 3 = 15 \times 1$	they are not balanced.	



Worksheet 1.9: Numerical equations whole numbers

Qu	estions	Answers
1)	If twelve is divided by three, we can write it as $12 \div 3 \text{ or } \frac{12}{3} \text{ or } 12 \times \frac{1}{3}$. Write these divisions in two other ways: a) $20 \div 5$ c) $3 \div 7$ b) $9 \div 2$ d) $4 \div 4$	1) a) $\frac{20}{5}$ or $20 \times \frac{1}{5}$ c) $\frac{3}{7}$ or $3 \times \frac{1}{7}$ b) $\frac{9}{2}$ or $9 \times \frac{1}{2}$ d) $\frac{4}{4}$ or $4 \times \frac{1}{4}$
2)	Write these multiplications as divisions: a) $7 \times \frac{1}{2}$ c) $\frac{1}{5} \times 8$ b) $9 \times \frac{1}{4}$ d) $\frac{1}{6} \times 6$	2) a) $\frac{7}{2}$ b) $\frac{9}{4}$ c) $\frac{8}{5}$ d) $\frac{6}{6}$
3)	Solve using inspection. The first one has been done for you: a) $18 \div 3 = \Box \div 6$ $6 = \Box \div 6$ Think: "What divided by 6 gives 6?" $\Box = 36$ b) Now try it this way: $\frac{18}{3} = \frac{\Box}{6}$ c) $\Box \div 8 = 4 \div 2$ You can choose the method used in Q3a or in Q3b.	3) b) $\frac{18}{3} = \frac{\Box}{6}$ $6 = \frac{\Box}{6}$ $\Box = 36$ b) $\frac{18}{3} = \frac{\Box}{6}$ $\Box \div 8 = 4 \div 2$ $\Box \div 8 = 2$ $\Box = 16$ c) $\Box = 16$ $\Box = 16$ $\Box = 16$ $\Box = 16$
4)	Give the multiplicative inverse of each number: a) $\frac{1}{6}$ b) $\frac{1}{2}$ c) $\frac{1}{5}$ d) $\frac{2}{3}$	4) a) 6 b) 2 c) 5 d) $\frac{3}{2}$
5)	Question and answers: Andrew solves the equation: $16 \div 2 \times 3 = \Box \div 4$ a)What has Andrew dome $16 \times \frac{1}{2} \times 3 = \Box \times \frac{1}{4}$ b)Show how Andrew got $24 = \Box \times \frac{1}{4}$ b)Show how Andrew got $24 \times 4 = \Box \times \frac{1}{4} \times 4$ c)Why did Andrew mult $96 = \Box$ side of the equal sign?	$16 \div 2 \times 3 = \Box \div 4$ using multiplicative inverses. a in this step? 24 on the left side. a iply by 4 on each a <t< th=""></t<>
6)	Solve the following equations using multiplicative inverses and then check using inspection: a) $\frac{\Box}{6} = \frac{12}{3}$ c) $30 \div 4 = \frac{\Box}{2}$ e) $\frac{12}{4} = \frac{\Box}{3}$ b) $\frac{\Box}{5} = \frac{12}{4}$ d) $\frac{15}{3} = \Box \div 5$	6) Answers Multiplicative inverses hint Results a) × each side by 6 $\Box = 24$ b) × each side by 5 $\Box = 15$ c) $\frac{30}{4} \times 2 = \frac{\Box}{2} \times 2$ $\Box = 15$ d) × each side by 5 $\Box = 25$ e) × each side by 3 $\Box = 9$
7)	Here is a set of threea)Which statemstatements:are not balanceA. $15 \div 3 \times 3 = 15 \times 1$ Re-write themB. $15 \div 3 = 5 \div 5 \times 1$ using a \neq signC. $\frac{23}{3} \times 3 = 15 \times 1$ b)Say why they a not balanced.	ents a) B. $15 \div 3 \neq 5 \div 5 \times 1$ b) $5 \neq 1$ red? C. $\frac{23}{3} \times 3 \neq 15 \times 1$ $23 \neq 15$ are



Worksheet 1.10: Numerical equations with whole numbers

This worksheet focuses on the equal sign as a balance for numerical equations. The equations have a mixture of multiplication and division where there are two or more whole numbers on each side of the equal sign.

Qu	estions	
1)	Write the question and select the correct	6) Copy and give the value for \Box in each equation.
	number for \Box .	A. $5 \times 4 \times 3 = \Box \times 4 \times 3$
	a) $5 \times 2 \times 3 = \Box \times 6$	B. $5 \times 4 \times 3 = \Box \times 2 \times 3$
	A. 10 B. 30 C. 5 D. 6	C. $5 \times 4 \times 3 = \Box \times 1 \times 6$
		D. $5 \times 4 \times 3 = 5 \times 2 \times \square$
	b) $6 \times \Box \div 2 = 4 \times 3$	E. $5 \times 4 \times 3 = 5 \times 1 \times \square$
	A 2 B 4 C 12 D 24	
	A. 2 D. 4 C. 12 D. 24	F. $5 \times \Box \times 3 = 3 \times 4 \times 5$
		G. $10 \times \Box \times 3 = 5 \times 4 \times 3$
	$c) \Box \times 5 \times 5 = 5 \times 10 \times 1$	H. $5 \times \Box \times 6 = 4 \times 5 \times 3$
	A. 25 B. 20 C. 5 D. 2	
2)	Copy and solve by inspection:	7) Look at the set of eight equations in Q6 and
	a) $\frac{\Box}{2} \times 5 \times 2 = 3 \times 5$	answer the questions:
	b) $F \times {}^{6} \times 4 = \Box$	a) Note that \Box is sometimes on the right as in
	$b_{j} J \land \frac{1}{4} \land 4 = \Box$	A to E. What is the product of the numbers
3)		on the left in A to E?
3)	a) Use the equation $10 \times 4 = 40$ to complete	b) Sometimes \Box is on the left of the equal sign
	a) Use the equation $10 \times 4 = 40$, to complete the following:	as in F to H. What is the product of the
	the following: $10 \times 4 \times 2 = 40 \times \Box$	numbers on the right in F to H?
	A. $10 \times 4 \times 2 = 40 \times \Box$	c) So, what should the product be on the sides
	B. $40 \times \frac{1}{5} = 4 \times 10 \div \square$	with a \Box in A to E? And in F to H?
	C. $10 \times 4 \div \Box = 40 \div 5$	
	b) Which answers in Q3 a are the same? Explain	8) Use the numbers 4, 5 and 6 to make
	your answer.	multiplication equations:
4)	Solve these equations by inspection. Rewrite the	a) two equations with \Box on the right
•,	division as multiplication if you find it is easier.	e.g. $4 \times 5 \times 6 = \Box \times 6 \times 20$
	a $F \times {}^{6} \times A = \Box$	
	a) $5 \times \frac{1}{4} \times 4 = \Box$	b) two equations with \Box on the left
	b) $5 \times 6 \div 4 \times 4 = \Box$	e.g. $\Box \times 6 \times 20 = 6 \times 5 \times 4$
	c) $\frac{\Box}{2} \times 4 \times 2 = 3 \times 4$	
	d) $\Box \div 2 \times 4 \times 2 = 3 \times 4$	
		-
5)	Solve these equations using inverses.	
1	a) $/ x_0 x_2 = \Box x_0$ b) $\Box \cdot 2 x_1 = 1 \times 10 \cdot 2$	
	$ \begin{array}{c} u_{1} \\ \vdots \\ $	
1	$U_1 \sqcup X \lor \forall 4 = 10 X \lor 2 \div 2$	
	$\begin{array}{c} u_{1} 10 \neq 5 \times 2 = 10 \times \Box \neq 2 \\ a_{1} 0 \times \Box \times 2 = 2 \times 5 \times 2 \end{array}$	



Worksheet 1.10: Numerical equations with whole numbers

Que	Questions and answers				
1)	Write the question and select the correct	6) Copy and give the value for \Box in each equation.			
	number for \Box .	A. $5 \times 4 \times 3 = \Box \times 4 \times 3$			
	a) $5 \times 2 \times 3 = \Box \times 6$	B. $5 \times 4 \times 3 = \Box \times 2 \times 3$			
	A. 10 B. 30 C. 5 D. 6	$C. 5 \times 4 \times 3 = \Box \times 1 \times 6$			
		D. $5 \times 4 \times 3 = 5 \times 2 \times \square$			
	b) $6 \times \Box \div 2 = 4 \times 3$	$E. 5 \times 4 \times 3 = 5 \times 1 \times \square$			
	A. 2 B. 4 C. 12 D. 24				
		$F. 5 \times \Box \times 3 = 5 \times 4 \times 5$			
	c) $\Box \times 5 \times 5 = 5 \times 10 \times 1$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$			
	A. 25 B. 20 C. 5 D. 2				
	Answers	Answers for 🗆			
	a) C b) B c) D	A. 5 C. 10 E. 12 G. 2			
		B. 10 D. 6 F. 4 H. 2			
2)	Conversion Annuar	7) Look at the set of eight equations in OC and ensues the			
2)	Copy and solve by inspection: Answers	7) Look at the set of eight equations in Q6 and answer the			
	a) $\frac{1}{2} \times 5 \times 2 = 3 \times 5$ a) $\Box = 3$	questions. a) Note that \Box is sometimes on the right as in A to E			
	b) $5 \times \frac{6}{4} \times 4 = \Box$ b) $\Box = 30$	What is the product of the numbers on the left in			
2)		A to E?			
3)		b) Sometimes \Box is on the left of the equal sign as in F			
	a) Use the equation $10 \times 4 = 40$, to complete the	to H. What is the product of the numbers on the			
	following: $10 \times 4 \times 2 = 40 \times \Box$	right in F to H?			
	A. $10 \times 4 \times 2 = 40 \times \Box$	c) So, what should the product be on the sides with a			
	B. $40 \times \frac{1}{5} = 4 \times 10 \div \Box$	\Box in A to E? And in F to H?			
	C. $10 \times 4 \div \Box = 40 \div 5$				
	b) which answers in Q3a are the same? Explain your	Answers			
		a) 60			
	a) b) D and C: Multiplying by ¹	b) 60			
	A. $\Box = 2$ the second sec	c) 60			
	B. $\Box = 5$ the same as dividing by 5				
	C. $\Box = 5$	8) Use the numbers 4, 5, and 6 to make multiplication			
		equations:			
4)	Solve these equations by inspection. Rewrite the division	a) two equations with \Box on the right			
	as multiplication if you find it is easier.	e.g. $4 \times 5 \times 6 = \Box \times 6 \times 20$			
	a) $5 \times \frac{3}{4} \times 4 = \Box$	b) two equations with \Box on the left			
	b) $5 \times 6 \div 4 \times 4 = \Box$	e.g. $\Box \times 6 \times 20 = 6 \times 5 \times 4$			
	c) $\frac{\Box}{2} \times 4 \times 2 = 3 \times 4$	Answers will differ			
	d) $\Box \div 2 \times 4 \times 2 = 3 \times 4$	a) $4 \times 5 \times 6 = \Box \times 2 \times 10$ and			
	Answers for	$2 \times 10 \times \Box = 6 \times 5 \times 4$			
	a) 30 b) 30 c) 3 d) 3	b) $\Box \times 3 \times 10 = 6 \times 5 \times 4$ and			
		$6 \times 4 \times 5 = 6 \times 20 \times \Box$			
5)	Solve these equations using inverses				
5)	Solve these equations using inverses. Answers 2^{1}	7 multiply each side by 1			
	a) $7 \times 3 \times 2 = \Box \times 6$ a) $\Box = 7$	$\frac{2}{6}$			
	b) $\Box \div 2 \times 5 = 1 \times 10 \div 2$ b) $\Box = 2$	2 multiply each side by $\frac{1}{1}$ and $\frac{1}{5}$ or by $\frac{1}{5}$			
	c) $\Box \times 6 \div 4 = 10 \times 3 \div 2$ c) $\Box = 1$	10 multiply each side by $\frac{4}{6}$			
	d) $10 \div 5 \times 2 = 16 \times \Box \div 2$ d) $\Box = \frac{1}{2}$	$\frac{1}{2}$ multiply each side by $\frac{2}{16}$			
	e) $9 \times \Box \times 2 = 3 \times 5 \times 2$	$\frac{16}{5}$ multiply each side by $\frac{1}{5}$			
		3 110 119 19 Court side by 18			



Worksheet 1.11: Numerical equations

This worksheet focuses on the equal sign as a balance for numerical equations. The questions involve a mixture of multiplication and division where there are two or more whole numbers on each side of the equal sign.

Questions

1)	Write the question and select the correct number for \Box . a) $9 \times 3 \times 2 = \Box \times 6$ A. 6 B. 9 C. 27 D. 54 b) $8 \times \Box \div 2 = 4 \times 3$ A. 12 B. 4 C. 3 D. 2 c) $\Box \times 6 \times 6 = 6 \times 12 \times 1$ A. 6 B. 2 C. 36 D. 12	6) Copy and write down the value for \Box in each equation. A. $7 \times 2 \times 3 = \Box \times 2 \times 3$ B. $7 \times 2 \times 3 = \Box \times 1 \times 3$ C. $7 \times 2 \times 3 = \Box \times 1 \times 6$ D. $7 \times 2 \times 3 = 7 \times 2 \times \Box$ E. $7 \times 2 \times 3 = 7 \times 1 \times \Box$ F. $7 \times \Box \times 3 = 3 \times 2 \times 7$ G. $14 \times \Box \times 3 = 7 \times 2 \times 3$
		H. $7 \times \Box \times 6 = 2 \times 7 \times 3$
2) 3) 4)	Copy and solve by inspection: a) $\frac{\Box}{3} \times 5 \times 3 = 3 \times 5$ b) $9 \times \frac{6}{7} \times 7 = \Box$ If $12 \times 4 = 48$ then use it to complete the following: a) $12 \times 4 \times 2 = 48 \times \Box$ b) $48 \times \frac{1}{12} = 4 \times 12 \div \Box$ c) $12 \times 4 \div \Box = 48 \div 12$ d) Which answers are the same? Why? Solve these equations by inspection. Rewrite the division as multiplication if you find it is easier. a) $9 \times \frac{3}{2} \times 2 = \Box$ b) $11 \times 6 \div 3 \times 3 = \Box$ c) $\frac{\Box}{3} \times 4 \times 3 = 9 \times 4$ d) $\Box \div 7 \times 4 \times 7 = 6 \times 2$	 7) Look at the set of eight equations in Q6 and answer these questions: a) Note that □ is sometimes on the right of the equal sign as in A to E. What is the product of the numbers on the left in A to E? b) Sometimes □ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H? c) So, what should the product be on the sides with □ in A to E? And in F to H?
5)	Solve these equations using inverses. a) $\Box \div 2 \times 7 = 1 \times 14 \div 2$ b) $8 \times 3 \times 2 = \Box \times 6$ c) $\Box \times 8 \div 4 = 10 \times 4 \div 2$	 8) Use the numbers 8, 9 and 10 to make multiplication equations with: a) two equations with □ on the right
	c) $\Box \times 6 \div 4 = 10 \times 4 \div 2$ d) $12 \div 6 \times 3 = 24 \times \Box \div 2$ e) $10 \times \Box \times 2 = 3 \times 5 \times 2$	b) two equations with \Box on the left e.g. $8 \times \Box \times 2 = 10 \times 8 \times 9$
Worksheet 1.11: Numerical equations



Questions and answers	
 Write the question and select the correct number for □. 	6) Copy and write down the value for \Box in each equation. A. 7 × 2 × 3 = \Box × 2 × 3
a) $9 \times 3 \times 2 = \Box \times 6$ A. 6 B. 9 C. 27 D. 54	B. $7 \times 2 \times 3 = \sqcup \times 1 \times 3$ C. $7 \times 2 \times 3 = \Box \times 1 \times 6$ D. $7 \times 2 \times 3 = 7 \times 2 \times \Box$
b) $8 \times \Box \div 2 = 4 \times 3$ A. 12 B. 4 C. 3 D. 2	E. $7 \times 2 \times 3 = 7 \times 1 \times \square$
c) $\Box \times 6 \times 6 = 6 \times 12 \times 1$ A. 6 B. 2 C. 36 D. 12	F. $7 \times \square \times 3 = 3 \times 2 \times 7$ G. $14 \times \square \times 3 = 7 \times 2 \times 3$ H. $7 \times \square \times 6 = 2 \times 7 \times 3$
Answers a) B b) C c) B	Answers for □ A. 7 C. 7 E. 6 G. 1 B. 14 D. 3 F. 2 H. 1
2) Copy and solve by inspection: a) $-\frac{1}{2} \times 5 \times 3 = 3 \times 5$	 7) Look at the set of eight equations in Q6 and answer these questions:
b) $9 \times \frac{6}{7} \times 7 = \Box$ Answers a) $\Box = 3$ b) $\Box = 54$	a) Note that □ is sometimes on the right of the equal sign as in A to E. What is the product of the numbers on the left in A to E?
3) If $12 \times 4 = 48$ then use it to complete the following: a) $12 \times 4 \times 2 = 48 \times \square$ b) $48 \times \frac{1}{12} = 4 \times 12 \div \square$ c) $12 \times 4 \div \square = 48 \div 12$ d) Which answers are the same? Why? Answers	 b) Sometimes □ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H? c) So, what should the product be on the sides with □ in A to E? And in F to H? Answers a) 42 b) 42
a) $\Box = 2$ d)Q3b and Q3c. Multiplying byb) $\Box = 12$ $\frac{1}{12}$ is the same as dividing by 12.c) $\Box = 12$	c) 42 and 42
 4) Solve these equations by inspection. Rewrite the division as multiplication if you find it is easier. a) 9 × ³/₂ × 2 = □ b) 11 × 6 ÷ 3 × 3 = □ c) □ ÷ 7 × 4 × 7 = 6 × 2 d) [□]/₃ × 4 × 3 = 9 × 4 	 8) Use the numbers 8, 9 and 10 to make multiplication equations: a) two equations with □ on the right e.g. 8 × 9 × 10 = 8 × □ × 1 b) two equations with □ on the left e.g. 8 × □ × 2 = 10 × 8 × 9
Answers for □ a) 27 b) 66 c) 3 d) 9	Answers will differ, here are two of each: a) $8 \times 9 \times 10 = 4 \times \square \times 2$ and $3 \times \square \times 3 = 8 \times 9 \times 10$ b) $4 \times \square \times 2 = 10 \times 8 \times 9$ and $10 \times 8 \times 9 = 5 \times 2 \times \square$
5) Solve these equations using inverses. a) $8 \times 3 \times 2 = \Box \times 6$ Answers for	
b) $\Box \div 2 \times 7 = 1 \times 14 \div 2$ a) 8 c) $\Box \times 8 \div 4 = 10 \times 4 \div 2$ b) 2	c) 10 e) $\frac{3}{2}$ d) $\frac{1}{2}$
d) $12 \div 6 \times 3 = 24 \times \Box \div 2$ e) $10 \times \Box \times 2 = 3 \times 5 \times 2$ <u>Note</u> : $\times \frac{2}{1}$ th	en $\times \frac{1}{7}$ is the same as $\times \frac{2}{7}$



Worksheet 1.12: Numerical equations with whole numbers

This worksheet focuses on solving numerical equations involving addition, subtraction, multiplication, and division of large numbers. There are two or more whole numbers on each side of the equal sign.

Questions	
 The equation 4630 + 490 = □ + 450 has two whole numbers on each side. Without doing any calculations, use this information to balance each of the following statements: a) 4630 + 490 - 450 = b) □ + 450 - 490 = c) 4630 = d) □ = 	 4) Given that 362 + 29 = □ + 31, which of the following statements are true? a) Because 31 is 2 more than 29, □ is 2 more than 362 b) Because 31 is 2 more than 29, □ is 2 less than 362 c) □ = 362
 2) Below is a set of four equations. A. 4863 + 2121 = □ + 2120 B. 4863 + 2121 + 70 = □ + 2120 + 70 C. 40 + 4863 + 2121 = □ + 2120 + 40 D. 2121 + 4863 + 64 = 64 + □ + 2120 a) Highlight the parts which are the same for each equation. b) Will the value of the box be the same for all the equations? Give reasons for your response. c) Solve all the equations. d) Was your answer in Q4b correct? Why/why not? 	5) Solve these equations using inverses: a) $74 \times 30 \times 20 = \Box \times 600$ b) $\frac{\Box}{40} \times 4 \times 20 = 3 \times 4$ c) $\Box \times 60 \div 4 = 10 \times 150 \div 10$ d) $200 \div 5 \times 2 = 400 \times \Box \div 5$
 3) Solve these equations using inverses. a) 512 + 347 = 510 +□ b) 794 - 56 = □ - 52 c) 1600 - □ = 1200 - 300 d) 1200 - □ + 50 = 1000 - 300 + 50 	6) Give a value for \Box to balance these statements: a) $1564 \times \frac{\Box}{100} = 1564 \times 1$ b) $\frac{\Box}{215} \times 215 = 307 \times 3$ c) $2222 \times \Box = 1111$ d) $5000 \times \frac{\Box}{1428} = 2500 \times 2$



Worksheet 1.12: Numerical equations with whole numbers

Qu	estions	Answers
1)	The equation $4630 + 490 = \Box + 450$ has two whole numbers on each side. Without doing any calculations, use this information to balance each of the following statements: a) $4630 + 490 - 450 =$ b) $\Box + 450 - 490 =$ c) $4630 =$ d) $\Box =$	1) a) \Box b) 4630 c) \Box + 450 - 490 d) 4630 + 490 - 450
2)	 Below is a set of four equations. A. 4863 + 2121 = □ + 2120 B. 4863 + 2121 + 70 = □ + 2120 + 70 C. 40 + 4863 + 2121 = □ + 2120 + 40 D. 2121 + 4863 + 64 = 64 + □ + 2120 a) Highlight the parts which are the same for each equation. b) Will the value of the box be the same for all the equations? Give reasons for your response. c) Solve all the equations. d) Was your answer in Q4b correct? Why/why not? 	 2) a) 4863 + 2121 = □ + 2120 b) Yes, because we are adding the same value to each side c) □ = 4864 for all equations d) Yes. The answer stayed the same
3)	Solve these equations using inverses. 3) a) $512 + 347 = 510 + \Box$ a b) $794 - 56 = \Box - 52$ b c) $1600 - \Box = 1200 - 300$ c d) $1200 - \Box + 50 = 1000 - 300 + 50$ d	$\square = 349$, subtract 510 from each side $\square = 798$, add 52 to each side $\square = 700$, add \square on each side, subtract 900 on each side $\square = 500$, add \square on each side, subtract 750 on each side
4)	 Given that 362 + 29 = □ + 31, which of the following statements are true? a) Because 31 is 2 more than 29, □ is 2 more than 362 b) Because 31 is 2 more than 29, □ is 2 less than 362 c) □ = 362 	4) b) is true
5)	Solve these equations using inverses: a) $74 \times 30 \times 20 = \Box \times 600$ b) $\frac{\Box}{40} \times 4 \times 20 = 3 \times 4$ c) $\Box \times 60 \div 4 = 10 \times 150 \div 10$ d) $200 \div 5 \times 2 = 400 \times \Box \div 5$	5) a) $\Box = 74$ b) $\Box = 6$ c) $\Box = 10$ d) $\Box = 1$
6)	Give a value for \Box to balance these statements: a) $1564 \times \frac{\Box}{100} = 1564 \times 1$ b) $\frac{\Box}{215} \times 215 = 307 \times 3$ c) $2222 \times \Box = 1111$ d) $5000 \times \frac{\Box}{1428} = 2500 \times 2$	6) a) $\Box = 100$ b) $\Box = 307 \times 3$ c) $\Box = \frac{1}{2}$ d) $\Box = 1428$



Worksheet 1.13: Numerical equations with whole numbers

This worksheet focuses on solving numerical equations involving addition, subtraction, multiplication, and division of large numbers. There are two or more whole numbers on each side of the equal sign.

Questions	
1) The equation $8570 - 4123 = \Box - 4124$ has two whole numbers on each side. Without doing any calculations, use this information to balance each of the following statements: a) $8570 - 4123 + 4124 =$ b) $\Box - 4124 + 4123 =$ c) $8570 =$ d) $\Box =$	 4) Given that 953 + 24 = □ + 30, which of the following statements are true? a) Because 30 is 6 more than 24, □ is 6 more than 953. b) Because 30 is 6 more than 24, □ is 6 less than 953. c) □ = 977
 2) Below is a set of four equations. A. 1234 - 56 - 78 - 0 = □ - 0 B. 1234 - 56 - 78 - 10 = □ - 10 C. 1234 - 56 - 78 - 20 = □ - 20 D. 1234 - 56 - 78 - 30 = □ - 30 a) Highlight the parts which are the same for each equation. b) Will the value of the box be the same for all the equations? Why or why not? c) Knowing that 1234 - 56 - 78 = 1100, without doing a calculation, what is the value of the box in each equation? d) Was your answer in Q2b correct? 	5) Solve these equations using inverses: a) $99 \times \square \times 2 = 33 \times 6 \times 3$ b) $\square \div 18 \times 22 = 11 \div 18$ c) $50 \times \frac{\square}{20} = 1 \times 1000 \div 20$ d) $400 \div 2 \times 5 = 400 \times \square \div 2$
 3) Solve these equations using inverses. a) 2 130 + □ = 7 130 + 3 130 b) □ + 915 = 472 + 910 c) 432 + 227 + 100 = 510 + □ + 100 d) 824 + 53 - 200 = □ + 52 - 200 	6) Give a value for \Box to balance these statements: a) $\frac{832}{15} \times \Box = 832 \times 1$ b) $873 \times \frac{201}{\Box} = 67 \times 3$ c) $6\ 666 \times \Box = 2\ 222$ d) $\frac{231}{51} \times \Box = 20 \div 20$



Worksheet 1.13: Numerical equations with whole numbers

Qu	estions	An	swers
1)	The equation $8570 - 4123 = \Box - 4124$ has two whole numbers on each side. Without doing any calculations, use this information to balance each of the following statements: a) $8570 - 4123 + 4124 =$ b) $\Box - 4124 + 4123 =$ c) $8570 =$ d) $\Box =$	1)	 a) □ b) 8 570 c) □ - 4 124 + 4 123 d) 8 570 - 4 123 + 4 124
2)	 Below is a set of four equations. A. 1 234 - 56 - 78 - 0 = □ - 0 B. 1 234 - 56 - 78 - 10 = □ - 10 C. 1 234 - 56 - 78 - 20 = □ - 20 D. 1 234 - 56 - 78 - 30 = □ - 30 a) Highlight the parts which are the same for each equation. b) Will the value of the box be the same for all the equations? Why or why not? c) Knowing that 1 234 - 56 - 78 = 1 100, without doing a calculation, what is the value of the box in each equation? 	2)	 a) 1234 - 56 - 78 and □ b) Yes, because we are subtracting the same value from each side c) □ = 1100 d) Yes
	d) Was your answer in Q2b correct?		
3)	Solve these equations using inverses. 3 a) $2 130 + \Box = 7 130 + 3 130$ b) $\Box + 915 = 472 + 910$ c) $432 + 227 + 100 = 510 + \Box + 100$ d) $824 + 53 - 200 = \Box + 52 - 200$) b) c) d)	$\Box = 8 130 \text{ subtract } 2 130 \text{ from each side}$ $\Box = 467 \text{ subtract } 915 \text{ from each side}$ $\Box = 149 \text{ subtract } 100 \text{ and } 510 \text{ from each side}$ $\Box = 825 \text{ add } 200 \text{ and subtract } 52 \text{ from each side}$
4)	 Given that 953 + 24 = □ + 30, which of the following statements are true? a) Because 30 is 6 more than 24, □ is 6 more than 953. b) Because 30 is 6 more than 24, □ is 6 less than 953. c) □ = 977 	4)	b) is true
5)	Solve these equations using inverses: a) $99 \times \square \times 2 = 33 \times 6 \times 3$ b) $\square \div 18 \times 22 = 11 \div 18$ c) $50 \times \frac{\square}{20} = 1 \times 1000 \div 20$ d) $400 \div 2 \times 5 = 400 \times \square \div 2$	5)	a) $\Box = 3$ multiply by $\frac{1}{99 \times 2}$ on each side b) $\Box = \frac{1}{2}$ multiply by $\frac{18}{2}$ on each side c) $\Box = 20$ multiply by $\frac{20}{50}$ on each side d) $\Box = 5$ multiply by $\frac{2}{400}$ on each side
6)	Give a value for \Box to balance these statements: a) $\frac{832}{15} \times \Box = 832 \times 1$ b) $873 \times \frac{201}{\Box} = 67 \times 3$ c) $6\ 6666 \times \Box = 2\ 222$ d) $\frac{231}{51} \times \Box = 20 \div 20$	6)	a) $\Box = 15$ b) $\Box = 873$ c) $\Box = \frac{1}{3}$ d) $\Box = \frac{51}{231}$



Worksheet 2.1: Numerical equations with integers

This worksheet focuses on solving numerical equations involving addition. There are two integers on each side of the equal sign.

Questions 1) Thabo solved these equations:				When solving a linear equation, we find		
				a value to balance the result on each		
A. $5 + 3 = 3 + \Box$	Answer:	$\Box = -5$				
B. $(-5) + 3 = \Box + 3$	Answer:	$\Box = -8$	a) b)	Correct any mistakes Thabo has made. Why do you think he made the		
C. $(-3) + \Box = 5 + (-3)$	Answer:	$\Box = 5$	2)	mistakes?		
D. $(-5) + \Box = (-3) + (-5)$	Answer:	$\Box = -3$				
Some of his answers are wrong.						
2)a) What is the additive inverse ofb) What is the additive inverse of	of —7? How of 3? How do	do you knov o you know?	v?			
3) Look at the equation $5 + (-8) =$ learners find that the solution is Read their methods carefully. Ma	$=$ \Box + 4. Sha -7. ake sure you	ahid and Mc can link the	ona solo words	ve the equation in different ways. Both and the statements.		
Shahid first works out the value or	n the side with	nout the \Box :				
He adds 5 and -8 to get -3 . $5 + (-8) = \Box + 4$ He now knows $\Box + 4$ must equal -3 . $5 - 8 = \Box + 4$ Shahid thinks 'What must I add to 4 to get -3 ?' This is the value of \Box . $-3 = \Box + 4$ Shahid gets: $\Box = -7$						
Mona uses solving using additive i	nverses.			$5 + (-8) = \Box + 4$		
Mona's first step is to simplify 5 +	(—8) just like	e Shahid did:		$5-8 = \Box + 4$ $-3 = \Box + 4$		
Mona then subtracts 4 from each side of the equation to get \Box on its own. (She uses the <i>additive inverse</i> of 4) $-3 - 4 = \Box + 4 - 4$ $-7 = \Box + 0$						
Mona then simplifies each side of	the equation	to get a value	for \Box .			
She gets:				$-7 = \Box$		
Which is the same as:						
Look at the equations below: A. $6 + (-3) = \Box + 5$	B. (-3) +	(−3) = □ -	⊦1	C. $\Box + (-7) = (-9) + 2$		
a) Quickly work out the answerb) Now try to solve the equatioc) Look at your response to equ	s using Shah ns using Moi iation B. Wri	id's method na's method te what you	. Write which did in	down only the answers. uses <i>additive inverses.</i> each step and say why you did it.		
4) If we know that $(-5) + \Delta = 0$, the second seco	nen which of	the followir	ng state	ements are TRUE? Give reasons.		
a) The value of Δ is -5 .		d)	The v	ralue of Δ is 5.		
b) Δ can be any whole number.		e)	Δ and	1-5 are additive inverses of each other.		
c) The value of Δ is 0.		f)	Δ is t	he additive identity for addition.		



Worksheet 2.1: Numerical equations with integers

Questions	Answers
1) Thabo solved these equations:	1)
A. $5+3=3+\Box$ Answer: $\Box = -5$	a) b)
B. $(-5) + 3 = \Box + 3$ Answer: $\Box = -8$	A. $\Box = 5$ He wrote the additive inverse of 5.
C. $(-3) + \Box = 5 + (-3)$ Answer: $\Box = 5$	B. $\Box = -5$ He added 5 and 3 and then attached the negative or be
D. $(-5) + \Box = (-3) + (-5)$ Answer: $\Box = -3$	thought the result on the
Some of his answers are wrong.	right must give – 5.
a) Correct any mistakes Thabo has made.	
b) Why do you think he made the mistakes?	
, , ,	
2)	2)
a) What is the additive inverse of -7 ? How do you know?	a) $+7$ because $-7 + 7 = 0$
b) What is the additive inverse of 3? How do you know?	b) -3 because $-3 + 3 = 0$
3) Look at the equation $5 + (-8) = \Box + 4$. Shahid and Mona	3)
solve the equation in different ways. Both learners find that the	a) A. $\Box = -2$ B. $\Box = -7$ C. $\Box = 0$
solution is -7 .	b)
Read their methods carefully. Make sure you can link the words	A. $6 + (-3) = \Box + 5$
and the statements.	$3-5=\Box+5-5$
Look at the equations below: $A = (-1)^{-1} = $	$-2 \equiv \Box$
A. $0 + (-3) = \Box + 5$ P. $(-3) + (-2) = \Box + 1$	B. $(-3) + (-3) = \Box + 1$
$\begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 $	$-6 = \Box + 1$
$(1)^{-1}(1)^$	$-6 - 1 = \Box + 1 - 1$
a) Ouickly work out the answers using Shahid's method.	$-7 = \Box$
Write down only the answers.	(-7) = (-9) + 2
b) Now try to solve the equations using Mona's method	$\Box + (-7) = -7$
which uses additive inverses.	$\Box = (-7) + 7$
c) Look at your response to equation B. Write what you did in	$\Box = 0$
each step and say why you did it.	c)
	Add -3 and -3 to see what $\Box + 1$ must
	equal. Subtract 1 from each side to get \Box on
	its own. Simplify each side to get a result for
	□.
4) If we know that $(-5) + \Delta = 0$, then which of the following	4)
statements are TRUE? Give reasons.	E: The value of Δ is 5 because $-5 + 5 = 0$
A. The value of Δ is -5 .	F: Δ and -5 are additive inverses of each other.
B. Δ can be any whole number.	Additive inverses add up to 0.
C. Δ is the additive identity for addition.	
D. The value of Δ is 0.	
E. The value of Δ is 5.	
F. Δ and -5 are additive inverses of each other.	



Worksheet 2.2: Numerical equations with integers

This worksheet focuses on solving numerical equations. Most equations involve subtraction and there are integers on each side of the equal sign.

	A statement that is balanced has the same result on both sides of the equal sign. So, a <i>balanced</i>
Questions	statement is an equation.
1) Solve the equations by inspection. Remember: Subtracting a negative number is the satisfied a positive number, e.g. $2 - (-3) = 2 + 3$ a) $7 - 0 = 8 - \Box$ b) $\Box - (-2) = 9 - 6$ c) $(-7) - 5 = \Box + (-4)$ d) $9 + (-3) = \Box - (-5)$	3) If you know that $-8 - (-4) = -7 + 3$, use it to solve these equations: a) $-8 - (-4) + 7 = \Box$ b) $-8 - (-4) - 3 = \Box$ c) $7 - 3 + 4 = \Box$ d) $\Box = -7 + 3 + 8 - (-4)$
 2) Lina and Nisha solve the equation 6 − (−3) = □ − different ways. Read them carefully and make sure you can see the between the words and the statements. 	- 4 in 4) If $-\Box - \Delta = -7 - 5$, give <i>four</i> sets of values for \Box and Δ that will balance the statement.
Lina uses inspection, as you did in Q1. $6 - (-3) = \Box$ Lina adds 3 and 6 to get: $9 = \Box - A$ Lina then thinks,'What subtract 4 gives 9?'She gets: $\Box = 13$	5) Look at these four statements and answer the questions that follow: A. $2 + (-3) = (-4) + 3$ B. $2 + (-3) + 5 = (-4) + 3$ C. $2 + (-3) - 1 + 1 = (-4) + 3$ D. $2 + (-3) - 2 = (-4) + 3$
Nisha uses additive inverses.Nisha subtracts -3 from 6 to get: $6 - (-3) = \Box - 4$ $9 = \Box - 4$ Nisha adds 4 to on each to get the \Box on its own. $9 + 4 = \Box - 4 + \Box$ $13 = \Box$ which is the same as:	 Highlight 2 + (-3) and (-4) + 3 in each equation. This will help you see the structure of the statements. a) Which statements are equations? b) Rewrite the statements that are not equations with the ≠ sign.
 Look at the equations below: A6-3 = □ - 2 B. □ - 4 = 10 - (-3) C. □ - (-7) = 13 - 11 a) Solve them using Lina's method. Just write to answers. b) Now try to solve the equations using Nisha's method which uses additive inverses. c) Write down what you did in each step of equation A in Q2b and why you did it. 	6) Here is a list of six statements: A. $\diamond + 2 = \diamond$ B. $\otimes + 2 = \diamond$ C. $\diamond + 6 = \otimes - 2 + 6$ D. $\diamond + 5 = \otimes + 3$ E. $\diamond + 5 = \otimes + 7$ F. $\diamond + 2 - 3 = \otimes + 3$ s You are now told that $\diamond = \otimes + (-2)$. Use this information to decide which statements are balanced. Write 'EQUATION' for those statements that are balanced.



Worksheet 2.2: Numerical equations with integers

Qu	estions	Ans	swers
1)	Solve the equations by inspection. Remember:	1)	Results
	Subtracting a negative number is the same as adding a		a) 1 b) 1 c) -8 d) 1
	positive number, e.g. $2 - (-3) = 2 + 3$		
	a) $7 - 0 = 8 - \Box$		
	b) $\Box - (-2) = 9 - 6$		
	c) $(-7) - 5 = \Box + (-4)$		
	d) $9 + (-3) = \Box - (-5)$		
2)	Lina and Nisha solve the equation	2)	
-/	$6 - (-3) = \Box - 4$ in different ways:	-/	a) $A \Box = -7 B \Box = 17 C \Box = -5$
	Boad them carefully and make cure you can see the links		b)
	he the second second the state second s		5)
	between the words and the statements.	Α.	. C.
	Look at the equations below:		$6 - 3 = \Box - 2$ $\Box - (-7) = 13 - 11$
	A. $-6 - 3 = \Box - 2$		$3 = \Box - 2 \qquad \Box + 7 = 2$
	B. $\Box - 4 = 10 - (-3)$		$3 + 2 = \Box + 2 - 2$ $\Box + 7 - 7 = 2 - 7$
	C. $\Box - (-7) = 13 - 11$		$5 = \Box + 0$ $\Box + 0 = -5$
	a) Solve them using Lina's method. Just write the		$5 = \square$ $\square = -5$
	answers.	R	
	b) Now try to solve the equations using Nisha's method	D.	$\Box = 4 - 10 = (-3) \text{Simplify 10} (-2) = 12 + 2$
	which uses additive inverses.		$\Box = 4 - 13 \qquad \text{supply } 10 - (-3) = 13 \text{ to}$
	c) Write down what you did in each step of equation A		$\Box = 4 = 13$ see what the left side must
	in Q2b and why you did it.		$\Box - 4 + 4 = 13 + 4$ equal; add 4 on each side to get
			$\Box - 0 = 1/ \qquad \Box \text{ on own.}$
			$\Box = 17$
3)	If you know that $-8 - (-4) = -7 + 3$, use it to solve	3)	Use $-8 - (-4) = -7 + 3$ i.e. $-8 + 4 = -7 + 3$
	these equations:		a) $\Box = 3$ Add 7 on each side
	a) $-8 - (-4) + 7 = \Box$		b) $\Box = -7$ Subtract 3 from each side
	b) $-8 - (-4) - 3 = \Box$		c) $\Box = 8$ Add 7 and subtract 3 on each side
	c) $7 - 3 + 4 = \Box$		d) $\Box = 0$ Add 8 and add (-4) to each side
	d) $\Box = -7 + 3 + 8 + (-4)$		
4)	If $-\Box = A = -7 - 5$ give four sets of values for \Box and	4)	Many possible answers will result in a difference of -12
-,	A that will balance the statement	,,	12 or 7 and 5 : 8 and 4 : -10 and 2 : 0 and 12
5)	Look at these four statements and answer the questions	5)	
	that follow:		a)
	A. $2 + (-3) = (-4) + 3$		A. $2 + (-3) = (-4) + 3$
	B. $2 + (-3) + 5 = (-4) + 3$		C. $2 + (-3) - 1 + 1 = (-4) + 3$
	C. $2 + (-3) - 1 + 1 = (-4) + 3$		D. $2 + (-3) - 2 = (-4) + 1$
	D. $2 + (-3) - 2 = (-4) + 3$		b)
High	hight $2 + (-3)$ and $(-4) + 3$ in each equation. This will		B. $2 + (-3) + 5 \neq (-4) + 3$
help	you see the structure of the statements.		(), ; ; (), ; 0
	a) Which statements are equations?		
	b) Rewrite the statements that are not equations with		
	the \neq sign.		
6)	Here is a list of six statements:		6) Given: $\diamond = \diamond + (-2)$
	A. $\diamond + 2 = \diamond$ D. $\diamond + 5 = \diamond + 3$		
	B. $\otimes +2 = \diamond$ E. $\diamond +5 = \otimes +7$		A. EQUATION
	C. $\diamond + 6 = \heartsuit - 2 + 6$ F. $\diamond + 2 - 3 = \heartsuit + 3$		B. Not an equation
			C. EQUATION
	You are now told that $\Diamond = \bigcirc + (-2)$. Use this information	to	D. FOUATION
	decide which statements are balanced. Write 'EQUATION' f	or	E Not an equation
	those statements that are balanced		
	mose statements that die Dalanceu.		r. Not an equation



Worksheet 2.3: Numerical equations with integers

This worksheet focuses on solving numerical equations that have addition on each side, or subtraction on each side of the equal sign. The equations involve integers.

Qu	estions					
1)	Solve these equations by inspection: a) $7 + 3 = \Box + 6$ b) $7 - 3 = 6 - \Box$ c) $7 - 3 = \Box - 6$ d) $7 - 3 = \Box - 6$	4)	Remember: A balanced statement has the same result on each side of the equal sign. e.g. $-9 + 3 = -10 + 4$. The result on each side of the equal sign is -6 .			
	a) $7-3 = 7 + \Box$ e) $\Box - 2 = 5 - 8$ f) $10 - \Box = 12 - (-2)$ g) $-4 + \Box = 5 + (-7)$		If the result on each side of the equal sign is not the same, the statement is not balanced. e.g. $-9 + 3 = -10 + 4 - 3$. The result on the left of the equal sign is -6 but the result on the right is -9 .			
2)	Select the correct option for \Box : a) $-10 - 2 = \Box - 4$ i) 8 ii) -12 iii) -8 iv) -16		Therefore, the statement is not balanced, and we write $-9 + 3 \neq -10 + 4 - 3$. We say the "the left side <i>is not equal to</i> the right side".			
	b) $7 + \Box = -9 + 2$ i) 0 ii) -7 iii) 7 iv) -14		Look at these three statements: A. $11 + (-2) = 4 - (-5)$			
3)	Solve these four equations using <i>inspection</i> and then using <i>additive inverses</i> .					B. $11 + (-2) - 5 = 4 - (-5)$ C. $11 + (-2) + (-3) = 4 - (-5) - 3$
	a) $-10 - (-3) = \Box - 1$ b) $\Box + 3 = (-8) + 9$ c) $\Box + (-6) = (-7) + (-3)$ d) $12 - (-4) = \Box - 6$		 a) Decide which statements are not balanced. Rewrite them using the ≠ sign. b) Show how you can balance the statement/s you wrote in Q4a by changing the right side. 			
		5)	The equation $-8 - (-4) = \Box - 3$ has two integers on each side. Use this information to decide whether the following equations are balanced or NOT balanced. If they are not balanced, re-write them with the \neq sign.			
			a) $-8 - (-4) + 3 = \Box + 3$ b) $\Box - 3 - 4 = -8$ c) $-8 = \Box - 3 + (-4)$ d) $\Box = -8 + 4 + 3$			



Worksheet 2.3: Numerical equations with integers

Qu	estions	An	swe	rs
2)	Solve these equations by inspection: a) $7 + 3 = \Box + 6$ b) $7 - 3 = 6 - \Box$ c) $7 - 3 = \Box - 6$ d) $7 - 3 = 7 + \Box$ e) $\Box - 2 = 5 - 8$ f) $10 - \Box = 12 - (-2)$ g) $-4 + \Box = 5 + (-7)$ Select the correct option for \Box : a) $-10 - 2 = \Box - 4$ i) 8 ii) -12 iii) -8 iv) -16 b) $7 + \Box = -9 + 2$ ii) $0 = \Box - 4$ iii) -12 iii) 7 iv) -14	2)	a) b) c) d) g) a) b)	$\Box = 4$ $\Box = 2$ $\Box = 10$ $\Box = -3$ $\Box = -1$ $\Box = -4$ $\Box = 2$ iii) -8 viii) -14
3)	Solve the equations using <i>inspection</i> and then using <i>additive</i> <i>inverses</i> . a) $-10 - (-3) = \Box - 1$ b) $\Box + 3 = (-8) + 9$ c) $\Box + (-6) = (-7) + (-3)$ d) $12 - (-4) = \Box - 6$	3)	a) b) c) d)	InspectionAdditive inverses hint $\Box = -6$ Add 1 to each side $\Box = -2$ Subtract 3 from each side $\Box = -4$ Subtract (-6) from each side or add 6 to each side $\Box = 22$ Add 6 to each side
4)	 Look at these three statements: A. 11 + (-2) = 4 - (-5) B. 11 + (-2) - 5 = 4 - (-5) C. 11 + (-2) + (-3) = 4 - (-5) - 3 a) Decide which statements are not balanced. Re-write them using the ≠ sign. b) Show how you can balance the statement/s you wrote in Q4a by changing the right side. 	4)	a) b)	B. $11 + (-2) - 5 \neq 4 - (-5)$ B. Subtract 5 from the right side: 11 + (-2) - 5 = 4 - (-5) - 5
5)	The equation $-8 - (-4) = \Box - 3$ has two integers on each side. Use this information to decide whether the following equations are balanced or NOT balanced. If they are not balanced, re-write them with the \neq sign. a) $-8 - (-4) + 3 = \Box + 3$ b) $\Box -3 - 4 = -8$ c) $-8 = \Box -3 + (-4)$ d) $\Box = -8 + 4 + 3$	5)	a) b) c) d)	$-8 - (-4) + 3 \neq \Box + 3 \text{ NOT}$ balanced $\Box - 3 - 4 = -8 \qquad \text{Balanced}$ $-8 = \Box - 3 + (-4) \qquad \text{Balanced}$ $\Box = -8 + 4 + 3 \qquad \text{Balanced}$



Worksheet 2.4: Numerical equations with integers

This worksheet focuses on solving equations involving: 1) addition only; or 2) subtraction only. There are two or more integers on each side of the equal sign.

Questions						
1) Three equations are given below. Lee's answers are given next to each equation. Copy the equations						
and answers. If Lee's answer is correct, give it a tick 🗸. If Lee's answer is incorrect, give it a cross 🗙						
and give the correct answer.						
a) $1 + (-3) + 2 = \Box + 2 + 1$ Answer:	a) $1 + (-3) + 2 = \Box + 2 + 1$ Answer: $\Box = 6$					
b) $8 - (-5) - \Box = 14 - 7 - (-6)$ Answer:	$\Box = 3$					
c) $-3-4-\Box = -10-(-4)+4$ Answer:] = -9					
2) Solve the following equations:						
a) $7 - (-3) = \Box + 7$	If you are not told which method to use					
b) $\Box + 4 = 6 + (-3)$	to solve an equation, you can use					
c) $12 - 2 - \Box + 9$	inspection or additive inverses.					
d) $\Box = 4 = 10 = (-5)$						
$u_{1} = 4 = 10 - (-3)$						
3) Below is a set of four equations:	a) Highlight another part which is the same					
11 - (-3) + (-5) - 0 1	in all the equations					
$\begin{array}{c} \begin{array}{c} & 11 \\ B \end{array} \\ \begin{array}{c} 11 \\ \end{array} \\ \begin{array}{c} (-3) \\ \end{array} \\ \begin{array}{c} + (-5) \\ \end{array} \\ \begin{array}{c} -1 \\ \end{array} \\ \end{array} \\ \begin{array}{c} -1 \\ \end{array} \\ \end{array} \\ \begin{array}{c} -1 \\ \end{array} \\ \begin{array}{c} -1 \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} -1 \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} -1 \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} -1 \\ \end{array} \\ $	h) Solve all the equations					
$\begin{array}{c} 0. & 11 & (-3) + (-5) & 1 - \Box & 1 \\ 0 & 11 - (-3) + (-5) - 2 - \Box & -1 \end{array}$	b) Solve all the equations.					
D $11 - (-3) + (-5) - 3 - \Box - 1$	c) compare your answers to the four					
Look at the highlighted part in equation A. Note	equations. What is the relation between					
that this expression appears in all the equations	the values of \Box in each case? What					
that this expression appears in an the equations.	causes this relation?					
4) Below is a set of four equations:	a) Highlight the parts which are the same					
A. $-5 + 3 + (-2) = \Box + (-8)$	for each equation.					
B. $-5+3+(-2)+7 = \Box + (-8)+7$	b) Predict TRUE or FALSE: The value of the					
C. $40 + (-5) + 3 + (-2) = \Box + (-8) + 40$	box will NOT be the same for all the					
D. $3 + (-2) + (-5) - 12 = -12 + \Box + (-8)$	equations. Give reasons for your					
	response					
To predict the answers: Look for what is the	c) Solve all the equations					
same and different in the equations and write	d) Was your prediction in O4b correct?					
what you expect the value of \Box to be.	Why/why not?					
	,					
5) You are given the equation: $\diamond + (-4) = - \diamond - 2$. This	s equation has two unknown values, \Diamond and \heartsuit .					
When you balance the statements below, both unkno	wn values must appear in the equation. They					
could be on different sides of the equal sign, or they c	ould be on the same side as in O5b.					
a) $(-4) + \Diamond =$						
b) $\Diamond + (-4) + \heartsuit =$						
(1) - (2) - (2) + (2) - (2)						
d) $\diamond \pm (-4) \pm 2 = -$						
$ \begin{array}{c} u_{1} & \forall + (-+) + 2 - \\ u_{2} & - 0 - 2 - (-4) - \end{array} $						
$e_{j} - 0 - 2 - (-4) =$						
,						



Worksheet 2.4: Numerical equations with integers

Qu	estions	Answers		
1)	Three equations are given below. Lee's answers are given next to each equation. Copy the equations and answers. If Lee's answer is correct, give it a tick \checkmark . If Lee's answer is incorrect, give it a cross \times and give the correct answer. a) $1 + (-3) + 2 = \Box + 2 + 1$ Answer: $\Box = 6$ b) $8 - (-5) - \Box = 14 - 7 - (-6)$ Answer: $\Box = 3$ c) $-3 - 4 - \Box = -10 - (-4) + 4$ Answer: $\Box = -9$	1) a) Answer: $\Box = 6$ × $\Box = -3$ b) Answer: $\Box = 3$ × $\Box = 0$ c) Answer: $\Box = -9$ × $\Box = -5$		
2)	Solve the following equations: a) $7 - (-3) = \Box + 7$ b) $\Box + 4 = 6 + (-3)$ c) $12 - 2 = \Box + 9$ d) $\Box - 4 = 10 - (-5)$	2) <u>Inspection</u> <u>Additive inverse hint</u> a) $\square = 3$ Subtract 7 from each side b) $\square = -1$ Subtract 4 from each side c) $\square = 1$ Subtract 9 from each side d) $\square = 19$ Add 4 to each side		
3)	Below is a set of four equations: A. $11 - (-3) + (-5) - 0 = \Box - 1$ B. $11 - (-3) + (-5) - 1 = \Box - 1$ C. $11 - (-3) + (-5) - 2 = \Box - 1$ D. $11 - (-3) + (-5) - 3 = \Box - 1$ Look at the highlighted part in equation A. Note that this expression appears in all the equations. a) Highlight another part which is the same in all the equations. b) Solve all the equations. c) Compare your answers to the four equations. What is the relation between the values of \Box in each case? What causes this relation?	 3) a) □ - 1 b) A. □ = 10 B. □ = 9 C. □ = 8 D. □ = 7 c) The value of box decreases by 1 each time. This happens because on the left side we decrease the value by 1 each time too. 		
4)	Below is a set of four equations: A. $-5 + 3 + (-2) = \Box + (-8)$ B. $-5 + 3 + (-2) + 7 = \Box + (-8) + 7$ C. $40 + (-5) + 3 + (-2) = \Box + (-8) + 40$ D. $3 + (-2) + (-5) - 12 = -12 + \Box + (-8)$ a) Highlight the parts which are the same for each equation. b) Predict TRUE or FALSE: The value of the box will NOT be the same for all the equations. Give reasons for your response. c) Solve all the equations. d) Was your prediction in Q4b correct? Why/why not?	 4) a) -5+3+(-2) = and □ + (-8) b) FALSE. The value of the box will be the same because the same number is added to or subtracted from each side of each equation. c) □ = 4 for all equations d) Prediction was correct because □ = 4 for all equations. Dependent on learners' response in 4b. 		
5)	You are given the equation: $\diamond + (-4) = -\heartsuit - 2$. This equation has two unknown values, \diamond and \heartsuit . When you balance the statements below, both unknown values must appear in the equation. They could be on different sides of the equal sign, or they could be on the same side as in Q5b. a) $(-4) + \diamond =$ d) $\diamond + (-4) + 2 =$ b) $\diamond + (-4) + \heartsuit =$ e) $-\heartsuit - 2 - (-4) =$ c) $-\heartsuit - 2 + 0 =$ f) $\heartsuit =$	5) Given: $\diamond + (-4) = -\heartsuit - 2$ a) $(-4) + \diamond = -\heartsuit - 2$ b) $\diamond + (-4) + \heartsuit = -2$ c) $-\heartsuit - 2 + 0 = \diamond + (-4)$ d) $\diamond + (-4) + 2 = -\heartsuit$ e) $-\heartsuit - 2 - (-4) = \diamond$ f) $\heartsuit = -2 - \diamondsuit - (-4)$		



Worksheet 2.5 Numerical equations with integers

This worksheet focuses on solving equations that contain a mixture of addition and subtraction. There are two or more integers on each side of the equal sign.

Questions						
1) Match the columns:						
COLUMN A	COLUMN B					
a) $9 + (-4) - \Box = 10 + (-4) - 7$	Ⅰ. □ = −5					
b) $7 - 4 + (-6) = 8 - 6 + \Box$	$\square = 4$					
c) $-3 + 9 - (-5) = 11 - (-4) - \Box$	$\square = 6$					
	\square IV. $\square = -1$					
2) Solve these four equations by inspection:	3) Solve these equations using additive inverses:					
a) $-5 + \Box = -10 - 3$	a) $8 + (-3) - 5 = \Box + (-2)$					
b) $9 - (-2) = \Box + (-2)$	b) $\Box + 4 - (-2) = 6 - 7 - 3$					
c) $8 - \Box = 7 - (-1)$	c) $-11 + (-1) - (-3) = \Box + (-4) + (-2)$					
d) $\Box - (-4) = -9 + (-8)$	To predict the answers: Look for what is the					
	same and different in the equations and write					
4) Given: $-4 + 3 + (-6) = \Box + (-5)$	what you expect the value of \Box to be.					
a) Look for what is the same and what is						
different in this set of four equations:						
A. $-4 + 3 + (-6) - 4 = \Box + (-5)$	b) Do you expect the values for \Box to be the					
B. $-4 + 3 + (-6) - 5 = \Box + (-5)$	same for each equation? Give a reason for					
C. $-4 + 3 + (-6) - 6 = \Box + (-5)$	your answer.					
D. $-4 + 3 + (-6) - 7 = \Box + (-5)$	c) Solve the equations.					
	d) Was your prediction in Q4b correct?					
5) Given: $9 - (-3) + (-2) = \Box + 2 + 4$						
a) Solve each equation:	c) Predict the answers to the following:					
A. $9 - (-3) + (-2) - 1 = \Box + 2 + 4 - 1$	C. $9 - (-3) + (-2) - 12 = \Box + 2 + 4 - 12$					
B. $9 - (-3) + (-2) + 2 = \Box + 2 + 4 + 2$	D. $9 - (-3) + (-2) + 40 - 40 = \Box + 2 + 4$					
b) You should have got the same answers to	E. $32 - 32 + 9 - (-3) + (-2) = \Box + 2 + 4$					
equations A and B. Why does this happen?	F. $-(-3) + (-2) + 9 - 14 = 2 + 4 + \Box - 14$					
	d) Use any method to check your predictions.					
6) Here is a set of five statements:						
A. $\diamond - (-2) + 2 = \heartsuit - 2$	You are now told that $\Diamond - (-2) = \bigcirc$					
B. $\diamond - (-2) + 7 = \odot + 7$	Use the equation $\Diamond - (-2) = \bigcirc$ to decide which of					
$C. \diamondsuit = \bigotimes + 2$	the statements (A to E) are balanced and why.					
D. $\diamond - (-2) + (-3) = \diamond + (-3)$						
$E. \Diamond - \heartsuit - (-1) = 1$						



Worksheet 2.5: Numerical equations with integers

Qu	estions	Answers			
1)	Match the columns:	1)			
	COLUMN A COLUMN B	, a) III			
	a) $9 + (-4) - \Box = 10 + (-4) - 7$ I. $\Box = -5$	b) I			
	b) $7 - 4 + (-6) = 8 - 6 + \Box$ II. $\Box = 4$	c)			
	c) $-3 + 9 - (-5) = 11 - (-4) - \Box$ III. $\Box = 6$	o,			
	$IV. \Box = -1$				
2)	Solve these four equations by inspection:	2)			
	a) $-5 + \Box = -10 - 3$	a) $\Box = -8$			
	b) $9 - (-2) = \Box + (-2)$	b) $\Box = 13$			
	c) $8 - \Box = 7 - (-1)$	c) $\Box = 0$			
	d) $\Box - (-4) = -9 + (-8)$	d) $\Box = -21$			
3)	Solve these equations using additive inverses:	3)			
	a) $8 + (-3) - 5 = \Box + (-2)$	a) $\Box = 2$ subtract (-2) from each side			
	b) $\Box + 4 - (-2) = 6 - 7 - 3$	b) $\Box = -10$ subtract 6 from each side			
	c) $-11 + (-1) - (-3) = \Box + (-4) + (-2)$	c) $\Box = -3$ subtract (-6) or add 6 from each side			
4)	Given: $-4 + 3 + (-6) = \Box + (-5)$	4)			
	a) Look for what is the same and what is different in this set of	a) $-4+3+(-6)$ and $\Box + (-5)$ are the same. What			
	four equations:	is different is that a different number is subtracted			
	A. $-4+3+(-6)-4 = \Box + (-5)$	from the left of each equation each time.			
	B. $-4+3+(-6)-5 = \Box + (-5)$	b) No \Box will not to be the same. because we are			
	C. $-4+3+(-6)-6 = \Box + (-5)$	subtracting different values from each side			
	D. $-4+3+(-6)-7 = \Box + (-5)$	c) A. $\Box = -6$ B. $\Box = -7$ C. $\Box = -8$ D. $\Box = -9$			
	b) Do you expect the values for \Box to be the same for	d) Yes. The answers were different			
	each equation? Give a reason for your answer.				
	c) Solve the equations.				
	d) Was your prediction in Q4b correct?	-)			
5)	Given: $9 - (-3) + (-2) = \Box + 2 + 4$	5)			
	a) Solve each equation:	a) A. $\Box = 4$ B. $\Box = 4$			
	A. $9 - (-3) + (-2) - 1 = \Box + 2 + 4 - 1$	b) We subtracted or added the same value on each			
	B. $9 - (-3) + (-2) + 2 = \Box + 2 + 4 + 2$	side of the given equation.			
	b) You should have got the same answers to equations A and B.	c) $\Box = 4$ for all equations			
	c) Prodict the answers to the following:	u) Check A. using inverses $2 - \Box = 6$			
	$C = 0 - (-3) + (-2) - 12 - \Box + 2 + 4 - 12$	$-2 - \Box = 0$			
	C. $9 = (-3) + (-2) = 12 = -1 + 2 + 4 - 12$ D $9 = (-3) + (-2) + 40 = 40 = -1 + 2 + 4$	$2 + 0 = \Box = 0 + 0$			
	F. $32 - 32 + 9 - (-3) + (-2) = \Box + 2 + 4$	or inspection: $-2 = \Box - 6$ so $4 = \Box$			
	$E_{-} = -(-3) + (-2) + 9 - 14 = 2 + 4 + \Box - 14$				
	d) Use any method to check your predictions				
6)	Here is a set of five statements:	6) Given $\Diamond - (-2) = \bigcirc$			
	A. $\diamond - (-2) + 2 = \heartsuit - 2$	B. The same number is added to each side so when it is			
	B. $\Diamond - (-2) + 7 = \heartsuit + 7$	subtracted from each side we get $\Diamond - (-2) = \bigcirc$ (which we			
	$C. \diamondsuit = \heartsuit + 2$	were given), so the statement is balanced $$ or $$ substitute $$ $$			
	D. $\diamond - (-2) + (-3) = \diamond + (-3)$	for $\diamond - (-2)$ into $\diamond - (-2) + 7 = \odot + 7$ and get $\odot + 7 =$			
	$E. \diamondsuit - \circlearrowright - (-1) = 1$	\odot + 7 so the statement is balanced.			
Υοι	are now told that $\Diamond - (-2) = \bigcirc$	D: Subtract (-3) from each side to get $\Diamond - (-2) = \bigcirc$ (Which			
Use	the equation $\Diamond - (-2) = \bigcirc$ to decide which of the statements (A	we were given), so the statement is balanced.			
to E	i) are balanced and why.	Note: substitution for \otimes gets complicated [substitute \diamond –			
		$(-2) = \odot$ for \odot into D. and get $0 = 0$ This type of			
		solution is mostly beyond Grade 8 and 9 level			

#equali = y matters

PRACTICE IN WORKING WITH LINEAR EQUATIONS



Worksheet 2.6: Numerical equations with integers

This worksheet focuses on the \Box in different positions in the equations with integers. The position of the \Box makes some equations more difficult than others. As in Worksheets 2.1 to 2.5, our aim is to get \Box on its own.

Questions 1) Look at this equation: $-5 + 8 = -9 + \Box$. Note that \Box is at the end of the equation. a) Solve the equation by inspection. b) Kate solved $-5 + 8 = -9 + \Box$ using additive inverses. Copy her response and answer the questions: Kate's response Questions $-5 + 8 = -9 + \Box$ Is Kate correct to re-write $-9 + \Box$ as $\Box + (-9)$? Why? i) $3 = \Box + (-9)$ ii) How does Kate get 3? $3 + 9 = \Box + (-9) + 9$ iii) Why does Kate add 9 on each side? $12 = \Box + 0$ iv) How does Kate get 0? v) And how does she get 12? $12 = \Box$ vi) Does Kate get the correct answer? 2) Here is another equation: $-7 - (+3) = -11 - \Box$. Note that it has "subtract \Box " at the end of the equation. a) Solve the equation by inspection. b) Tina solved the equation using additive inverses. Copy her response and answer the questions: Tina's response Questions $-7 - (+3) = -11 - \Box$ $-10 = -11 - \Box$ i) How does Tina get -10? $-10 + \Box = -11 - \Box + \Box$ ii) What has Tina done on each side? $-10 + \Box = -11 + 0$ iii) How did Tina get 0? $\Box = -11 + 10$ iv) How does Tina get +10 on the right of the equal sign? $\Box = -1$ v) This should be the same answer you got for Q2a. Is it? 3) Solve these equations using additive inverses. Refer to Kate and Tina's responses if you need help. a) $5 - (-3) = 9 - \Box$ also write what you did in each step in Q3a, and why you did it. b) $-7 - (-5) = 1 - \Box$ c) $16 - \Box = -1 - (-19)$ d) $-2 + \Box = -7 - 3$ also write what you did in each step in Q3d, and why you did it. e) $9 + (-3) = -8 - \Box$



Worksheet 2.6: Numerical equations with integers

Questions	Answers		
 Look at this equation: -5 + 8 = equation. a) Solve the equation by inspect b) Kate solved -5 + 8 = -9 + Copy her response and ans Kate's response 	1) a) $\Box = 12$ b) i) Yes, because addition is commutative ii) Adds -5 and 8 iii) To get \Box on its own. iv) (-0) + 0 = 0		
$-5 + 8 = -9 + \square$ $3 = \square + (-9)$ $3 + 9 = \square + (-9) + 9$ $12 = \square + 0$ $12 = \square$) Is Kate correct to re-write -9 + □ as □ + (-9)? Why? i) How does Kate get 3? ii) Why does Kate add 9 on each side? v) How does Kate get 0? v) And how does she get 12? vi) Does Kate get the correct answer? 	v) $(-9) + 9 = 0$ v) $9 + 3 = 12$ vi) Yes	
 2) Here is another equation: -7 - Note that it has "subtract □" at a) Solve the equation by inspective b) Tina solved the equation us Copy her response and ans 	$(+3) = -11 - \Box$. the end of the equation. ection. sing additive inverses. wer the questions:	 2) a) □ = -1 b) i) She "has" -7 and subtracts 3 more which gives -10. ii) Added box iii) - □ is the addition improve of 	
Tina's response $-7 - (+3) = -11 - \Box$ $-10 = -11 - \Box$ $-10 + \Box = -11 - \Box + \Box$ $-10 + \Box = -11 + 0$ $\Box = -11 + 10$ $\Box = -1$	Questions i) How does Tina get -10? ii) What has Tina done on each side? iii) How did Tina get 0? iv) How does Tina get +10 on the right of the equal sign? v) This should be the same answer you got for Q2a. Is it?	 iii) +□ is the additive inverse of -□ so together they sum to zero. iv) She added the additive inverse of -10 to each side 	
 3) Solve these equations using additive inverses. Refer to Kate and Tina's responses if you need help. a) 5 - (-3) = 9 - □ also write what you did in each step in Q3a, and why you did it. b) -7 - (-5) = 1 - □ c) 16 - □ = -1 - (-19) d) -2 + □ = -7 - 3 also write what you did in each step in Q3d, and why you did it. e) 9 + (-3) = -8 - □ Answers to Q3a and Q3d's steps a) Simplify the right side: 5 - (-3) = 5 + 3 = 8. Add box to each side to get +□ on the left. Add the inverse of 8 to each side and get 9 - 8 which is 1. d) Simplify the left side: -7 - 3 = -10. Add the inverse of -2 to each side to get 		 3) Final answers to Q3a to Q3e a) □ = 1 b) □ = 3 c) □ = -2 d) □ = -8 e) □ = -14 See steps for Q3a and Q3d below the question. 	



Worksheet 2.7: Numerical equations with integers

This worksheet focuses on solving equations that contain a mixture of addition and subtraction There are two or more whole numbers on each side of the equal sign with \Box in a variety of positions. As in Worksheets 2.1 to 2.6, our aim is to get \Box on its own.

Que	estions						
1)	Look at this equation: $(-7) + (-8) = 3 - \Box$. Note that it has $-\Box$ at the end of the equation.						
	a) Solve the equation by using additive inverses.						
	b) Based on your answer to Q1a), answer the following questions:						
	i) Could we rewrite $3 - \Box$ as $\Box - 3$? Explain.						
	ii) How do you get $+\Box$ on the left of the equal sign?						
2)	Given: $(-10) + (-13) + 4 = \Box + (-11)$						
1	a) Look for what is the same and different in this set of four equations:						
	A. $(-10) + (-13) + 4 - 4 = \Box + (-11)$						
	B. $(-10) + (-13) + 4 - 5 = \Box + (-11)$						
	C. $(-10) + (-13) + 4 - 6 = \Box + (-11)$						
	D. $(-10) + (-13) + 4 - 7 = \Box + (-11)$						
1	b) Do you expect the results for \Box to be the same for each equation? Give a reason for your answer.						
	c) Solve the equations.						
	d) Was your prediction in Q2b correct?						
3)	Solve these equations using additive inverses.						
	a) $10 - (-3) = 9 - \Box$						
	b) $11 - \Box = -12 - 4$						
	c) $-2 + \Box = -5 + 3$						
	d) $9 + (-5) = 2 - \Box$						
	e) $12 + (-3) = 18 - \Box$						
4)	Here is a set of five statements:						
,	A. $\diamond + (-4) + (-7) = \diamond + (-7)$						
	B. $\diamond + (-4) + 7 = \diamond + 7$						
	C. $\diamond = \otimes + (-4)$						
	D. $\diamond + (-4) + 3 = \diamond$						
	E. $\diamond - \diamond = -4$						
,	You are now told that $(-4) = 0$						
	Use the equation $(-4) = 0$ to decide which statements (A to E) are not belanced. Source show why						
	Use the equation $\vee + (-4) = \odot$ to decide which statements (A to E) are not balanced. Say of show why they are not belanced and write them using a \pm sign						
	they are not balanced and write them using a \neq sign.						



Worksheet 2.7: Numerical equations with integers

Que	Questions			Answers			
1)	1) Look at this equation: $(-7) + (-8) = 3 - \Box$. Note that it has			1)			
	$-\Box$ at the end of the equation.			a) $(-7) + (-8) = 3 - \Box$			
	a) Solve the equation by using additive inverses.				$-15 = 3 - \Box$		
	b) Based on your answer to Q1a), answer the fo	llowing			$-15 + \Box = 3 - \Box + \Box$		
	questions:	_		_	$-15 + 15 + \Box = 3 + 15$		
	i) Could we rewrite $3 - \Box$ as $\Box - 3$? Expl	ain.			$\Box = 18$		
	ii) How do you get $+\Box$ on the left of the e	qual sign?		b)			
		1			i) No. Subtraction is not commutative.		
					$3 - \Box \neq \Box - 3$		
					ii) Add \Box to each side of the equation		
2)	$C_{1} = (10) + (12) + 4 = \Box + (11)$		2)				
2)	Given: $(-10) + (-13) + 4 = \Box + (-11)$		2)	,			
	a) Look for what is the same and different in thi	s set of four		a)	$(-10) + (-13) + 4$ and $\Box + (-11)$ are the		
	equations:				same. What is different is that in each		
	A. $(-10) + (-13) + 4 - 4 = \Box + (-11)$.)			equation we subtract one more on the left in		
	B. $(-10) + (-13) + 4 - 5 = \Box + (-11)$.)			the previous equation.		
	C. $(-10) + (-13) + 4 - 6 = \Box + (-11)$.)		b)	No. The left side gets smaller by one each time		
	D. $(-10) + (-13) + 4 - 7 = \Box + (-11)$)			so the value for \square would also get smaller by 1		
					each time.		
	b) Do you expect the results for \Box to be the sar	ne for each		c)			
	equation? Give a reason for your answer.				A) $\Box = -12$		
	c) Solve the equations.				$B) \Box = -13$		
	d) Was your prediction in Q2b correct?				$\begin{array}{c} C \\ \Box \\ \end{array} = -14 \\ \end{array}$		
				-1)	D) $\Box = -15$		
2)			2)	a)	res. Dependent on learners response in Q2b.		
3)	Solve these equations using additive inverses.		3)	-)	$\Box = 4$		
	a) $10 - (-3) = 9 - \Box$			d) b)	$\Box = -4$		
	b) $11 - \Box = -12 - 4$			(U	$\Box = 27$		
	c) $-2 + \Box = -3 + 3$ d) $9 + (-5) = 2 - \Box$			c)	$\Box = 0$ $\Box = -2$		
	a) $12 \pm (-3) = 18 = \square$			u)	$\Box = -2$		
				C)			
4)	Here is a set of five statements:	4) Given ◊ ·	+ (-	4) =	0		
	A. $\diamond + (-4) + (-7) = \diamond + (-7)$	$C \diamondsuit = \bigotimes -$	+ (4) bu	It we were given $\diamond + (-4) = \bigcirc$ if we subtract		
	B. $\diamond + (-4) + 7 = \diamond + 7$	(-4) fro	om ea	ch si	de we get $\Diamond - (-4) = \bigcirc$ and		
	C. $\diamond = \diamond + (-4)$	◊ - (-4	1)≠	⊗ or			
	D. $\diamond + (-4) + 3 = \diamond$	substitut	te 🛇	+ (-	4) for \otimes into C:		
	E. $\diamond - \diamond = -4$	$\diamond = [\diamond]$	+ (-	4)] -	$(-4) = \Diamond - 8$ and $\Diamond \neq \Diamond - 8$		
		Conclusi	on: t	he sta	atement is not balanced		
	You are now told that $\diamond + (-4) = \diamond$	$\Diamond \neq \bigcirc$	+ (4)			
	Use the equation $\diamond + (-4) = \diamond$ to decide	D 3 is adde	ed to	♦ +	(-4) and nothing is added to or subtracted		
	which statements (A to E) are not balanced.	from \heartsuit ,	chan	ges c	on each side are different or by subtracting		
	Say or show why they are not balanced and	, 3 from b	oth s	ides	of D we get $\diamond + (-4) = \diamond - 3$ but		
	write them using $a \neq sign.$	$\odot \neq \odot$	– 3 c	or			
		substitut	te 🛇	for 🗘	$> + (-4)$ on the left of D to get $\otimes + 3$ but		
		$\otimes + 3 \neq \otimes$					
		Conclusi	on: t	he sta	atement is not balanced: $\Diamond + (-4) + 3 \neq \bigcirc$		
		E The give	n car	be v	vritten as $\Diamond - 4 = \bigcirc$ by adding 4 to each side		
		we get <	$\Diamond - \heartsuit = 4$ comparing this to E. $4 \neq -4$.				
		Conclusi	on: t	he sta	atement is not balanced: $\Diamond - \heartsuit \neq -4$		



Worksheet 2.8: Numerical equations with integers

This worksheet focuses on solving equations involving multiplication. There are two integers on each side of the equal sign. Some answers are fractions.

Qu	Questions						
1)	Give the multiplicative inverse of each number: a) 3 b) 2 c) -5 d) -7 When we multiply <i>multiplicative inverses</i> , we get a product of 1. e.g. $-4 \times -\frac{1}{4} = 1$						
2)	Work out the value of \Box to balance these statements: Note: when we multiply a negative integer by another negative integer, we get a positive integer. a) $4 \times \frac{1}{\Box} = 4 \times 1$ b) $-8 \times -\frac{1}{\Box} = 8 \times 1$ c) $5 \times \frac{1}{\Box} = -5 \times (-1)$						
3)	Lina and Thabo solve the equation $10 \times (-2) = \Box \times 4$ in different ways: Lina uses inspection: Lina uses inspection: Lina first simplifies the equation $-20 = \Box \times 4$ She then thinks: "What multiplied by 4 gives -20 ?" and gets: $\Box = -5$ Thabo uses multiplicative inverses: $10 \times (-2) = \Box \times 4$ Thabo also simplifies the equation $-20 = \Box \times 4$ He then multiplies each side by $\frac{1}{4}$ to get \Box on its own. $-20 \times \frac{1}{4} = \Box \times 4 \times \frac{1}{4}$ Thabo then simplifies $-20 \times \frac{1}{4}$ like this: $-\frac{20}{1} \times \frac{1}{4} = -\frac{20}{4} = -5$ And gets: $-5 = \Box$ Solve these equations using multiplicative inverses in the way Thabo did. Then check your answers using inspection. a) $(-4) \times 6 = \Box \times 3$ b) $\Box \times 5 = (-10) \times (-4)$						
4)	Lina made up this question to try Thabo's method. She found the answer was a fraction. Copy her response and answer the questions: $3 \times (-8) = (-7) \times \square$ $3 \times (-8) = \square \times (-7)$ $3 \times (-8) = \square \times (-7)$ $3 \times (-8) \times \left(-\frac{1}{7}\right) = \square \times (-7) \times \left(-\frac{1}{7}\right)$ b) Why does Lina multiply each side by the multiplicative inverse of -7 ? $\frac{3}{1} \times \left(-\frac{8}{1}\right) \times \left(-\frac{1}{7}\right) = \square$ c) What is Lina doing in line 4? $\frac{24}{7} = \square$ d) Write Lina's answer as mixed number.						
5)	a) $\Box \times (-6) = 5 \times 12$ b) $(-3) \times (-5) = \Box \times 2$ c) $(-4) \times \Box = (-7) \times (-5)$ d) $\Box \times 6 = (-5) \times 5$ e) $9 \times 2 = (-3) \times \Box$						



Worksheet 2.8: Numerical equations with integers

Questions	Answers
 1) Give the multiplicative inverse of each number: a) 3 b) 2 c) -5 d) -7 	1) a) $\frac{1}{3}$ c) $-\frac{1}{5}$ b) $\frac{1}{2}$ d) $-\frac{1}{7}$
 2) Work out the value of □ to balance these statements: Note: when we multiply a negative integer by another negative integer, we get a positive integer. a) 4 × 1/□ = 4 × 1 b) -8 × -1/□ = 8 × 1 c) 5 × 1/□ = -5 × (-1) 	2) a) $\Box = 1$ b) $\Box = 1$ c) $\Box = 1$
3) Lina and Thabo solve the equation $10 \times (-2) = \Box \times 4$ in different ways: Lina uses inspection: $10 \times (-2) = \Box \times 4$ Lina first simplifies the equation $-20 = \Box \times 4$ She then thinks: "What multiplied by 4 gives $\Box = -5$ -20?" and gets: Thabo uses multiplicative inverses: $10 \times (-2) = \Box \times 4$ Thabo also simplifies the equation $-20 = \Box \times 4$ He then multiplies each side by $\frac{1}{4}$ to get \Box on its $-20 \times \frac{1}{4} = \Box \times 4 \times \frac{1}{4}$ own. $-20 \times \frac{1}{4} = \Box \times 4 \times \frac{1}{4}$ Thabo then simplifies $-20 \times \frac{1}{4}$ like this: $-\frac{20}{1} \times \frac{1}{4} = -\frac{20}{4} = -5$ And gets: $-5 = \Box$ Solve these equations using multiplicative inverses in the way Thabo did. Then check your answers using inspection. a) $(-4) \times 6 = \Box \times 3$ b) $\Box \times 5 = (-10) \times (-4)$	3) a) $(-4) \times 6 = \Box \times 3$ $-24 = \Box \times 3$ $-24 \times \frac{1}{3} = \Box \times 3 \times \frac{1}{3}$ $-8 = \Box$ b) $\Box \times 5 = (-10) \times (-4)$ $\Box \times 5 = 40$ $\Box \times 5 \times \frac{1}{5} = 40 \times \frac{1}{5}$ $\Box = 8$
4) Lina made up this question to try Thabo's method. She found the answer was a fraction. Copy her response and answer the questions: $3 \times (-8) = (-7) \times \square$ a) Lina rewrote $(-7) \times \square$ as $\square \times (-7)$. Explain why she can do this. $3 \times (-8) \times (-\frac{1}{7}) = \square \times (-7) \times (-\frac{1}{7})$ b) Why does Lina multiply each side by the multiplicative inverse of -7 ? $\frac{24}{7} = \square$ c) What is Lina doing in line 4? d) Write Lina's answer as mixed number	 4) a) Multiplication is commutative (-7) × □ = □ × (-7) b) To get box on its own c) She is writing all the values in fraction form. d) ²⁴/₇ = 3 ³/₇
5) Solve the following equations using multiplicative inverses: a) $\Box \times (-6) = 5 \times 12$ b) $(-3) \times (-5) = \Box \times 2$ c) $(-4) \times \Box = (-7) \times (-5)$ b) Solve the following equations using multiplicative inverses: a) $\Box \times 6 = (-5) \times 5$ b) $\Box \times 6 = (-5) \times 5$ c) $(-4) \times \Box = (-7) \times (-5)$	5) Value of \Box a) -10 d) $-\frac{25}{6}$ b) $\frac{15}{2}$ e) -6 c) $-\frac{35}{4}$



Worksheet 2.9: Numerical equations with integers

This worksheet focuses on solving equations involving division where there are two integers on each side of the equal sign.

	A statement that is balanced has the same result on each side of the equal sign.
Questions	So, a <i>balanced statement</i> is an equation.
1) If negative nine is divided by three, we can write it as $-9 \div$ Write these divisions in two other ways:a) $20 \div (-5)$ b) $-9 \div (-2)$ c) $-3 \div$	3 or $-\frac{9}{3}$ or $-9 \times \frac{1}{3}$. -7 d) $4 \div (-4)$
a) Write these multiplications as divisions: a) $7 \times \left(-\frac{1}{2}\right)$ b) $-9 \times \frac{1}{4}$ c) $-\frac{1}{5} \times \frac{1}{5}$	(-8) d) $\frac{1}{6} \times (-6)$
 3) Solve using inspection. The first one has been done for you a) 18 ÷ (-3) = □ ÷ 6 -6 = □ ÷ 6 Think: "What divided by 6 give □ = -36 b) Now try it this way: -¹⁸/₃ = [□]/₆ c) □ ÷ (-8) = (-4) ÷ 2 You can choose the method 	s —6?" used in Q3a or in Q3b.
4) Give the multiplicative inverse of each number: a) $\frac{1}{6}$ b) $-\frac{1}{2}$ c) $-\frac{1}{7}$ d) $\frac{3}{5}$	When we multiply <i>multiplicative inverses,</i> we get a product of 1. e.g. $-\frac{1}{6} \times (-6) = 1$
 5) a) Solve this equation using <i>inspection</i>: (-6) ÷ 2 × (-3) b) Ravi solves the equation: (-6) ÷ 2 × (-3) = □ ÷ (-4) response and answer the questions: 	$= \Box \div (-4)$ 4) using <i>multiplicative inverses</i> . Copy his
$(-6) \div 2 \times (-3) = \Box \div (-4)$ $(-6) \times \frac{1}{2} \times (-3) = \Box \times \left(-\frac{1}{4}\right) \qquad \text{i) What has}$ $9 = \Box \times \left(-\frac{1}{4}\right) \qquad \text{ii) Show how}$ $9 \times (-4) \qquad \text{iii) Why did F}$ $= \Box \times \left(-\frac{1}{4}\right) \times (-4) \qquad \text{sign?}$	Ravi done in this step? v Ravi got 9 on the left side. Ravi multiply by —4 on each side of the equal
6) Solve the following equations using <i>multiplicative inverses</i> a) $\frac{\Box}{\Box} = -\frac{12}{2}$ b) $-\frac{\Box}{\Box} = \frac{12}{2}$ c) $30 \div (-4) = \frac{12}{2}$	and then check using <i>inspection</i> : $\frac{\Box}{2} d) \frac{15}{2} = \Box \div (-5) e) -\frac{12}{2} = -\frac{\Box}{2}$
7) Here is a set of three statements: a) Wr A. $15 \div 3 \times (-3) = 15 \times (-1)$ the B. $15 \div (-3) = 5 \div 5 \times (-1)$ usi C. $-\frac{23}{3} \times 3 = 23 \times (-1)$ b) Chasta	$\frac{2}{2}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{4}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{3}{3}$ $\frac{3}{3}$ $\frac{4}{3}$ $\frac{3}{3}$ $\frac{3}$



Worksheet 2.9: Numerical equations with integers

Qu	Questions and answers						
1)	1) If negative nine is divided by three, we can write it as $-9 \div 3$ or $-\frac{9}{2}$ or $-9 \times \frac{1}{2}$.						
	Writ	e these divisions in two	other ways:			5 5	
	a)	$20 \div (-5)$	b) −9÷((-2)	c)	$-3 \div 7$	d) $4 \div (-4)$
Ans	wers						
	a)	$\frac{20}{-5}$ or $20 \times \frac{1}{-5}$	b) $\frac{9}{2}$ or 9 ×	$\left(\frac{1}{2}\right)$	c)	$-\frac{3}{7}$ or $-3 \times \frac{1}{7}$	d) $\frac{4}{-4}$ or $4 \times \frac{1}{-4}$
2)	Writ	e these multiplications	as divisions:				
	a)	$7 \times \left(-\frac{1}{2}\right)$	b) $-9 \times \frac{1}{4}$	-	c)	$-\frac{1}{5} \times (-8)$	d) $\frac{1}{6} \times (-6)$
Ans	wers	(2)					
	a)	7 ÷ (-2)	b) −9÷4	4	c)	$-\frac{1}{5} \div \frac{1}{-8}$	d) $\frac{1}{6} \div \frac{1}{-6}$
3)	Solv	e using inspection. The	first one has be	en done for yo	u:		
	a)	$18 \div (-3) = \Box \div 6$					American
		$-6 = \Box \div 6$	Think: "What	divided by 6 gi	ves –6	5?"	h) $\Box = -36$
		$\Box = -36$	18 🗆				c) $\Box = -30$
	b)	Now try it this way:	$\frac{10}{3} = \frac{11}{6}$				$\Box = 16$
	c)	$\Box \div (-8) = (-4) \div 2$	You can ch	oose the meth	od use	d in Q3a or in Q3b.	
4)	Give	the multiplicative inve	rse of each num	iber:			
	a)	$\frac{1}{6}$ b) $-\frac{1}{2}$	c) $-\frac{1}{7}$	d) = 3			
Ans	wers	6 Z	1	5			
	a)	6 b) -2	c) —7	d) $\frac{5}{2}$			
5)				3			
5)	a) Solve this equation using inspection: $(-6) \div 2 \times (-3) = \Box \div (-4)$						
	b) Ravi solves the equation: $(-6) \div 2 \times (-3) = \Box \div (-4)$ using <i>multiplicative inverses</i> . Copy his response and answer the						
		questions:					
		$(-6) \div 2 \times (-3) = \Box \div (-4)$					
		$(-6) \times \frac{1}{2} \times (-3) =$	$\Box \times \left(-\frac{1}{1}\right)$		i)	What has Ravi done	e in this step?
			(4)		ii)	Show how Ravi got	9 on the left side
		9 =	$\Box \times \left(-\frac{-}{4}\right)$,		s on the left side.
		$9 \times (-4) =$	$= \Box \times \left(-\frac{1}{-}\right) \times$	(-4)	iii)	Why did Ravi multi	bly by -4 on each side of the equal
			- (4) ^ ·			sign?	
		-36 =	= []				
Ans	wers	□ - 26 h) [:] \	Chaparad	livicion	-	() , , 1	iii) 4 is the multiplicative inverse
	a)	$\Box = -36 \text{D} $	to multiplicati	aivision ii)	-3	\times (-3) or 18 $\times \frac{1}{2}$	$\frac{1}{10} -4 \text{ is the multiplicative inverse}$
			to multiplicati	UII			$OT = \frac{1}{4}$
6)	Solv	e the following equatio	ns using <i>multipl</i>	icative inverses	s and t	hen check using <i>insp</i>	ection:
	a)	$\frac{\Box}{6} = -\frac{12}{3}$ b)	$-\frac{\Box}{5} = \frac{12}{4}$	c) 30	÷(-4	$(4) = \frac{1}{2}$ d) $\frac{1}{2}$	$\frac{3}{3} = \Box \div (-5)$ e) $-\frac{12}{4} = -\frac{\Box}{3}$
Ans	wers	□ - 24 b) [] _ 1[a) 🗆	_ 10	رام ۱	
	a)	$\Box = -24$) [] = -15	C) 🗆	= -13	6 u)	$\Box = -25 \qquad \text{e)} \Box = 9$
7)	Here	e is a set of three staten	nents: a)	Which stater	nents	are not balanced?	Answers
	A.	$15 \div 3 \times (-3) = 15$	× (-1)	Re-write the	m usin	g a ≠ sign.	a) B. $15 \div (-3) \neq 5 \div 5 \times (-1)$
	В.	$15 \div (-3) = 5 \div 5 \times$	(-1) b)	Change the r	ight si	de of the	b) $15 \div (-3) = 5 \div 5 \times (-1) \times 5$
	C.	$-\frac{23}{2} \times 3 = 23 \times (-1)^{-1}$)	unbalanced	statem	ents so that they	
		3		are balanced	Ι.		

#equali = y matters

PRACTICE IN WORKING WITH LINEAR EQUATIONS



Worksheet 2.10: Numerical equations with integers

This worksheet focuses on the equal sign as a balance for numerical equations involving a mixture of multiplication and division where there are two or more integers on each side of the equal sign.

Q	ue	es	ti	0	n	S
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1)	Write the question and select the correct number for \Box . a) $(-5) \times 2 \times (-3) = \Box \times 6$ A. -30 B. 30 C. -5 D. 5 b) $6 \times \Box \div (-2) = 4 \times (-3)$ A. -12 B. 4 C. -1 D. 1 c) $\Box \times (-5) \times 5 = (-5) \times (-10) \times (-1)$ A. -25 B. -50 C. 2 D. -2 Copy and solve by inspection:	6) Copy and give the value for \Box in each equation A. $(-5) \times (-4) \times 3 = \Box \times 4 \times (-3)$ B. $(-5) \times (-4) \times 3 = \Box \times 2 \times (-3)$ C. $(-5) \times (-4) \times 3 = \Box \times (-1) \times 6$ D. $(-5) \times (-4) \times 3 = 5 \times 2 \times \Box$ E. $(-5) \times (-4) \times 3 = (-5) \times (-1) \times \Box$ F. $(-5) \times \Box \times 3 = 3 \times 4 \times 5$ G. $10 \times \Box \times 3 = (-5) \times (-4) \times 3$ H. $5 \times \Box \times (-6) = (-4) \times (-5) \times 3$ 7) Look at the set of eight equations in Q6 and answer the questions:
3)	a) $-\frac{1}{2} \times 5 \times (-2) = (-3) \times 5$ b) $5 \times \frac{6}{4} \times (-4) = \Box$ a) Use the equation $(-10) \times 4 = -40$ to complete the following: A. $(-10) \times 4 \times 2 = (-40) \times \Box$ B. $(-40) \times \frac{1}{5} = 4 \times (-10) \div \Box$ C. $(-10) \times 4 \div \Box = (-40) \div 5$ b) Which answers in Q3a are the same? Explain your answer.	 a) Note that □ is sometimes on the right as ir A to E. What is the product of the numbers on the left in A to E? b) Sometimes □ is on the left of the equal sig as in F to H. What is the product of the numbers on the right in F to H? c) So, what should the product be on the side with a □ in A to E? And in F to H?
4)	Solve these equations by inspection. Rewrite the division as multiplication if you find it is easier. a) $3 \times \left(-\frac{6}{4}\right) \times (-4) = \Box$ b) $3 \times 6 \div (-4) \times 4 = \Box$ c) $\frac{\Box}{2} \times (-4) \times 3 = (-3) \times 4$ d) $\Box \div 2 \times (-4) \times 2 = (-3) \times 4$	
5)	Solve these equations using inverses. a) $(-7) \times 3 \times 2 = \Box \times (-6)$ b) $\Box \div (-2) \times 5 = 1 \times 10 \div 2$ c) $\Box \times (-6) \div 4 = (-10) \times 3 \div (-2)$ d) $10 \div 5 \times (-2) = 16 \times \Box \div (-2)$ e) $(-9) \times \Box \times 2 = (-3) \times (-5) \times (-2)$	 8) Use the numbers 4, 5 and -6 to make four multiplication equations: a) two equations with □ on the right e.g. 4 × 5 × (-6) = □ × (-6) × 10 b) two equations with □ on the left e.g. □ × (-6) × 10 = -6 × 5 × 4

Worksheet 2.10: Numerical equations with integers

Answers



Questions and answers	
Questions and answers 1) Write the question and select the correct number for \Box . a) $(-5) \times 2 \times (-3) = \Box \times 6$ A. -30 B. 30 C. -5 D. 5 b) $6 \times \Box \div (-2) = 4 \times (-3)$ A. -12 B. 4 C. -1 D. 1 c) $\Box \times (-5) \times 5 = (-5) \times (-10) \times (-1)$ A. -25 B. -50 C. 2 D. -2 Answers a) D b) B c) C	6) Copy and give the value for \Box in each equation. A. $(-5) \times (-4) \times 3 = \Box \times 4 \times (-3)$ B. $(-5) \times (-4) \times 3 = \Box \times 2 \times (-3)$ C. $(-5) \times (-4) \times 3 = \Box \times (-1) \times 6$ D. $(-5) \times (-4) \times 3 = 5 \times 2 \times \Box$ E. $(-5) \times (-4) \times 3 = (-5) \times (-1) \times \Box$ F. $(-5) \times \Box \times 3 = 3 \times 4 \times 5$ G. $10 \times \Box \times 3 = (-5) \times (-4) \times 3$ H. $5 \times \Box \times (-6) = (-4) \times (-5) \times 3$ Answers for \Box A) -5 C) -10 E) 12 G) 2 B) -10 D) 6 F) -4 H) -2
2) Copy and solve by inspection: a) $-\frac{\Box}{2} \times 5 \times (-2) = (-3) \times 5$ b) $5 \times \frac{6}{4} \times (-4) = \Box$ Answers for \Box a) -3 b) -30 3) a) Use the equation $(-10) \times 4 = -40$ to complete the following: A. $(-10) \times 4 \times 2 = (-40) \times \Box$ B. $(-40) \times \frac{1}{5} = 4 \times (-10) \div \Box$ C. $(-10) \times 4 \div \Box = (-40) \div 5$ b) Which answers in Q3a are the same? Explain your answers a) A) 2 B) 5 C) 5 b) B and C. multiplying by $\frac{1}{5}$ is the same as dividing by 5.	 7) Look at the set of eight equations in Q6 and answer the questions: a) Note that □ is sometimes on the right as in A to E. What is the product of the numbers on the left in A to E? b) Sometimes □ is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H? c) So, what should the product be on the sides with a □ in A to E? And in F to H? swer. a) 60 b) 60 c) 60
4) Solve these equations by inspection. Rewrite the division as multiplication if you find it is easier. a) $3 \times \left(-\frac{6}{4}\right) \times (-4) = \Box$ b) $3 \times 6 \div (-4) \times 4 = \Box$ c) $\frac{\Box}{2} \times (-4) \times 3 = (-3) \times 4$ d) $\Box \div 2 \times (-4) \times 2 = (-3) \times 4$ Answers for \Box a) 18 b) -18 c) 2 d) 3 5) Solve these equations using inverses. a) $(-7) \times 3 \times 2 = \Box \times (-6)$ b) $\Box \div (-2) \times 5 = 1 \times 10 \div 2$ c) $\Box \times (-6) \div 4 = (-10) \times 3 \div (-2)$ d) $10 \div 5 \times (-2) = 16 \times \Box \div (-2)$ e) $(-9) \times \Box \times 2 = (-3) \times (-5) \times (-2)$ Answers for \Box a) 7 c) -10 e) $\frac{5}{3}$ b) -2 d) $\frac{1}{2}$ Note: $\times \frac{2}{1}$ then $\times \frac{1}{5}$ is the same as $\times \frac{2}{5}$	 8) Use the numbers 4, 5 and -6 to make four multiplication equations: a) two equations with □ on the right e.g. 4 × 5 × (-6) = □ × (-6) × 10 b) two equations with □ on the left e.g. □ × (-6) × 10 = -6 × 5 × 4 Answers will differ e.g. a) 4 × 5 × (-6) = □ × (-3) × 2 and 4 × 5 × (-6) = □ × (-5) × 2 b) □ × (-2) × 5 = -6 × 5 × 4 and □ × (-3) × 10 = -6 × 5 × 4

#equali = y matters

PRACTICE IN WORKING WITH LINEAR EQUATIONS



Worksheet 2.11: Numerical equations with integers

This worksheet focuses on the equal sign as a balance for numerical equations involving a mixture of multiplication and division where there are two or more integers on each side of the equal sign.

Remember We can write multiplication in different ways using \times or . or (...). For example, 3×2 or 3.2 or 3(2).

Questions

1) Write the question and select the correct number for \Box . a) $(-2) \times 3 \times (-2) = \Box \times (-6)$ A. -12 B. 12 C. -2 D. 2 b) $3(\Box) \div 2 = 4 \times (-3)$ A. -2 B. -8 C. 8 D. -7 c) $\Box . (-6)(-6) = (-6) \times 12 \times 1$ A. -72 B. -6 C. -2 D. 2	6) Copy and write down the value for \Box in each equation. A. $7 \times (-2) \times 3 = \Box \times (-2) \times 3$ B. $7 \times (-2) \times 3 = \Box \times (-1) \times 3$ C. $7 \times (-2) \times 3 = \Box \times 1 \times (-6)$ D. $7 \times (-2) \times 3 = 7 \times 2 \times \Box$ E. $7 \times (-2) \times 3 = (-7) \times 1 \times \Box$ F. $7 \times \Box \times (-3) = 3 \times (-2) \times (-7)$ G. $14 \times \Box \times (-3) = 7 \times (-2) \times 3$ H. $(-7) \times \Box \times 6 = 2 \times 7 \times 3$
2) Copy and solve by inspection: a) $\frac{\Box}{3}(-7)(-3) = 3 \times 7$ b) $(-8)\left(-\frac{6}{4}\right)(-5) = \Box$ 3) You know that $12(-3) = -36$. a) Use this fact to complete the following: i) $12(-3).2 = (-36).\Box$ ii) $(-36)\left(-\frac{1}{12}\right) = (-3).12 \div \Box$ iii) $12(3) \div \Box = (-36) \div 12$ b) Which answers are the same? Why? 4) Solve these equations by inspection. Rewrited division as multiplication if it is easier for your a) $9\left(-\frac{3}{2}\right) \times 2 = \Box$ b) $(-11) \times 6 \div (-3) \times (-3) = \Box$ c) $\frac{\Box}{3}(-4)(3) = (-9)(4)$ d) $\Box \div 7 \times (-4)(-7) = 6 \times (-2)$	7) Look at the set of eight equations in Q6 and answer these questions: a) Note that \Box is sometimes on the right of the equal sign as in A to E. What is the product of the numbers on the left in A to E? b) Sometimes \Box is on the left of the equal sign as in F to H. What is the product of the numbers on the right in F to H? c) So, what should the product be on the sides with \Box in A to E? And in F to H?
5) Solve these equations using inverses. a) $8 \times (-3) \times 2 = \Box \times (-6)$ b) $\Box \div 2 \times (-7) = 1 \times (-14) \div 2$ c) $\Box \times (-8) \div 4 = 10 \times 4 \div (-2)$ d) $12 \div 6 \times (-3) = (-24) \times \Box \div (-2)$ e) $(-10) \times \Box \times (-2) = (-3) \times (-5) \times (-5) \times (-5)$	8) Use the numbers -7 , 8 and -5 to make multiplication equations: a) Two equations with \Box on the right e.g. $(-5)(8)(-7) = 8 \times \Box \times 1$ b) Two equations with \Box on the left e.g. $(8)(\Box)(1) = (-7) \times 8 \times (-5)$

Worksheet 2.11: Numerical equations with integers

Answers



Questions and answers	Questions and answers				
1) Write the question and select the correct	6) Copy and write down the value for \Box in each				
number for \Box .	equation.				
a) $(-2) \times 3 \times (-2) = \Box \times (-6)$	A. $7 \times (-2) \times 3 = \Box \times (-2) \times 3$				
A12 B. 12 C2 D. 2	B. $7 \times (-2) \times 3 = \Box \times (-1) \times 3$				
	$(-7 \times (-2) \times 3 = \Box \times 1 \times (-6)$				
b) $3(\Box) \div 2 = 4 \times (-3)$	D $7 \times (-2) \times 3 = 7 \times 2 \times \square$				
A2 B8 C. 8 D12	$F = 7 \times (-2) \times 3 = (-7) \times 1 \times \square$				
	$E = 7 \times (-3) = 3 \times (-2) \times (-7)$				
c) $\Box . (-6)(-6) = (-6) \times 12 \times 1$	$6 14 \times [-3] = -7 \times (-2) \times 3$				
A72 B6 C2 D. 2	H $(-7) \times \Box \times 6 = 2 \times 7 \times 3$				
Answers	Answers for				
a) C b) B c) C					
	R 14 D -3 E -2 H -1				
2) Copy and solve by inspection:	7) Look at the set of eight equations in Q6 and answer				
a) $\frac{1}{3}(-7)(-3) = 3 \times 7$	these questions:				
b) $(-8)(-\frac{6}{-})(-5) = \Box$	a) Note that \Box is sometimes on the right of the				
$\frac{2}{4} \left(\frac{3}{4} \right) \left(\frac{3}{4} \right) = \frac{1}{4}$	equal sign as in A to E. What is the product of				
Answers $(0, 1)$	the numbers on the left in A to E?				
a) 5 b) -60	b) Sometimes □ is on the left of the equal sign as				
3) You know that $12(-3) = -36$.	in F to H. What is the product of the numbers				
a) Use this fact to complete the following:	on the right in F to H?				
i) $12(-3), 2 = (-36), \square$	c) So, what should the product be on the sides				
i) $(-26)(-\frac{1}{2}) - (-2)(12 \div \Box)$	with \Box in A to E? And in F to H?				
(i) $(-30)(-\frac{1}{12}) = (-3)(12 \div 1)$	Answers				
(iii) $12(3) \div \Box = (-36) \div 12$	a) -42				
b) which answers are the same? why?	b) 42				
Answers	$ \begin{array}{c} \text{For A to E.} -42 \\ \text{For E to H:} 42 \end{array} $				
a) 1) 2 11) -12 111) -12	FOLF 10 11. 42				
b) ii) and iii) multiplying by $\frac{1}{12}$ is the same as dividing by 12					
4) Solve these equations by inspection. Rewrite the division as	8) Use the numbers -7 , 8 and -5 to make				
multiplication if it is easier for you.	multiplication equations:				
a) $9\left(-\frac{3}{2}\right) \times 2 = \Box$	a) Two equations with \Box on the right				
b) $(-11) \times 6 \div (-3) \times (-2) = \Box$	e.g. $(-5)(8)(-7) = 8 \times \Box \times 1$				
$\Box_{j} = (-11) \land \Box_{\tau} (-3) \land (-3) = \Box$	b) Two equations with \Box on the left				
c) $\frac{1}{3}(-4)(3) = (-9)(4)$	e.g. $(8)(\Box)(1) = (-7) \times 8 \times (-5)$				
d) $\Box \div 7 \times (-4)(-7) = 6 \times (-2)$	Answers will differ e.g.				
Answers for	a) $(-5)(8)(-7) = 2 \times \Box \times 2$ and				
a) -27 b) -66 c) 9 d) -3	$(-5)(8)(-7) = 4 \times 2 \times \square$				
	b) $(4)(\Box)(1) = (-7) \times 8 \times (-5)$ and				
	$(2)(\Box)(-2) = (-7) \times 8 \times (-5)$				
5) Solve these equations using inverses					
a) $8 \times (-3) \times 2 = \Box \times (-6)$	ers for \Box				
h) $\Box \div 2 \times (-7) = 1 \times (-14) \div 2$	2 2 10 3				
c) $\Box \times (-8) \div 4 = 10 \times 4 \div (-2)$ a) 8	b c) 10 e) $-\frac{1}{2}$				
b) 2 d) $12 \div 6 \times (-3) = (-24) \times \Box \div (-2)$	$\frac{2}{2}$ d) $-\frac{1}{2}$				
e) $(-10) \times \Box \times (-2) = (-3) \times (-5) \times (-2)$ Note:	Multiplying by $\left(-\frac{2}{2}\right)$ then multiplying by $\left(-\frac{1}{2}\right)$				
<u></u>					
is the	same as multiplying by $\frac{1}{7}$				



Worksheet 3.1: Algebraic equations

This worksheet provides a recap of numeric expressions involving integers and the commutative law for addition. We use a box or a letter to represent an unknown number. ٦

Questions				An <i>expression</i> consists of numbers, letters	;		
1) The tables contain workal expressions and numeric expressions				by operations $(+, -, \times, \div)$.	╞		
1)	ine	tabl	es contain verbai expressions and numeric	expri	25510	Shs.L	
		Ve	bal expressions	ſ	Jum	eric expressions	
		A.	Add 2 and 13.		1.	-13 + (-2)	
		B.	Add 2 to negative 13		2	$\frac{10}{2-13}$	
		с.	Add negative 2 and negative 13		2. 2	$\frac{13}{13-(-2)}$	
		D.	Subtract 2 from 13). 1	$\frac{13}{2+(-13)}$	
		F.	Subtract 2 from negative 13		т. 5	$\frac{13}{13} = 2$	
		с. Е	Subtract pegative 2 from pegative 13		5. S	-2 - 13	
		١.	Subtract negative 2 nonn negative 15.		J. 7	-2 - 13	
					/. >	-2 + (-13)	
	-)				5.		
	a)	iviato	in the columns. There may be more than or	ne co	rrect	t answer for some options!	
	b)	IT any	/ items in the verbal expressions column do	o not	nave	e partners, provide the numeric expression.	•
	C)	If any	ritems in the numeric expressions column	do n	ot ha	ave partners, provide the verbal expression.	•
2)	Con	side	the expression $(-5) + 3$. Match it with the	e exp	oress	sions and results in A to F below. There is	
•	moi	re th	an one match!				
		A.	-8 B. 2 C2	D.	-1	.5 E. $3 + (-5)$ F. $3 - 5$	
3)	Con	side	the expression $6 - (-1)$.				
-,	a)	Writ	e the expressions and results from A to F th	hat d	o no	bt match $6 - (-1)$.	
		A.	5 B. 7 C. 6	D.	6 –	-1 E.6+1 F7	
	b)	Expl	ain why they do not match.				
4)	Wri	te nu	meric expressions to match the verbal exp	ressi	ons b	below. Use a box (\Box) as a placeholder for	
	the	unkr	nown number. There may be more than one	e wa	y to v	write some expressions.	
	e.g.	Add	5 to a number: $5 + \Box$ or $\Box + 5$				
	0						
	a)	Ado	13 to a number.	d)	Add	a number to negative 5.	
	b)	Sub	tract 2 from a number.	e)	Subt	tract a number from 2.	
	c)	Sub	tract a number from negative 13.	f)	Subt	tract negative 5 from number.	
5)	The	lette	er n represents any number in the expression n	ons b	elow	w. State whether each statement is TRUE or	
	FAL	SE. If	the statement is false, give a numeric exan	nple	to sh	how why the statement is false.	
	2)	י 7	n is the same as $n \pm 7$	c)	7 _ ·	n is the same as $n = 7$	
	d) L)	/ +	$-n$ is the same as $n \pm 7$	c) d)	/ — 1 _ 7	n is the same as $n = 7$	
	נט	-/	-n is the same as $-n - n$	u)	_/-	+ (n) is the same as $n + (-1)$	



Worksheet 3.1: Algebraic equations

Que	estion	ns and	Answers							
1)	Th	ne tab	les contain verbal expressions an	d num	neric express	ions				
								_		
		Vei	bal expressions		Nur	mer	ic expressions	An	swers	to a)
		Α.	Add 2 and 13.		1.	_	-13 + (-2)	А.	8	see note
		В.	Add 2 to negative 13.		2.		2 – 13	В.	4	see note
		С.	Add negative 2 and negative 13	8.	3.	_	13 - (-2)	C.	1; 7	see note
		D.	Subtract 2 from 13		4.		2 + (-13)	D.	5	
		Ε.	Subtract 2 from negative 13.		5.		13 - 2	Ε.	1	
		F.	Subtract negative 2 from negat	ive 13	. 6.		-2 - 13	F.	Non	e
					7.		-2 + (-13)			
					8.		2 + 13	_		
	a)	Mato optio	h the columns. There may be mo ns!	re tha	n one correc	t an	swer for some		Note We cou	Id include 4.: $13 - (-2)$ for
	b)	If any	v items in the verbal expressions of expression.	colum	n do not hav	e pa	rtners, provide the		A. since 2 + 13.	it gives the same result as
	c)	If any	items in the numeric expression	s colu	mn do not ha	ave	partners, provide		For sim $2 \cdot 2 = 1$	ilar reasons we could include
		the v	erbal expression.						22 - 6.: −2 -	– 13 for C.
A	nswe	ers	•							
	b)	F: -	-13 - (-2)							
	c)	2: 1	3 subtracted from 2 or subtract 1	3 fron	n 2 (but see r	ote)			
2)	Con	cidor	the expression $(-5) \pm 3$ Match	it with	the expression	ions	and results in A to F	bolo	w The	oro is moro than one matchl
2)	COIL	Juci				10113		5010	w. m.	
		Α.	-8 В. 2	C.	-2	D.	-15 E.	3 +	- (-5)	F. 3 – 5
A	nswe	ers								
		C; E	and F							
4)	Con	sider	the expression $6 - (-1)$.							
	2)	\M/rit	e the expressions and results from	m A to	E that do no	t m	-(-1)			
	aj	۸ vviit		п д เо С	6		6-1 F	6 -	. 1	F _7
	h)	Expl	ain why they do not match	с.	0	υ.	0 I L.	01	1	1. /
Δ	~, Inswe	ers								
-	a)	A: C:	D and F							
	b)	Beca	use $6 - (-1) = 6 + 1 = 7$							
	- /									
5)	Writ	te nur	meric expressions to match the ve	erbal e	expressions b	elo	w. Use a box (\Box) as	s a pla	acehol	der for the unknown
	num	nber.	There may be more than one way	to w	rite some exp	ores	sions.			
	e.g.	Add S	5 to a number: $5 + \Box$ or \Box	+ 5						
	,		140.			N				
	a)	Add	13 to a number.			d)	Add a number to r	negati	ive 5.	
	b)	Sub	stract 2 from a number.			e)	Subtract a number	from	n 2.	
	c)	Sub	ptract a number from negative 13	i.		f)	Subtract negative	5 fror	m num	ber.
A	nswe	ers	. 12	,	10 🗆			,	2	
	a)		+ 13	c)	$-13 - \square$			e) f)	Z —	□ - (5)
	5)		- Z	u)	-3+□			1)		- (3)
6)	The	letter	r n represents any number in the	expre	ssions below	. St	ate whether each sta	ateme	ent is T	RUE or FALSE. If the
	state	emen	t is false, give a numeric example	to sh	ow why the s	state	ement is false.			
	a)	7 +	n is the same as $n + 7$	c)	7-n is the	e sar	ne as $n-7$			
	b) $-7 - n$ is the same as $-n - 7$ d) $-7 + (n)$ is the same as $n + (-7)$									
A	nswe	ers								
	a)	Τrι	ie	c)	False. Subt	ract	ion is not commutat	tive.		
	b)	Τrι	ie	d)	True					



Worksheet 3.2: Algebraic equations

This worksheet recaps algebraic simplification of like and unlike terms in preparation for algebraic equations. It also provides practice in linking verbal and algebraic expressions.

An *algebraic* expression consists of letters (variables), numbers and operations.

	An exp	ression de	escribed	d in word	s is called a ve	<i>rbal</i> expression.
Question	ns					
1) In ea	ach cluster, identify the unlik	e term. If	there	is no un	like term, pro	ovide a term that would be unlike
the r	est of the cluster.			F		
a) ($6x 6x^2 3x$				Like terms ha	ave the same variables with the
b) -	-x 3x x				same expone	ents for the variables. Unlike terms
c) (6 6p -6				have differen	It variables or different exponents
d) 4	4ab 7ba ab			L	even in they i	lave the same variables.
e) 8	8x 8p x					
2) The	tables below contain verbal	and algeb	oraic ex	nressio	ns. The letter	h represents 'a number'
–, Mate	ch the columns. There may b	e more t	han on	le correc	t answer for	some options!
	arbal avaracsions		Algo	hraicar	proceione	In algebra the multiplication sign
Ve			Aige		pressions	(×) is usually not written. So we
1.	Add 6 to a number		А.	6 +	h	write $5 \times h$ as 5. <i>h</i> or 5 <i>h</i> or 5(<i>h</i>).
2.	A number multiplied by 6)	В.	6 <i>h</i>		The convention is that we write
3.	6 less than a number		C.	h +	6	that are being multiplied. So, we
4.	A number subtract 6		D.	h –	6	write $5h$ rather than $h5$.
5.	A number subtracted fro	m 6	Ε.	6(h	.)	Did you identify both algebraic
6.	A number divided by 6		F.	$\frac{6}{h}$		options for verbal expression 2?
			G.	6 –	h	
			Н.	g ÷	· 6	
3) Simp	blify the following expression	s. If the e	express	sion canı	not be simpli	fied further, say so.
a)	8n - 5n		e) 9 <i>p</i>) — 9		
⊆) b)	6 + 6y + 10 - 5y		c, , , , , , , , , , , , , , , , , , ,	7b + 4b	+ 6	
,~ c)	3 - 2c + c		g) 5d	l + 3e +	12f + 2d -	-e-2f
(5 d)	k - m + m - k + k		h) —[5m - 4m	n + 3m - 2r	n + m
- ,			,			
4) The	five examples show learners	' answers	and th	heir expl	anations. Th	ere is an error in each explanation.
Say v	what is wrong with the learn	ers' reaso	oning a	and give	the correct a	nswer.
	Learners' answers	Learnei	rs' exp	lanation	S	
А.	$8p + 5p = 13p^2$	There are and we g	e like te get p^2 . T	rms so we The answe	can add them r is $13p^2$.	. 8 and 5 is 13. Then we add p and p ,
В.	8p - 5p = 3	8 subtra	ct 5 is 3.	. Then p s	ubtract p is 0. S	to the answer is 3.
С.	6 + 6y + 10 = 12y + 10	6 add 6 i	s 12 the	en you wri	te the y. So you	u get 12y. But 12y and 10 are not
	9a + b = 9ab = 0	like term	is so you	$\frac{1}{2} \cos \frac{2\pi h}{2}$	plify further.	subtract 8ab which gives me area
D.	$\delta u + v - \delta uv = 0$	ou auu b	gives n	ne oub. Ir	ien i nave odb	Subtract oub which gives the zero.
Ε.	5x + 3x - 11x + 4x = 7x	I add 5 a 15 <i>x</i> subt	nd 3 wh tract 8 <i>x</i>	nich gives which is 7	me 8 <i>x</i> . Then I a 7 <i>x</i> .	add 11 and 4 which is $15x$. Then it's



Worksheet 3.2: Algebraic equations

Answers

Qu	estions					Answers
1)	In each cluster, identify the	unlike term. If there	e is no u	nlike term, provide a	term that	1)
	would be unlike the rest of t	he cluster.				$a) \epsilon x^2$
	a) 6 <i>x</i> 6 <i>x</i> ² 3 <i>x</i>					d) OX^{-}
	b) $-x \ 3x \ x$					b) No unlike term, $3y$
	c) 6 6 <i>p</i> −6					c) op d) No unlike torm $4a^2h$
	d) 4ab 7ba ab					a) No unike term, $4a^{-}b$
	e) 8 <i>x</i> 8 <i>p x</i>					e) 8p
	-					
2)	The tables below contain ve	rbal and algebraic e	expressio	ons. The letter <i>h</i> rep	resents 'a	2)
	number'. Match the column	s. There may be mo	ore than	one correct answer	for some	1. A and C
	options!					2. B and E
	Verbal expressions		Alge	braic expressions		3. D
	1. Add 6 to a number		Α.	6+h		4. D
	2. A number multiplied	by 6	В.	6h		5. G
	3. 6 less than a numbe	r	C.	h+6		6 Eand H
	4. A number subtract 6		D.	$\frac{h-6}{(l)}$		
	5. A number subtracte	a from 6	E.	$\frac{6(h)}{6}$		
	6. A number divided by	/ 6	F.	$\frac{1}{h}$		
			G.	6-h		
			Н.	$g \div 6$		
3)	Simplify the following expres	ssions. If the expres	ssion car	nnot be simplified fu	rther, 3	
	say so.					a) 3p e) Can't
	a) 8 <i>p</i> – 5 <i>p</i>	e) 9 <i>p</i> –	- 9			b) $16 + \alpha = 6$
	b) $6 + 6y + 10 - 5y$	f) —7 <i>b</i>	+ 4b +	6		b = 10 + y $1 = 3b + 0$
	c) $3 - 2c + c$	g) 5 <i>d</i> +	- 3e + 1	2f + 2d - e - 2f		c) $3 - c$ g) $7d + 2e + 10f$
	d) $k - m + m - k + k$	h) —5 <i>n</i>	n-4m	+3m - 2m + m		d) k h) $-7m$
		, ,				
4)	The five examples show lear	ners' answers and	their exp	planations. There is a	an error in ea	ach explanation. Say what is wrong
	with the learners' reasoning	and give the corre	ct answe	er.		
	Loorpors' onswors	Loornors' ovnlon	ations		Corroct an	swor with explanation
-			ations			
	A. $8p + 5p = 13p^2$	I here are like te	rms so v	ve can add them.	A. $8p$ and	5p are like terms. When we add like
		8 and 5 is 13. In	ien we a	dd p and p , and	terms the	letters do not get multiplied. The
		we get p^2 . The a	answer i:	s 13p ² .	answer sho	buid be 13p.
	B. $8p - 5p = 3$	8 subtract 5 is 3	. Then p	subtract <i>p</i> is 0.	B. We do r	not take out the letters from loke term
		So the answer is	53.		and subtra	ict them. The answer should be $3p$.
	C. $6 + 6y + 10$	6 add 6 is 12 the	en you w	rite the y. So you	C. 6 and 6	y are not like terms so you can't add
	= 12y + 10	get 12 <i>y</i> . But 12	y and 10) are not like	them. 6 a	nd 10 are like terms. The answer
.		terms so you cai	n't simp	lify further.	should be	16 + 6y
	D. $8a + b - 8ab = 0$	8 <i>a</i> add <i>b</i> gives n	ne 8 <i>ab</i> .	Then I have 8 <i>ab</i>	D. 8 <i>a, b</i> ar	nd $8ab$ are unlike terms so we can't
		subtract 8ab wh	nich give	s me zero.	add them.	The expression cannot be simplified.
	E. $5x + 3x - 11x + 4x$	I add $5x$ and $3x$	which g	ives me 8x. Then I	E. We mus	t subtract $11x$ from $8x$ then add $4x$.

add 11x and 4x which is 15x. Then it's

15x subtract 8x which is 7x.

The correct answer is x

=7x



Worksheet 3.3: Algebraic equations

This worksheet focuses on solving algebraic equations. It involves whole numbers and addition and/or subtraction.

			An <i>equation</i> is a statement indicating		
Qı	Jestions				
1)	Recall how we solved: $2 + \Box = 5 + 3$, using inspection	and inv	verses.		
	a) Solve using <i>inspection</i> : b)) Solv	ve using <i>inverses</i> :		
	Work out the value on the side with no \Box	Wor	rk out the value on the side with no		
	(the right side in this case) to see what the	□. N	Now think: To get 🗆 on its own, I must add		
	result of $2 + \Box$ must be. Now think:	the	of 2 to each side of the		
	"What must I add to 2 to get?" Then	equa	equation. Do this and give the value of \Box .		
	work out the value of \Box .				
	A <i>solution</i> to an equation is the value				
	that gives the same result on both sides	3	3) Solve the following equations using		
	of the equal sign, i.e. it balances the lef	t	additive inverses.		
	and right sides of the equation.		a) $n+7=9$		
2)	Look at the equation: $2 + y = 5 + 3$.		b) $g + 5 = 9 + 5$		
	It has an unknown, y, on the left side of the equation.		c) $k - 6 = 7 - 5$		
	Amy and Pindi solve the equation in different ways.		d) $8 \times 2 = h - 5$		
	Read their methods carefully and answer the questions		e) $7 - 3 = s - 3$		
	Amy: <u>solving by inspection</u> .		f) $7 - x = 5 \times 6$		
	Amy first works out the result on the side with no y.				
	2 + y = 5 + 3	4	4) Look at these five equations:		
	2 + y = 8		A. $r + 5 = 7 + 2$		
	Amy thinks 'What must I add to 2 to get 8?'		B. $r + 5 = 7 - 2$		
	Amy gets: $y = 6$		C. $5 + r = 7 + 2$		
	Pindi: <u>solving using additive inverses.</u>		D. $r - 5 = 7 + 2$		
	Pindi also first works out the result on the side with no y.		E. $r - 5 = 7 - 2$		
	2 + y = 8				
	She rewrites the left side to make it easier for her to see		a) Which equations have the same		
	what to do to both sides. y + 2 - 8		left side?		
	y + 2 = 0 Pindi then adds the additive inverse of 2 to each side		b) Which equations have the same		
	v + 2 - 2 = 8 - 2		right side?		
	Pindi gets: $y = 6$		c) Work out the solutions to all		
l]	equations using inverses.		
Qı	uestions:		d) Which equations have the same		
	a) Why do Amy and Pindi work out the results on the si	ide	solution? Why does this happen?		
	with no y?		Challenge		
	b) Why can Pindi rewrite $2 + y$ as $y + 2$?		e) Use your answers to predict the		
	c) Why does Pindi add the additive inverse of 2 to each	n	answer to:		
	side?		50 + r = 70 + 20		
	d) Complete this substitution to check that 6 is the		f) Now check by solving the equation		
	correct answer: $2 + y = 2 + (_) = 8$ and $5 + 3 =$		using inverses.		
	so $y = .$				

Worksheet: 3.3: Algebraic equations



Г	
Questions and answers	
 Recall how we solved: 2 + □ = 5 + 3, using inspection and inverses. a) Solve using <i>inspection</i>: Work out the value on the side with no □ (the right side in this case) to see what the result of 2 + □ must be. Now think: "What must I add to 2 to get?" Then work out the value of □. Answers a) 8 b) Additive inverse 	 b) Solve using <i>inverses</i>: Work out the value on the side with no □. Now think: To get □ on its own, I must add the of 2 to each side of the equation. Do this and give the value of □.
2) Look at the equation: $2 + y = 5 + 3$.	4) Look at these five equations:
It has an unknown, v , on the left side of the equation.	A. $r + 5 = 7 + 2$
Amy and Pindi solve the equation in different ways.	B $r + 5 - 7 - 2$
Read their methods carefully and answer the questions.	5.7+5-7+2
Amy: solving by inspection.	C. 5+r = 7+2
Amy first works out the result on the side with no γ .	D. $r-5 = 7+2$
2 + y = 5 + 3	E. $r - 5 = 7 - 2$
2 + y = 8	Questions and answers
Amy thinks 'What must I add to 2 to get 8?'	a) Which equations have the same
Amy gets: $y = 6$	left side?
Pindi: solvina usina additive inverses.	A, B and C
Pindi also first works out the result on the side with no y.	b) Which equations have the same
2 + y = 8	right side?
She rewrites the left side to make it easier for her to see what to do to	A, C and D
both sides.	B and E
y + 2 = 8	c) work out the solutions to all
Pindi then adds the additive inverse of 2 to each side.	equations using inverses.
y + 2 - 2 = 8 - 2	$\begin{array}{c} \mathbf{A} \cdot \mathbf{r} = \mathbf{F} \\ \mathbf{B} \cdot \mathbf{r} = 0 \end{array}$
Pindi gets: $y = 6$	$\begin{array}{c} \mathbf{C}, r = 4 \end{array}$
ии	D. $r = 14$
Questions:	E. $r = 10$
a) Why do Amy and Pindi work out the results on the side with no y?	d) Which equations have the same
b) Why can Pindi rewrite $2 + y$ as $y + 2$?	solution? Why does this happen?
c) Why does Pindi add the additive inverse of 2 to each side?	A and C. This happens because
d) Use substution to check that 6 is the correct answer.	addition is commutative
Answers	i.e. $r+5=5+r$ and both A
a) To make it easier to see what $y + 2$ should equal.	and C have 7 + 2 on the right
b) Addition is commutative	side.
c) To get y on its own d) $2 + y = 2 + (6) = 9$ and $5 + 2 = 9$ so $y = 6$ is the correct answer	e) Use your answers to predict the
u) $2 + y = 2 + (0) = 8$ and $3 + 3 = 8$ so $y = 0$ is the correct answer	answer to: $50 + r = 70 + 20$.
3) Solve the following equations using additive inverses.	This equation has the same
Questions Answers	structure as C but each constant
a) $n+7=9$ a) $n=2$	has been multiplied by 10. So the
b) $g + 5 = 9 + 5$ b) $g = 9$	solution to C should be
c) $k-6=7-5$ c) $k=8$	multiplied by 10. $\therefore r = 40$
d) $8 \times 2 = h - 5$ d) $h = 21$	t) Now check by solving the equation
e) $7-3 = s-3$ e) $s = 7$	using inverses.
t) $7 - x = 5 \times 6$ t) $x = -23$	50 + r - 50 = 70 + 20 - 50
	r = 40



Worksheet 3.4: Algebraic equations

This worksheet focuses on solving equations with one variable and one or more constants by applying additive and multiplicative inverses.

Qu	estions	;				
1)	Use th	ne information given	in the table to fill the bl	ank spaces. Q1	a is done for you.	
		Constant	Additive inverse	Mul	tiplicative inverse	
	a)	7	-7 (because 7 + (-7)	$(7) = 0) \qquad \frac{1}{7}$ (be	cause $7 \times \frac{1}{7} = 1$)	
	b)	-3				
	c)	$\frac{1}{7}$				
	d)		5			
	e)				-5	
2)	Solve	the following equation	ons using multiplicative	inverses. Q2a i	s done for you.	
	a) 51	u = 20				
	М	ultiply each side by	$\frac{1}{5}: 5u \times \frac{1}{5} = 20 \times \frac{1}{5}$	b)	3t = 15	
	\٨/	e can re-write these ،	$\frac{5u}{20} = \frac{20}{20}$	c)	99 = 11k	
	vv	e can re-write these	5 - 5	d)	10x = 3	
			u = 4	e)	2d = 14	
3)	Consid Read 1 St	der the equation $4x$ - the steps below and a ep 1: $4x - 2 = 6$ en 2: $4x - 2 + 2 = -2$	-2 = 6. We will solve if answer the questions fr a) 1 6 ± 2	t using additive om 3a to 3e. n Step 2, why c	and multiplicative inv do we add 2 on each si	de?
	St St	ep 2: $4x - 2 + 2 =$	0+2 b) I	n Step 4, why d	and we multiply each si	de by $\frac{-2}{4}$?
	5L C+	ep 3. $4x = 0$	c) l	s $8 \times \frac{1}{4}$ the sam	te as $\frac{8}{4}$? Explain.	
	51		$\frac{1}{4}$ d) I	n which step (1	L – 6) did we use additi	ive inverses?
		$4x \times \frac{1}{4} = 8 \times \frac{1}{4}x + \frac{1}{8}x$		n which step d	id we use multiplicativ	e inverses?
	St	ep 5: $\frac{1x}{4} = \frac{3}{4}$				
	St	ep 6: $x = 2$				
4)	Solve	these algebraic equa	tions using additive and	l multiplicative	inverses.	
	a) 3s	s + 9 = 15				
	b) 2	+5t = 11 - 4				
	c) 10	0+4=7u+7				
	d) 9	-3 = 4 + 5m				
5)	Look a	At the following two each A . $15 = 2t + 3$	equations and answer q B B. $30 = 4t + t$	uestions 5a to 6	5d.	
	a) W	hat is the same and v	what is different in equa	ations A and B?		
	b) Sil	bu predicts that both	equations will have the	e same answers	5. Do you agree?	
	c) Sc	olve equations A and	В.			
	d) W	as your prediction in	Q5b correct? If not, wh	at was wrong v	with your thinking?	



Worksheet 3.4: Algebraic equations

Que	estions	and answers			
1)	Use the	information given in the	table to fill the blank spa	ces. Q1a is c	done for you.
		Constant	Additive inverse		Multiplicative inverse
	a)	7	-7 (because 7 +	(-7) = 0)	$\frac{1}{7}$ (because 7 × $\frac{1}{7}$ = 1)
	b)	-3	3 (because $-3 + 3 = 0$))	$-\frac{1}{3}$ (because $-3 \times -\frac{1}{3} = 1$)
	c)	1 7	$-\frac{1}{7}$ (because $\frac{1}{7} - \frac{1}{7} = 0$)	7 (because $7 \times \frac{1}{7} = 1$)
	d)	-5	5		$-\frac{1}{5}$
	e)	$-\frac{1}{5}$	$\frac{1}{5}$		-5
2)	Solve th	ne following equations us	ing multiplicative inverses	s. Q2a is dor	ne for you.
	a) 5 <i>u</i>	a = 20			
	M	ultiply each side by $\frac{1}{5}$: 5	$u \times \frac{1}{5} = 20 \times \frac{1}{5}$		b) $3t = 15$
	W	e can re-write these as:	$\frac{5u}{5} = \frac{20}{5}$		c) $99 = 11k$ d) $10r = 3$
			u = 4		e) $2d = 14$
	Answer	s			-,
	b) <i>t</i>	$= 5$ d) $x = \frac{3}{10}$			
	c) <i>k</i>	c = 9 e) $d = 7$			
	,				
3)	Conside	er the equation $4x - 2 =$	6. We will solve it using a	additive and	multiplicative inverses.
	Read th	e steps below and answe	r the questions from 3a t	0 3e.	
	C+/	2 - 6	Que	estions and a	answers
	Ste Ste	4x - 2 = 0	d) 2	Because it	t is the additive inverse of -2
	Ste	2 + 2 + 2 = 0 + 2 = 0	2 b)	In Stop 4	why did we multiply each side by 1^{2}
	Ste Ste	2p 3: $4x = 0$	by $\frac{1}{2}$	in Step 4,	why did we multiply each side by $\frac{1}{4}$?
	510	$4 \dots 1^{1} 0 \dots 1^{1}$		Because in	t's the multiplicative inverse of 4.
		$4x \times \frac{1}{4} = 8 \times \frac{1}{4}$	C)	$18.8 \times \frac{-}{4}$ th	le same as $\frac{1}{4}$? Explain.
	Ste	ep 5: $\frac{4x}{4} = \frac{3}{4}$		Yes. Multi	iplying by a number is the same as dividing by its
	Ste	ep 6: $x = 2$	۲۱,	reciprocal	l.
			a)	Stop 2	(1 - 6) did we use additive inverses?
			ല	In which s	ten did we use s multiplicative inverses?
			C)	Step 4	
4	C -1 ''		antine and distantion of the first of	•	A
4)	inversor	iese algebraic equations i	using additive and multipl	licative	$\begin{array}{c c} \text{Answers} \\ \hline \\ \text{a)} & \text{s} = 2 \\ \hline \\ \text{c)} & u = 1 \\ \hline \\ \end{array}$
	a) 3	15 + 9 = 15	c) $10 + 4 = 7u + 7$		b) $t = 1$ d) $m = \frac{2}{3}$
	b) 2	$\frac{1}{2} + 5t = 11 - 4$	d) $9-3 = 4 + 5m$		$u_{1}^{2} = u_{1}^{2} = \frac{1}{5}$
	- /		,]	
5)	Look at	the following two equati	ons and answer question	s 5a to 5d.	
		A. $15 = 2t + 3$	B. $30 = 4t + 6$		
	a) Wl	hat is the same and what	is different in equations A	A and B?	
	Sa	me: A and B have one nu	mber on left of the equa	l sign and a	variable (t) and number added together on right of
	the	e equal sign. Different: C	onstants and coefficients	are differe	nt.
	b) Sib	ou predicts that both equa	ations will have the same	answers. Do	o you agree? Explain
	۲) د) در	es.ii we multiply each sid	- 6 for both aquations	ger ednatio	וו D.
	d) W:	ive equations A and B. t as your prediction in OSh	– o ior both equations correct? If not what was	wrong with	your thinking? Depends on your answer .
	~, •••		serreet. It not, what was		



Worksheet 3.5: Algebraic equations

This worksheet focuses on solving equations with the variable on one side of the equal sign by applying the additive inverse of the constant or the variable.

Qu	estion	IS					
1)	Give	the additive inverse	of the following:				
	a) !	5 b) -5	c) 3 <i>x</i> c	d) $-8x$			
2)	Yvon	ne has simplified five rectly, write the corr	e expressions below rect expression.	v. Check her re	sponses	. If she has simplified the expression	
		Expression	Simplification	Correct/In	correct	Correct simplification	
	a)	f+3f-2	2 <i>f</i>				
	b)	n-2+3	n —5				
	c)	7 + t - 3	5 <i>t</i>				
	d)	3a + 5 - 5a	-2a + 5				
3)	a)Mike solved the equation $14 = -x - 6$. He applied the additive inverse of the constant to solve for x: $14 = -x - 6$ $14 + 6 = -x - 6 + 6$ $20 = -x$ $-20 = x$ c)Mike could have applied the additi 						
5)	 i) Consider the following two equations: A. 4x = 9 B. 4x + 1 = 9 a) What is the same and what is different about the equations? b) Which inverse must we apply to equation A? Why? c) We must apply two inverses to equation B. Which inverse must be applied first? Why? d) Solve the two equations using inverses. 						
6)	Solve	using inverses:					
	a) b)	9x = 7 7 = 2x + 3	c) $3x - d$ $6x =$	10 = -1 -4			


Worksheet 3.5: Algebraic equations

Que	Questions					Answers
1)	Give t	he additive inverse of the	following:		1)	
						a) -5 c) $-3x$
	a) :	5 b) -5	c) $3x$ d)	-8x		b) 5 d) 8x
2)	Yvonn	e has simplified five expr	essions below. Check he	responses. If	she has sime	l plified the expression incorrectly, write
,	the co	prrect expression.				
				Answers		
		Expression	Simplification	Correct/ Inc	orrect	Correct simplification
	a)	f + 3f - 2	2 <i>f</i>	Incorrect		4f - 2
	b)	n - 2 + 3	n –5	Incorrect		n+1
	c)	7 + t - 3	5 <i>t</i>	Incorrect		t+4
	d)	3a + 5 - 5a	-2a + 5	Correct		
2)	N 4''	1 1.1				
3)	IVIIKe s	solved the equation				
	Hean	14 - x - 0.	c) Mike cou	a nave applied	a the	a) $14 + 6 = -x - 6 + 6$ b) On the right side of the
	of the	constant to solve for x .	instead or	the constant	Convand	b) On the light side of the
	oj ene	14 = -x - 6	complete	to show what	he could	-(-20) - 6 = 20 - 6
	14 -	+6 = -x - 6 + 6	have don	2.		= 14
		20 = -x	14	x = -x - 6		And 14 is on the left side
	-	-20 = x	14+	$_{-} = -x + _{}$	- 6	c) $14 = -x - 6$
	a) C	Copy Mike's solution and	14 +	x = -6		$14 + \mathbf{x} = -\mathbf{x} + \mathbf{x} - 6$
	h	ighlight where he applied	d) Now solve	e for x.		14 + x - 14 = -6 - 14
	t	he additive inverse.	e) Do you ge	et the same re	sult for <i>x</i>	d) $x = -20$
	b) U	Jse substitution to check	as in Q3a	?		e) Yes
	ti	hat his solution is correct				
4)	Here a	are five equations:				4)
	Α.	В.	C. D.		E.	a)
	24 =	= x + 6 $-32 = a - 5$	7 - b = 17 -18	3 + p = 11	30 = -m +	6 A. $x = 18$ D. $p = 29$
	a) S	olve the equations. Apply	/ the additive inverse to t	he variable.		B. $a = -27$ E. $m = -24$
	b) S	Solve each equation agair	n by applying the additive	inverse to the	e constant th	$\begin{array}{c} \text{C. } b = -10 \\ \text{C. } b = -10 \\$
	is	s on the same side as the	variable.			b) Same as above.
	<u> </u>	lander faller in t			-	
5)	Lonsic	der the following two equipses $4x = 0$	lations:		5)	oth aquations have the as the left as the
	A. R	4x + 1 = 9			a) B	both equations have $4x$ on the left and
	υ.	1				bas an additional ± 1 on the left side
	a) V	Vhat is the same and wha	at is different about the e	quations?	h) N	Automatic for the fert side. Multiplicative inverse. To get γ on its
	b) V	Vhich inverse must we ap	pply to equation A? Why?	1		own.
	c) V	Ve must apply two invers	es to equation B. Which	inverse	с) Т	he additive inverse to collect the like
	n	nust be applied first? Wh	y? 		t t	erms
	d) S	olve the two equations u	sing inverses		A (b	$x = \frac{9}{2}$ B, $x = 2$
					<i>.,,</i> ,,	4
6)	Solve	using inverses:			I	6)
	a)	9x = 7 c)	3x - 10 = -1			a) $x = \frac{7}{2}$ c) $x = 3$
	b)	7 = 2x + 3 d)	6x = -4			b) $r = 2$ d) $r = -\frac{4}{3}$
	-					$\begin{bmatrix} J & J & J & J & J & J & J & J & J & J $



Worksheet 3.6: Algebraic equations

This worksheet focuses on solving equations with the variable or the constant on both sides of the equal sign by applying the additive inverse of the constant or the letter.

Qu	Questions					
1)	1) Make 5 pairs of additive inverses from the list of terms below. If the additive inverse does not appear in					
	the list, provide it.					
	$-4 \frac{1}{4} -x 6x$	$6 \frac{1}{6} -\frac{1}{2} 4 x 0,5$				
2)	In the table below, a	pply the additive inverse that is in	dicated. Then write down the new form c	f the		
	equation after applyi	ing the inverse. The first one has k	been done for you.			
	Equation	Apply additive inverse of	Equation after applying inverse			
		-4	x = 2x + 4			
	x-4=2x	x				
		2 <i>x</i>				
3)	This equation has k 's	s on both sides of the equal sign: 4	4k = k + 6			
	a) If you apply the a	additive inverse of 6 to both sides	, will there still be k 's on both sides?			
	b) Remember that t	the additive inverse of k is $-k$.				
	i) Apply this ad	Iditive inverse to both sides.				
	ii) Are there stil	ll <i>k</i> 's on both sides?				
	iii) What remain	is on the right side?				
	c) Continue to solve	e the equation and show that the	solution is 2.			
	d) What type of inve	erse did you use to continue solvi	ng the equation in Q3c?			
4)	Solve the following e	quations by applying the additive	inverse of the variable			
	a) $3n = n - 2$ b)	b) $p = 3p - 2$ c) $7h = 5h + 4$	d) $20b = 50b - 10$			
5)						
5)	I his equation has cor	nstants on both sides of the equal	sign: $16 = 5m + 6$			
	 a) Apply the additiv b) Apply the additiv 	e inverse of 16 to both sides. Are	there constants on only one side?			
	 a) Change the method 	re inverse of o to both sides. Are t	de te colve the equation			
	d) Why is it not a go	and idea to apply the additive inw	are to solve the equation. f_{m} to both sides?			
	d) why is it not a go	Jou luea to apply the additive live	erse of Smillo both sides?			
6)	Solve the following e	equations using inverses.				
	a) $5d = d - 20$	b) $s = 5s - 4$	c) $30x = 70x - 10$			
	d) $5d + 8 = d - 2$	e) $s + 6 = 5s - 4$	f) $70x + 30 = 30x - 10$			



Worksheet 3.6: Algebraic equations

Que	estions		Answers		
1)	Make 5 pairs of ad	ditive inverses from the list of terms b	elow. If the additive	1) Here are 7 pairs:	
	inverse does not a	ppear in the list, provide it.		-4;4	
	$-4 \frac{1}{2} - x = 6$	$x 6 \frac{1}{2} -\frac{1}{2} 4 x 0,5$	-x;x		
	4	6 2		6x; -6x	
				$\frac{1}{4}; -\frac{1}{4}$	
				$-\frac{1}{2}$, $\frac{1}{2}$	
				2'2	
				6; -6 0 F 0 F	
2)				0,5; -0,5	
2)	In the table below,	, apply the additive inverse that is indic	cated. Then write down th	he new form of the equation after	
	apprying the invers	se. The first one has been done for you	Answers		
	Fauntien	Angle addition income of	Answers		
	Equation	Apply additive inverse of	Equation after applying	g inverse	
		-4	x = 2x + 4		
	x - 4 = 2x	x	-4=2x-x		
		2 <i>x</i>	x-2x-4=0		
3)	This equation has	k's on both sides of the equal sign: $4k$	-k + 6	3)	
5,	a) If you apply the	r = additive inverse of 6 to both sides y	will there still be k 's on	a) Ves	
	both sides?			(b) i) $3k = 6$	
	b) Bomombor th	bat the additive inverse of k is $-k$		i) No, the terms with k are	
	i) Apply thi	is additive inverse to both sides		on left side of equal sign	
	i) Apply th	$r_{\rm s}$ still $k_{\rm s}$ on both sides?		iii) c	
	iii) Alether	mains on the right side?		(k) = 0	
	c) Continuo to se	name on the right side:	olution is 2	k - 2	
	d) What type of	inverse did you use to continue solving	σ the equation in O3c?	d) Multiplicative inverse	
4)	Columnation following				
4)	Solve the following	g equations by applying the additive in	verse of the variable	(4)	
	$2 \sum_{n=n}^{\infty} 2^n = n = 2$	d) $7h - Fh + 4$		a) $p = -1$	
	a) $3p = p - 2$ b) $n = 3n - 2$	$\begin{array}{ccc} d \\ r \\$		p = 1	
	$b_{1} p = 5p 2$	u, 200 – 500 10		b = 2	
				d) $D = \frac{1}{3}$	
5)	This equation has o	constants on both sides of the equal sig	gn: $16 = 5m + 6$	5)	
	a) Apply the add	litive inverse of 16 to both sides. Are the	here constants on only	a) $0 = 5m + 6 - 16$; Yes	
	one side?			b) $10 = 5m$; Yes	
	b) Apply the add	litive inverse of 6 to both sides. Are the	ere constants on only	c) $5m = 10$	
	one side?			m = 2	
	c) Choose the method that gives constants on one side to solve the equation			d) Because then we would have	
	d) Why is it not a	a good idea to apply the additive inver	se of 5 <i>m</i> to both sides?	-5m on the left.	
6)	Solve the following	g equations using inverses.		6)	
	a) $5d = d - 20$	c) $s = 5s - 4$ e) 30:	x = 70x - 10	a) $d = -5$ c) $s = 1$ e) $x = \frac{1}{4}$	
	b) $5d + 8 = d - 3d = 3d =$	-20 d) $s + 6 = 5s - 4$ f) 70	x + 30 = 30x - 10	b) $d = -7$ d) $s - \frac{5}{2}$ f) $v - 1$	
	b) $a = -7$ d) $s = \frac{1}{2}$ f) $x = \frac{1}{2}$				



Worksheet 3.7: Algebraic equations

This worksheet focuses on solving equations with variables and constants on both sides of the equal sign.

Qu	Questions							
1)	1) The equation: $x + 5 = 17 - 2x$ has x's and constants on both sides of the equal sign:							
	a)	Thandi solved the equation this way. Read her solution carefully and then answer the questions.						
		x + 5 = 17 - 2x						
		Step 1: $x = 12 - 2x$ i) How did Thandi get 12 in step 1?						
		Step 2: $x + 2x = 12 - 2x + 2x$ ii) Why did she add $2x$ in step 2?						
		Step 3: $3x = 12$ iii) How did she get $x = 4$ in step 4?						
		Step 4: $x = 4$ iv) Use substitution to check that her solution $x = 4$ is correct.						
	b)	Solve the equation by first applying the additive inverse of 17. Do you get the same answer as Thandi?						
	c)	Now solve the equation by first applying the additive inverse of x . Do you get the same answer as Thandi?						
	d)	Lastly, solve the equation again by first applying the additive inverse of 5. Do you get the same						
		answer as Inandi?						
	e)	what other type of inverse did you apply to solve each equation?						
2)	Foc	us on how we solve $4 - 3x = 12 + x$ by answering the following questions:						
	a)	We apply the additive inverse of x. Explain why.						
	b)	Then we apply the additive inverse of 4. Explain why.						
	c)	When we have applied both additive inverses above, we will be left with $-4x = 8$. Do you agree?						
	d)	Which inverse do we apply to isolate x?						
	e)	What is the solution to the equation?						
	f)	Now solve the equation again by gathering all the terms with x on the right side of the equal sign						
		(and the constants on the left side).						
3)	We	are going to solve the equation $3x - 10 = x + 6$ in using two different strategies.						
	a)	Apply inverses so that you collect the terms with variables on the left side, and the terms with						
		constants on the right side.						
	b)	Now apply inverses so that you collect the terms with variables on the <i>right</i> side, and the terms with						
		constants on the <i>left</i> side.						
4)	Solv	e the following using inverses.						
''	a)	2y = y + 9 - 3 b) $2x + 3 = x + 9$ c) $r + 9 = 2 - 6r$ d) $r + 9 = 2r - 6$						
	~)							
5)	Her	e is David's response to solving the						
	equ	ation: $2x + 3 = x - 4$						
		2x + 3 = x - 4 a) Copy David's response and ring any errors he made.						
	St	ep 1: $2x + 3 - 3 = x + 3 - 4$ b) Correct his errors.						
	St	ep 2: $2x = x + 1$ c) Use substitution to confirm that your solution is						
	St	ep 3: $2x - x = x - x + 1$						
	St	ep 4: $x = 1$ d) What happens on the right of the equal sign in Step 3						
	<u>I</u>	when he added – x to both sides of the equation?						



Worksheet 3.7: Algebraic equations

Qu	Questions and answers						
1)	1) The equation: $x + 5 = 17 - 2x$ has x's and constants on both sides of the equal sign:						
	a)	Thandi solved the equation this way. Read her solution carefully and then answer the questions.					
		x + 5 = 17 - 2x					
		Step 1: $x = 12 - 2x$ i) How did Thandi get 12 in step 1?					
		Step 2: $x + 2x = 12 - 2x + 2x$ ii) Why did she add $2x$ in step 2?					
		Step 3: $3x = 12$ iii) How did she get $x = 4$ in step 4?					
		Step 4: $x = 4$ iv) Use substitution to check that her solution $x = 4$ is corr	ect.				
		Answers					
		i) She applied the additive inverse of 5; $17 - 5 = 12$					
		ii) $2x$ is the additive inverse of $-2x$, adding it to both sides gets the terms with x on the left si	de of the				
		equation.					
		iii) She used the multiplicative inverse of 3 on both sides of the equal sign.					
		iv) On the left: $(4) + 5 = 9$. On the right: $17 - 2(4) = 17 - 8 = 9$ So her solution $x = 4$ is con-	rect				
	b)	Solve the equation by first applying the additive inverse of 17. Do you get the same answer as Thandi	?				
		Answer $x - 12 = -2x$; $3x = 12$; $x = 4$ Yes					
	c)	Now solve the equation by first applying the additive inverse of x . Do you get the same answer as That	ndi?				
		Answer $5 = 17 - 3x$; $-12 = -3x$; $4 = x$ Yes					
	d)	Lastly, solve the equation again by first applying the additive inverse of 5. Do you get the same answe	r as Thandi?				
		Answer $x = 12 - 2x$; $3x = 12$; $x = 4$ Yes					
	e)	What other type of inverse did you apply to solve each equation? Answer Multiplicative inverse					
2)	_						
2)	Foci	cus on <u>how</u> we solve $4 - 3x = 12 + x$ by answering the following questions:					
	a)	We apply the additive inverse of x. Explain why. Answer To collect the terms with x on the left side of	the equation.				
	b)	I nen we apply the additive inverse of 4. Explain why. Answer To collect the constants on the right sid	e of the				
	-	equation.					
	c)	when we have applied both additive inverses above, we will be left with $-4x = 8$. Do you agree? An: Which inverse do we apply to icolate x2 Applier Multiplicative inverse.	swer res				
	u)	What is the colution to the equation? Answer $x = -2$					
	e)	What is the solution to the equation: Answer $x = -2$ Now solve the equation again by gathering all the terms with x on the right side of the equal sign (and	the constants				
	e)	now solve the equation again by gathering all the terms with x on the right side of the equal sign (and on the left side). Answer $x = -2$					
2)	14/-	on the left side). Answer $x = 2$					
3)	we	are going to solve the equation $3x - 10 = x + 6$ in using two different strategies.	nto on the right				
	a)	Apply inverses so that you collect the terms with variables on the left side, and the terms with constant c_{ide} Answer $2w = 16 + w = 9$	ints on the right				
	b)	Side. All swell $2x - 10$; $x - 0$ Now apply inverses so that you collect the terms with variables on the <i>right</i> side, and the terms with	constants on				
	D)	the left side Answer $-16 = -2x \cdot x = 8$					
		the left side. Answer $-10 = -2\lambda$, $\lambda = 0$					
4)	Solv	ve the following using inverses.					
	a)	b) $2y = y + 9 - 3$ b) $2x + 3 = x + 9$ c) $r + 9 = 2 - 6r$ d) $r + 9 = 2r$	- 6				
	An	nswers					
	a)	y = 6 b) $x = 6$ c) $r = -1$ d) $r = 15$					
5)	Her	re is David's response to solving the Answers	\bigcirc				
	equ	uation: $2x + 3 = x - 4$ a) Copy David's response and ring a) $2x + 3 - 3$	=x(+3)-4				
		2x + 3 = x - 4 any errors he made. $2x$	= x + 1				
	S	Step 1: $2x + 3 - 3 = x + 3 - 4$ b) Correct his errors. b) $2x + 3 - 3$	= x - 4 - 3				
	S	Step 2: $2x = x + 1$ c) Use substitution to confirm $2x$	= x - 7				
	S	Step 3: $2x - x = x - x + 1$ that your solution is correct. $x = x - x + 1$	= -/				
	S	Step 4: $x = 1$ d) What happens on the right of c) Left: $2(-7)$	+ 3 = -11 - 4 = -11				
		the equal sign in Step 3 when Right: (-/)	=4 = -11				
		x he added – x to both sides of x ide of the side					
		the equation?	-quation.				

#equaliTymatters

PRACTICE IN WORKING WITH LINEAR EQUATIONS



Worksheet 3.8: Algebraic equations

This worksheet focuses mostly on equations with variables on both sides of the equal sign.

Qu	esti	ons			

1) Consider the following 2 equations

```
A. 4 = 2n + 6
```

```
B. 4 + n = 2n + 6
```

- a) What is the same and what is different about the two equations?
- b) Without solving the equations, try to decide whether they will have the same solution. Justify your answer.
- c) Sizwe says we apply only the additive inverse of 6 for equation A. Do you agree?
- d) Which inverse would you apply first for equation B? Why?
- e) Solve the two equations.
- f) Was your prediction in Q2b correct? If not, what was wrong with your thinking?

2)

a) In the table below, apply the additive inverse and write down the equation after applying the inverse. The first one has been done for you.

Equation	Apply additive inverse of	Equation after applying inverse	
	-3	2a = 5a - 1	
2a = 2 - Ea = 4	-4		
2u - 5 = 5u - 4	2 <i>a</i>		
	5 <i>a</i>		

b) Solve the equation in Q2a.

3) The six equations below are arranged in pairs.

A. 5x - 3 = 2x + 6B. 3 - 5x = 2x + 6

C. 3y = 6 - 2y

D.
$$3y - 5 = 6 - 2y - 5$$

E. -5a = 6 - 2a

F.
$$0 = 5a - 2a + 6$$

a) Without solving the equations, try to predict whether each pair will have the same solution.

b) Now solve all six equations using inverses.

- c) Were your predictions correct? If not, was there something you didn't pay attention to?
- 4) Solve the following equations using inverses.

a) 5x - 3 = 4x + 6

- b) 5a 3 = 2a + 6c) 5b = 2b + 6
- d) 5c 3 = 2c



Worksheet 3.8: Algebraic equations

Qu	Questions			Answers		
1)	1) Consider the following 2 equations					
	A. $4 = 2n + 6$			Same: Both equations have 4 or	n the left side and	
	B. $4 + n = 2n + 6$			2n + 6 on the right side.		
	a) What is the same and what is different about the two			Different: Equation B. has $+n$ of	n the left side.	
	equations?		b)	No, because n was added to on	ly one side, the	
	b) Without solving the equations try t	o decide whether		left.		
	they will have the same solution. Iu	stify your answer.	c)	Yes, $-2 = 2n$ is easy to solve by	/ inspection.	
	c) Sizwe says we apply only the additiv	ve inverse of 6 for	d)	Additive inverse of n to collect t	he like terms.	
	equation A. Do you agree?		e)	A. $n = -1$ B. $n = -2$		
	d) Which inverse would you apply first	for equation B?	f)	May not have recognised that n	is added only on	
	Whv?			left side in B, and right side is id	entical.	
	e) Solve the two equations.					
	f) Was your prediction in Q2b correct	? If not, what was				
	wrong with your thinking?	,				
2)						
2)	a) In the table below, apply the additive	invarca and write de	we the ear	ution ofter applying the inverse	The first and	
	a) In the table below, apply the additive	e inverse and write do	own the equ	ation after applying the inverse.	. The first one	
	has been done for you.			Annua 10		
	Equation	Apply additive inve	rea of	Allsweis What romains		
	Equation		1SE 01	2a - 5a = 1		
		-3		2u - 3u - 1		
	2a - 3 = 5a - 4	29		-3 - 3a - 4		
		5 <i>a</i>		-3a - 3a - 4		
	b) Solve the equation in O2a Answer	2a - 3 = 5a - 4		54 5 - 1		
	20	$a^{-3} - 5a = 5a - 4$	-5a			
		-3a - 3 = -4	54			
	_	3a - 3 + 3 = -4 + 3	3			
		-3a = -1				
		$a-\frac{1}{2}$				
		$u = \frac{1}{3}$				
3)	The six equations below are arranged in	pairs.	3) Ans	wers		
	A. $5x - 3 = 2x + 6$		a)	Pair 1: No; Pair 2: Yes; Pair 3: \	/es	
	B. $3 - 5x = 2x + 6$		b)			
				A. $x = 3$		
	C. $3y = 6 - 2y$			B. $x = -\frac{3}{2}$		
	D. $3y - 5 = 6 - 2y - 5$			- ⁷		
			($y = \frac{1}{5}$		
	E. $-5a = 6 - 2a$		1	D. $y = \frac{6}{5}$		
	F. $0 = 5a - 2a + 6$			E. $a = -2$		
	a) Without solving the equations trut	o predict whether	1	F. $a = -2$		
	a) without solving the equations, if y to predict whether			May have not recognised that:		
	b) Now solve all six equations using inverses			A & B: $5x - 3 \neq 3 - 5x$;		
	c) Were your predictions correct? If no	nt was there		D & E: Same number added to b	ooth sides;	
	something you didn't now attention	to?		E & F: additive inverse of $-5a$ h	as been applied	
	something you durn t pay attention	10 :		in F, as well as commutative law	· · ·	
۸۱	Solve the following equations using invo	rses	 4) Anc. 	wars		
(4)	a) $5r - 3 = 4r + 6$ c) $5h - 5h - 6h - 6h - 6h - 6h - 6h - 6h - $	= 2h + 6	+) AIIS	r = 9 c) $h = 7$		
	b) $5a - 3 = 2a + 6$ d) $5c - 3a - 3$	-3 = 2c	a)	x = y $y = 2$		
	2, 54 5 <u>2</u> 4 6 4, 50		b)	a = 3 d) $c = 1$		



Worksheet 3.9: Algebraic equations

This worksheet focuses on comparing solutions and on shorter methods of solving equations using inverses.

Qu	esti	ons						
1)	Consider the equation $4x + 1 = 7 + 2x$. Both Anu and Andrew solved it using additive and							
	multiplicative inverses. Read the steps below carefully and answer the questions that follow.							
		Anu's response	Andrew's response					
		Step 1: $4x + 1 = 7 + 2x$	Step 1: $4x + 1 = 7 + 2x$					
		Step 2: $4x + 1 - 1 = 7 + 2x - 1$	Step 2: $4x + 1 - 4x = 7 + 2x - 4x$					
		Step 3: $4x = 7 - 1 + 2x$	Step 3: $1 = 7 - 2x$					
		Step 4: $4x = 6 + 2x$	Step 4: $1 - 7 = 7 - 2x - 7$					
		Step 5: $4x - 2x = 6 + 2x - 2x$	Step 5: $-6 = -2x$					
		Step 6: $2x = 6$	Step 6:					
	Ļ	Step 7:						
	a)	Complete Step 7 for Anu and complete Step	p 6 for Andrew.					
	b)	Explain why they get the same solution.						
	c)	Now go back and look at the responses aga	in.					
		i) What is the difference in the two resp	oonses in Step 2?					
		ii) In which step did Anu use the multipli	cative inverse?					
		iii) In which step did Andrew use the mul	tiplicative inverse?					
	d)	Whose response do you prefer? Why?						
	e) Try to go from Step 1 to step 6 in your head in Anu's solution and Step 1 to step 5 in your head							
		Andrew's solution. If you can, you will be a	ble to work faster. If you can't, keep practicing the steps.					
2)	Co	nsider the equation: $3x - 10 = x + 6$						
-,	al	If you want to collect like terms with the va	riable on the left side what inverse must you apply and					
	u,	which term must you apply it to?	music on the left side, what inverse must you upply, and					
	h)	If you want to collect constants on the left	side what inverse must you apply and which term must					
	5)	you apply it to?	side, what inverse must you apply, and which term must					
		Solve the equation using inverses						
	() 	Solve the equation using inverses.						
	a)	How many times did you apply the additive	inverse? which inverses did you apply?					
3)	Со	nsider the equation: $1 - 3x = 5 - x$	Roger's response Vas's response					
	Ro	ger and Vas both solved it incorrectly.	1 - 3x = 5 - x $1 - 3x = 5 - x$					
	a)	Spot the errors in their responses.	-2x = 4x $1 - 3x + 3x = 5 - x + 3x$					
	b)	Solve the equation correctly.	$x = -2 \qquad \qquad 1x = 7x$					
			x = 7					
4)	Yo	u are given the following equation: $3x + 5 =$	x - 9. Write TRUE or FALSE for each statement.					
	a)	The solution is a fraction.						
	b)	If you apply the additive inverse of 5, you v	will get -14 on the right side.					
	c)	If you apply the additive inverse of x , you	will get -14 on its own.					
	d)	If you apply the additive inverse of -9 , the	e equation won't be balanced.					
	, e)	If you apply the additive inverse of 5, and t	hen add – x to both sides. the result on the left side will					
	- /	be $-3x - x$.	······································					
	f)	The solution is $x = -7$.						



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Worksheet 3.9: Algebraic equations

Qu	esti	ons and answers				
1)	1) Consider the equation $4x + 1 = 7 + 2x$. Both Anu and Andrew solved it using additive and multiplicative inverses. Read					
	the steps below carefully and answer the questions that follow.					
		Anu's response		Andrew's r	esponse	
	Step 1: $4x + 1 = 7 + 2x$ Step 1: $4x + 1$			x = 7 + 2x		
		Step 2: $4x + 1 - 1 = 7 + 2x - 1$	Step 2:	4x + 1 - 4x	x = 7 + 2x - 4x	
		Step 3: $4x = 7 - 1 + 2x$	Step 3:	1	l = 7 - 2x	
		Step 4: $4x = 6 + 2x$	Step 4:	1 – 1	7 = 7 - 2x - 7	
		Step 5: $4x - 2x = 6 + 2x - 2x$	Step 5:	-6	b = -2x	
		Step 6: $2x = 6$	Step 6:			
		Step 7:				
	a)	Complete Step 7 for Anu and complete Step	p 6 for Andrev	/. Ans	Swer Step 7: $x = 3$ S	tep 6: $x = 3$
	b)	Explain why they get the same solution.		An	swer $-\frac{1}{-2} = \frac{1}{2}$	
	c)	Now go back and look at the responses aga	in.			
		i) What is the difference in the two	responses in t	Step 2? Ans	swer Anu applied th	e additive inverse of 1 and
		ii) In which stop did Applied the additive inve	rse or 4x.		swor Stop 7	
		iii) In which step did Andrew use the	multiplicative	eise: An	swer Step 7	
	d)	Whose response do you prefer? Why?	manapheative	An	swer Anu's, he deal	s with positives
	e)	Try to go from Step 1 to step 6 in your head	d in Anu's solu	tion and Step 2	1 to step 5 in your h	ead in Andrew's solution. If
	-	you can, you will be able to work faster. If y	/ou can't, keej	practicing the	e steps.	
2)	Cor	nsider the equation: $3x - 10 = x + 6$			2) Answers	
	a)	If you want to collect like terms with the va	riable on the	eft side,	a) Additiv	e inverse of x, i.e. add $-x$
		what inverse must you apply to terms with	variables?		b) Additiv	e inverse of 6, i.e. add -6
	b)	If you want to collect constants on the left	side, what inv	erse must	c) Solution	1: x = 8
	-)	you apply to constant terms?			d) Will dej	pend on approach. Will
	c)	Solve the equation using inverses.	inverse2 W/bi	ah invarias	need to	apply additive inverse to
	u)	did you apply?	inverser win	ch inverses		and to letter. Will then
					inverse	
3)	Cor	nsider the equation:			3)	Answers
	1 -	-3x = 5 - x Rog	er's response	Vas's res	sponse	a) See circles
	Rog	ger and Vas solved it incorrectly. $1 - \frac{1}{2}$	3x = 5 - x -2x = 4x	1 - 3x = 1 - 3x + 3x =	5 - x 5 - x + 3x	b) $1 - 3x = 5 - x$
	a)	Spot the errors in their responses.	x = -2	$1 0 \\ 1 \\ 1 \\ x \neq 1$	7x $7x$	-2x = 4
	b)	Solve the equation correctly.		(<i>x</i> =	7	$x = -\frac{1}{2}$
4)	Υοι	are given the following equation: $3x + 5 =$	x - 9. Write	RUE or FALSE	for each	4) Answers
	statement.					
	a) The solution is a fraction. a) False					
	b) If you apply the additive inverse of 5, you will get -14 on the right side. b) True					
	c) If you apply the additive inverse of x , you will get -14 on its own. c) True					
1	വ) ല	If you apply the additive inverse of 5 and t'	$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{1}{2} + \frac{1}$	hoth sides th	e result on the left	e) Falco
	C)	side will be $-3x - x$.		sour sides, th		f) True
	f)	The solution is $x = -7$.				.,
	,					



Worksheet 3.10: Algebraic equations

This worksheet provides practice in solving equations using inverses. The equations have variables on one side or on both sides. Some equations have integer coefficients.

Qu	Questions								
1)	Sol	lve the e	equations using additive ar	nd multiplicativ	/e ir	iverses.			
	a)	5 <i>m</i> –	6 = 3m + 2		d)	5 <i>m</i> + 6	= 3m - 2		
	b)	6 – 5 <i>1</i>	n = -2 - 3m		e)	10m -	12 = 6m + 4		
	c)	5 <i>m</i> –	2 = 3m + 6		f)	m + 1 -	-2m + 2 = 3 +	-m+4	
2)	Со	nsider t	he equation: $-9 + t = -3$	3 <i>t</i> + 3					
,	a)	Read t	he statements A to E and c	decide if they a	are 1	RUE or F	ALSE.		
	,	Α.	The value of t (i.e. the solution	, ution) is positi	ve.				
		В.	The value of t will not cha	inge if we mult	iply	each sid	e of the equatio	on by 3.	
		C.	The value of t will not cha	inge if we divid	le e	ach side (of the equation	by 3.	
		D.	The value of t will not cha	inge if we mult	tiply	by 3 on	the left side and	l divide by 3 on the	right
			side of the original equation	on.					
		Ε.	The value of t will change	if we add 9 to	eac	h side.			
	b)	Solve t	he equation.						
	c)	Now te	est any of the statements y	you are unsure	of,	i.e. perfo	orm the require	d operations and the	en
		solve t	he new equation.						
3)	Sol	ve these	e algebraic equations using	g additive and	mul	tiplicative	e inverses.		
	a)	<i>d</i> – 9	-2d = 7						
	b)	5p = -	-3 + p - 5						
	c)	<i>n</i> – 3	= 7 - n						
	d)	-s-6	6 + 2s = 7 - s						
	e)	3 - 2i	n = 2m + 3						
	T)	$\kappa = 2I$	(K + K +)						
4)	The	ere are i	five equations in the table.						
	a)	Predic	t whether the solution to e	each equation	will	be: a pos	sitive integer, or	a negative integer	or a
		fractio	n.						
	b)	Solve t	he equations to check you	ir predictions.					
			Equation	My prec	licti	on	Solution	Was I correct?	
		A.	a + 5 = 2a - 3						
		В.	5a + 7 = 2a - 3						
		C.	5a - 7 = 2a - 3						
		D.	6 - 3a = 2a - 3						
		с.	5u + 5 - 7u = 2u - 5						
5)	Six	equatio	ons are given below.						
	a)	Witho	ut solving the equations, p	redict which e	qua	tions will	NOT have the s	solution $t = 7$.	
	b) Solve each equation to check your predictions.								
		Δ 8-	- <i>t</i> + 1	ח	-2	1 – 9 =	-3t + 9		
		В. —2	21 = -3t	E.	42	= 6t	5017		
		C2	21 + 4 = -3t + 4	 F.	21	= -3t +	. 3		
L									

Worksheet 3.10: Algebraic equations



Qu	estions	Answers
1)	Solve the equations using inverses.	1)
	a) $5m-6 = 3m+2$ d) $5m+6 = 3m-2$	a) $m = 4$ d) $m = -4$
	b) $6-5m = -2-3m$ e) $10m - 12 = 6m + 4$	b) $m = 4$ e) $m = 4$
	c) $5m-2 = 3m+6$ f) $m+1-2m+2 = 3+m+4$	c) $m = 4$ f) $m = -3$
2)	Consider the equation: $-9 + t = -2t + 3$	2)
	a) Read the statements A to E and decide if they are TRUE or FALSE.	a)
	A. The value of <i>t</i> (i.e. the solution) is positive.	A. True
	B. The value of t will not change if we multiply each side of the	B. True
	equation by 3.	C. True
	C. The value of t will not change if we divide each side of the	D. False
	equation by 3.	E. False
	D. The value of t will not change if we multiply by 3 on the left	b) <i>t</i> = 3
	side and divide by 3 on the right side of the original	c) $-9+t = -3t+3$
	equation.	$(-9+t) \times 3 = (-3t+3) \times 3$
	E. The value of t will change if we add 9 to each side.	-27 - 27t = -9t + 9
	 b) Solve the equation. c) Now test equation. 	12t = 36
	c) Now test any of the statements you are unsure or, i.e. perform	t = -3 [Testing B.]
	the required operations and then solve the new equation.	
3)	Solve these algebraic equations using additive and multiplicative	3)
	inverses.	a) $d = 16$
	a) $d - 9 - 2d = 7$	b) $p = -2$
	b) $5p = -3 + p - 5$	c) $n = 5$
	d) $n-3=7-n$	d) $s = \frac{1}{2}$
	e) $3 - 2m = 2m + 3$	e) $m = 0$
	f) $k = 2k + k + 7$	f) $k = -\frac{1}{2}$
	These are first and the shift	
4)	I here are five equations in the table.	toger or a possible integer or a fraction
	 a) Predict whether the solution to each equation will be: a positive in b) Solve the equations to check your predictions 	teger, of a negative integer of a fraction.
Ans	by Solve the equations to thete your predictions.	
,	Equation My prediction Sol	ution Was I correct?
	A. $a + 5 = 2a - 3$ $a = 8$ a	= 8
	B. $5a + 7 = 2a - 3$ $a = -\frac{10}{a}$ $a = -\frac{10}{a}$	- 10 Depends on
	$\begin{array}{c c c c c c c c c c c c c c c c c c c $	3 learners'
	c. $3u^{-1} - 2u^{-3}$ $a = \frac{1}{3}$ a	= 3 predictions
	D. $6 - 3a = 2a - 3$ $a = \frac{9}{5}$ a	$=\frac{9}{5}$
	E. $5a + 5 - 7a = 2a - 3$ $a = 2$ a	= 2
5)	Six equations are given below.	5)
	a) Without solving the equations, predict which equations will NOT	a) D; F
	have the solution $t = 7$.	b)
	b) Solve each equation to check your predictions.	A. $t = 7$ D. $t = \frac{37}{2}$
	A. $8-t+1$ D. $-21-9=-3t+9$	B. $t = 7$ E. $t = 7$
	B. $-21 = -3t$ E. $42 = 6t$	C. $t = 7$ F. $t = -6$
1	C. $-21 + 4 = -3t + 4$ F. $21 = -3t + 3$	



Worksheet 3.11: Algebraic equations

This worksheet provides practice in solving equations with fractions, using inverses.

Qu	Questions							
1)) Copy and complete the following products.							
	a) $\frac{1}{2} \times 9 = $	e) $\frac{3x}{4} \times \frac{1}{3} = $						
	b) $6 \times \frac{2}{3} = $	f) $\frac{3x}{4} \times 4 = $						
	c) $\frac{3}{5} \cdot \Box = 6$	g) $\frac{3x}{4} \times \frac{4}{3} = $						
	d) $\frac{2}{5} \cdot \Box = -4$	h) $\frac{-x}{2} \times \Box = x$						
2)	The following equations have a fractio	n as a coefficient. Solve the equations. Q2a is done for you.						
	a) $\frac{1}{2}a + 7 = 5$	b) $4 - \frac{2}{3}c = 7$						
	$\frac{1}{2}a + 7 - 7 = 5$	c) $5 = \frac{1}{4}d - 2$						
	$\frac{1}{2}a = -2$	d) $9 = 6 + \frac{5}{4}d$						
	$\frac{1}{2}a \times 2 = -2 \times 2$							
	<i>u</i> = -4	_ 2						
3)	The following equation has a fraction a	as a constant: $7x - \frac{1}{3} = 1 + 5x$						
	Read Siya's response. Then, answer th	e questions from 3a to 3e.						
	Siya's response	a) In Step 1, Siya used an additive inverse of						
	$7x - \frac{2}{3} = 1 + 5x$	b) In which step did Siya use a multiplicative inverse?						
	Step 1: $7x - \frac{1}{3} - 5x = 1 + 5x - 5x$ Step 2: $7x - 5x - \frac{2}{3} = 1$	c) Explain what Siya did in Step 6.						
	Step 3: $2x - \frac{2}{3} = 1$	d) Siya used the multiplicative inverse of in Step 8.						
	Step 4: $2x - \frac{2}{3} + \frac{2}{3} = 1 + \frac{2}{3}$	e) Try to use a different way to solve this equation.						
	Step 5: $2x = 1 + \frac{2}{3}$	f) Compare the steps of your approach with Siya's steps.						
	Step 6: $2x = \frac{1 \times 312}{3}$							
	Step 7: $2x = \frac{1}{3}$ Step 8: $2x \times 3 = \frac{5}{3} \times 3$							
	Step 9: $6x = 5^{3}$							
	Step 10: $x = \frac{5}{6}$							
4)	Find the solutions to the following equ	intions						
+)	a) $E = 2t + \frac{1}{2} = 7t$	d) ${}^{3}m$ 0 - m + 0						
	a) $5 - 5i + \frac{1}{2} = /i$ b) $-9 - \frac{3}{2}h$	a) $\frac{1}{4}m - 9 = -m + 9$ a) $\frac{1}{4}a - 3a$						
	c) $-27 + 4 = -3t + 14t$	f) $4 = -9t + \frac{3}{2}$						
	· · · · · · · · · · · · · · · · · · ·	1/ 1 - 20 6						

Worksheet 3.11: Algebraic equations



Questions		Answers
1) Copy and complete the following products.		1)
a) $\frac{1}{2} \times 9 = $ b) $6 \times \frac{2}{3} = $ c) $\frac{3}{5} \cdot \Box = 6$ d) $\frac{2}{5} \cdot \Box = -4$	e) $\frac{3x}{4} \times \frac{1}{3} = $ f) $\frac{3x}{4} \times 4 = $ g) $\frac{3x}{4} \times \frac{4}{3} = $ h) $\frac{-x}{2} \times \Box = x$	a) $\frac{2}{2}$ e) $\frac{-}{4}$ b) 4 f) $3x$ c) 10 g) x d) -10 h) -2
2) The following equations have a fraction as a is done for you. a) $\frac{1}{2}a + 7 = 5$ $\frac{1}{2}a + 7 - 7 = 5$ $\frac{1}{2}a = -2$ $\frac{1}{2}a \times 2 = -2 \times 2$ a = -4	coefficient. Solve the equations. Q2a b) $4 - \frac{2}{3}c = 7$ c) $5 = \frac{1}{4}d - 2$ d) $9 = 6 + \frac{5}{4}d$	2) b) $-\frac{9}{2}$ c) $d = 28$ d) $d = \frac{12}{5}$
3) The following equation has a fraction as a concerned Read Siya's response. Then, answer the quest $7x - \frac{2}{3} = 1 + 5x$ Step 1: $7x - \frac{2}{3} - 5x = 1 + 5x - 5x$ Step 2: $7x - 5x - \frac{2}{3} = 1$ Step 3: $2x - \frac{2}{3} = 1$ Step 4: $2x - \frac{2}{3} + \frac{2}{3} = 1 + \frac{2}{3}$ Step 5: $2x = 1 + \frac{2}{3}$ Step 6: $2x = \frac{1 \times 3 + 2}{3}$ Step 7: $2x = \frac{5}{3}$ Step 8: $2x \times 3 = \frac{5}{3} \times 3$ Step 9: $6x = 5$ Step 10: $x = \frac{5}{6}$	anstant: $7x - \frac{2}{3} = 1 + 5x$ stions from 3a to 3e. a) In Step 1, Siya used an additive inverse of b) In which step did Siya use a multiplicative inverse? c) Explain what Siya did in Step 6. d) Siya used the multiplicative inverse of in Step 8. e) Try to use a different way to solve this equation. f) Compare the steps of your approach with Siya's steps.	3) a) $5x$ b) Step 8 c) created a single fraction, changing 1 to be $\frac{3}{3}$ d) $\frac{1}{3}$ e) $7x - \frac{2}{3} - 5x = 1 + 5x - 5x$ $7x - 5x - \frac{2}{3} = 1$ $2x - \frac{2}{3} = 1$ $2x - \frac{2}{3} + \frac{2}{3} = 1 + \frac{2}{3}$ $2x = 1 + \frac{2}{3}$ $2x = \frac{1 \times 3 + 2}{3}$ $2x = \frac{5}{3}$ $x = \frac{5}{3} \times \frac{1}{2}$ $x = \frac{5}{6}$ f) I used the multiplicative inverse of 2 instead of $\times 3$ on each side (Step 9]
4) Find the solutions to the following equations a) $5 - 3t + \frac{1}{2} = 7t$ d) $\frac{3}{4}n$ b) $-9 = -\frac{3}{5}h$ e) 42 c) $-27 + 4 = -3t + 14t$ f) 4 =	5. a - 9 = -m + 9 $+ \frac{1}{5}g = 3g$ $= -9t + \frac{3}{6}$	4) a) $t = \frac{11}{20}$ b) $h = 15$ c) $t = -\frac{23}{11}$ d) $t = \frac{72}{7}$ e) $g = 15$ f) $t = -\frac{17}{18}$



Worksheet 3.12: Algebraic equations

This worksheet provides practice in solving equation using inverses. This will require dealing with the four arithmetic operations and brackets.

Qu	Questions								
1)	1) Solve following equations using inverses.								
	a) $\frac{9}{2}h$	h + 5 = 27	d)	$y + \frac{7}{6} = -y$	$+\frac{7}{6} = -y - \frac{7}{6}$				
	b) 5 <i>t</i>	t - 7 = -27 + 7t	e)	b - 11 = 1	3				
	c) 1	$-\frac{3}{2}m = m - 9$	f)	4 - 3n + 9	+5n = 21				
2)	This q	uestion focuses on the	distributive law. Cor	nplete the ta	ble. Q2a is done for you	J.			
		Expression	Using distribut	ive law	Simplification]			
	a)	2(a+7)	$2 \times a + 2$	× 7	2a + 14	_			
	b)	2(a-7)				-			
	c)		$2 \times (-a) + 2$	2 × 7		-			
	d)				-2a - 14				
3)	Seven	equations are given be	elow.						
	a) W	ithout solving the equa	ations, predict which	equations w	ill have the same solution	on as $3(s - 2) = 9$.			
	b) Sc	olve each equation to c	heck your prediction	IS.					
	A.	3s - 2 = 9	C. $3s - 6$	= 27	F. $(s-2)3 =$	= 9			
	В.	3s - 6 = 9	D. $9 = 3(s$	s — 2)	G. $s - 3 = \frac{9}{6}$				
			E. $9 = 2(s$	s – 3)	6				
4)	Six eq	uations are given below	v.						
	a) W	ithout solving the equa	ations, predict which	equations D	O NOT have the same so	olution as			
	_	-3(s-2)=9.							
	b) Sc	olve each equation to c	heck your prediction	IS.					
	A.	-3s - 2 = 9	C. $-3s + 6$	6 = 9	E. $-9 = 3(s)$	- 2)			
	В.	-3s + 2 = 9	D. $-3s - 6$	6 = 9	F. $3(s-2) = 0$	= -27			
5)	Solvo	the following equation			$G. \ S - 2 = -$	\sim 3			
5)	O5a h	as been done for you	s using inverses.	6) Lee solv	red the equation: $-3($	$(2-n) = \frac{1}{4}$			
	a) 13	3 = 2(k - 4) + 5		Her res	ponse is given alongside	2.			
	13	$3 = 2 \times k - 2 \times 4 + 5$		Spot the	e error(s) and write the $\frac{3}{3}$	correct solution.			
	1	3 = 2k - 8 + 5			$-3(2-n) = \frac{1}{4}$				
	1	3 = 2k - 3		Step 1:	$-3 \times 2 - 3 \times n = \frac{3}{4}$				
	1	3+3=2k		Step 2:	$-6 - 3n = \frac{3}{4}$				
	1	6 = 2k		Step 3:	$-3n = \frac{3}{4} + 6$				
	{	3 = k or k = 8		Step 4:	$-3n = \frac{3+6}{4}$				
	b) 5/	(-6 - i) - 2		Step 5:	$-3n = \frac{9}{4}$				
	c) 27	(-0-3) = 2 (-3(2) = 5		Step 6:	$n = \frac{9}{4} \times \frac{1}{2}$				
	d) 2((n-3) = 5(4)		Sten 7 [.]	$n = \frac{3}{2}$				
	e) —	4(f-3) = 7			4				

wits maths connect

Worksheet 3.12: Algebraic equations

Qu	estions		Answers			
1)	Solve fo	ollowing equations using	g inverses.	1)		
	$a) = \frac{9}{h}$	$1 \pm 5 - 27$	d) $y + \frac{7}{2} - y - \frac{7}{2}$	a) $h = \frac{44}{9}$ d) $y = -\frac{7}{6}$		
	b) E_{t}	7 - 27 + 7t	$y + \frac{y}{6} - \frac{y}{6}$	b) $t = 10$ e) $b = 24$		
	b) 5 <i>l</i>	-7 = -27 + 70	e) $b - 11 = 15$ f) $4 - 3n + 9 + 5n - 21$	c) $m = 4$ f) $n = 4$		
	C) I -	$-\frac{1}{2}m = m - 9$				
2)	This que	estion focuses on the di	stributive law. Complete the table. Q2a is done f	or you.		
	Answer	S Evenession		Cimulification		
	2)	$2(a \pm 7)$		$\frac{2a \pm 14}{2}$		
	a) b)	2(a - 7)	$2 \times a - 2 \times 7$	2a - 14		
	c)	2(-a+7)	$2 \times (-a) + 2 \times 7$	-2a + 14		
	d)	-2(a+7)	$-2 \times a - 2 \times 7$	-2a - 14		
	۵,	=(a + 7)		200 11		
3)	Seven e	equations are given belo	w.	3)		
	a) Wi	ithout solving the equat	ions, predict which equations will have the	a) B; D; F		
	sai	me solution as $3(s-2)$	= 9.	b)		
	b) So	lve each equation to ch	eck your predictions.	s = 5 for B; D and F		
	Α.	$3s - 2 = 9 \qquad \qquad$	3s - 6 = 27 F. $(s - 2)3 = 9$	A. $s = \frac{11}{3}$		
	В.	3s - 6 = 9	0. $9 = 3(s-2)$ G. $s-3 = \frac{9}{6}$	C. <i>s</i> = 11		
		E	9 = 2(s - 3)	E. $s = \frac{15}{2}$		
				G. $s = \frac{9}{9}$		
4)	Six equa	ations are given below.		<u> </u>		
,	a) Wi	ithout solving the equat	ions, predict which equations DO NOT have the	a) A; B; D; F		
	sai	me solution as		b) $s = -1$ for C; E; G		
	-	3(s-2)=9.		A. $-\frac{11}{2}$		
	b) So	lve each equation to ch	eck your predictions.	$B - \frac{7}{2}$		
	A. $-3s - 2 = 9$ C. $-3s + 6 = 9$ E. $-9 = 3(s - 2)$			-3 = -5		
	B. $-3s + 2 = 9$ D. $-3s - 6 = 9$ F. $3(s - 2) = -27$			E.s = -7		
			G. $s - 2 = -3$			
5)	Solve th	ne following	6) Lee solved the equation: $-3(2-n) = \frac{3}{2}$	5)		
	equatio	ons using inverses.	Her response is given alongside	b) $j = -\frac{32}{\pi}$		
	Q5a has	s been done for you.	Spot the error(s) and write the correct	c) $z = \frac{11}{12}$		
	a) 13	3 = 2(k-4) + 5	solution.	$c_{1}^{2} = \frac{1}{2}$		
	13	$= 2 \times k - 2 \times 4 + 5$	$-3(2-n) = \frac{3}{2}$	d n = 13		
	13	5 = 2k - 8 + 5	Step 1: $-3 \times 2 - 3 \times n - \frac{3}{2}$	$e_{j} = \frac{1}{4}$		
	13	k = 2k - 3 k + 3 = 2k	Step 1: 3×2 $3 \times n = \frac{4}{4}$			
	16	b = 2k	Step 2: $-6 - 3n = \frac{3}{4}$	6) See circled errors		
	8	= k or k = 8	Step 3: $-3n = \frac{3}{2} + 6$	Stop 4: $-2n - \frac{3+24}{4}$		
			4 3+6	Step 4. $-3n = \frac{4}{4}$		
	b) 5(-6-j)=2	Step 4: $-3n = \frac{3}{4}$	Step 5: $-12\pi = 27$		
	c) $2z$	(-3(2) = 5)	Step 5: $-3n = \frac{9}{4}$	Step b: $n = -\frac{1}{12}$		
	u) 2(1 e) -4	n - 3j = 3(4) 4(f - 3) = 7	Step 6: $n = \frac{9}{2}\sqrt{\frac{1}{1}}$	Lee's Step 6 from $-3n = \frac{1}{4}$ should		
			Step 0. $n = \frac{1}{4} \sqrt{\frac{3}{3}}$	have been: $\times -\frac{1}{3}$		
			Step 7: $n = \frac{3}{4}$			

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PRACTICE IN WORKING WITH LINEAR EQUATIONS



Worksheet 3.13: Algebraic equations

This worksheet provides practice in solving equations using inverses. In the last question, you will need to create the equations from verbal expressions.

Questions	
1) Solve following equations using inverses.	
a) $h + 8 = 15$ b) $h + 15 = 8$ c) $h + 15 = 8h$ d) $8m - 11 = 10 + m$ e) $2b - 8b = 3(5 - 3)$ f) $2h - 8 = 3(h - 3)$	g) $6s + \frac{2}{3} = 3 - s$ h) $y + \frac{7}{6} - 3y = 1 + \frac{7}{6}$ i) $\frac{1}{2}(2x - 4) = 7$ j) $\frac{1}{2}(2x + 4) = 7x$

2) Check whether each of the given values is the solution of the equation.

	Equation	Value	Is it the solution: Yes/ No?	Evidence
a)	v - 7 = 0	v = -7		
b)	2(a-5) = -5	$a = -\frac{5}{2}$		
c)	9m-7=2m-7	$m = \frac{2}{9}$		
d)	$\frac{b}{3} - 1 = 1$	b = 6		

3) Look at the pairs of equations.

- a) Predict whether the solution to the two equations will be the same for each pair. Give reasons for your answers.
- b) Solve the equations and check your predictions.

	Equation 1	Equation 2	Same solution?	Reason
Α.	3f + 7 = 11	3f + 10 = 14		
В.	g - 8 = 15 - g	g - 8 = -15 - g		
С.	g + 8 = 15 - g	2g + 16 = 30 - 2g		
D.	2p - 3 = 3(p - 7)	$\frac{1}{3}(2p-3) = p-7$		
E.	$\frac{20b}{3} - 7 = 49$	$\frac{20b}{3} - 7 - b = 49 - b$		

4) Write equations for the following statements. Solve the equations after forming them.

- a) The sum of 5 and a number is -1.
- b) The product of a number and -7 is 140.
- c) A number decreased by 8 is 11.
- d) A number divided by 5 gives 10 times 7.
- e) Three times a number added to 7 gives 19.
- f) Seven times a number added to 4 gives -15.
- g) The sum of one-third of a number and 6 gives 10.
- h) Three-quarters of a number gives 5.

Worksheet 3.13: Algebraic equations



Qu	estic	ons and answers						
1)	Sol	ve following equations usi	ing inverses.		1) Answ	vers		
	a	a) $h+8=15$ g) $6s+\frac{2}{2}=3-s$			a)	a) $h = 7$ g) $s = \frac{1}{2}$		
	b) $h + 15 = 8$	h) $v + \frac{7}{2}$	-3v = 1 + 1	<u>7</u> b)	b) $h = -7$ b) $y = -\frac{1}{2}$		
	c)	h + 15 = 8h	i) ¹ (2		⁶ c)	$h = \frac{15}{7}$	i) r-	- Q 2
	d) $8m - 11 = 10 + m$	1) $\frac{1}{2}(2x)$	– 4) = 7	d)	m = 3	i) x	= 5
	e) $2b - 8b = 3(5 - 3)$	j) $\frac{1}{2}(2x)$	+ 4) = 7	e)	b = -1]/ 70	C C
	f)	2b - 8 = 3(b - 3)			f)	b = 1		
2)	Che	eck whether each of the g	iven values is th	ne solution of	the equation.			
	Ans	swers						
		Equation	Value	Is it the so	lution: Yes/ No?	Evide	nce	
	a)	v - 7 = 0	v = -7		Νο	v =	7	
	b)	2(a-5) = -5	$a = -\frac{5}{2}$		No	<i>a</i> =	$\frac{5}{2}$	
	c)	9m-7=2m-7	$m = \frac{2}{9}$		No	<i>m</i> =	0	
	d)	$\frac{b}{1} - 1 - 1$	h - 6		Voc	<i>b</i> –	3 = 3	
	u)	$\frac{1}{3}$ - 1 - 1	D = 0		165	b =	6	
3)	Loc	ok at the pairs of equation	s.					
	a)	Predict whether the solu	ution to the two	o equations v	vill be the same	for each pair. G	ive reasons	s for your answers.
	b) And	Solve the equations and	Check your pre	dictions.				
	Alla	Equation 1	Equation 2		Same solution)	Re	ason
	Α.	3f + 7 = 11	3f + 10 =	14	Yes	11 – 7 and	14 – 10 a	re both 4
	В.	g - 8 = 15 - g	g - 8 = -1	15 – <i>g</i>	No	15 is negat	ive in equa	tion 2
	C.	a + 8 = 15 - a	2a + 16 =	30 - 2q	Yes	Each term	Each term in equation 1 is multiplied by 2 to	
		3.5 _5 3	-9			get correspon		m in equation 2
	D.	2p - 3 = 3(p - 7)	$\frac{1}{2}(2p-3)$	= n - 7	Yes	If you mult	iply both si	des of equation 2 by 3,
			3 1 - 7	Ľ		then you g	et equatior	1.
	Ε.	$\frac{20b}{3} - 7 = 49$	$\frac{20b}{3} - 7 - b$	= 49 - b	Yes	Subtracted	b on each	side of equation 2
4)	Wr	ite equations for the follo	wing statement	s. Solve the	4) Ansv	vers		
	equ	The sum of F and a num	n. Maria 1		Equ	ation	Solu	tion
	d) h)	The product of a number	iden is -1 .	n	a)	5 + x = -1	<i>x</i> =	-6
	c)	A number decreased by			b)		<u></u>	
	-/		8 is 11.		D)	-7y = 140	y =	-20
	d)	A number divided by 5 g	8 is 11. gives 10 times 7	7.	b) c)	-7y = 140 p - 8 = 11	y = p =	-20 19
	d) e)	A number divided by 5 و Three times a number a	8 is 11. gives 10 times 7 dded to 7 gives	7. 5 19.	c)	$-7y = 140$ $p - 8 = 11$ $\frac{a}{2} = 10 \times 7$	y = p = a =	-20 19 350
	d) e) f)	A number divided by 5 g Three times a number a Seven times a number a	8 is 11. gives 10 times 7 dded to 7 gives dded to 4 gives	7. 5 19. 5 –15.	c) d)	$-7y = 140$ $p - 8 = 11$ $\frac{a}{5} = 10 \times 7$ $3x + 7 = 19$	y = p = a = x =	-20 19 350 4
	d) e) f) g) h)	A number divided by 5 g Three times a number a Seven times a number a The sum of one-third of Three-quarters of a num	8 is 11. gives 10 times 7 dded to 7 gives dded to 4 gives a number and ber gives 5.	7. 5 19. 5 —15. 6 gives 10.	c) d) e) f)	-7y = 140 p - 8 = 11 $\frac{a}{5} = 10 \times 7$ 3x + 7 = 19 7y + 4 = -11	y = p = a = x = 5	-20 19 350 4 $-\frac{19}{7}$
	d) e) f) g) h)	A number divided by 5 g Three times a number a Seven times a number a The sum of one-third of Three-quarters of a num	8 is 11. gives 10 times 7 dded to 7 gives added to 4 gives a number and aber gives 5.	7. 5 19. 5 –15. 6 gives 10.	c) d) e) f)	-7y = 140 p - 8 = 11 $\frac{a}{5} = 10 \times 7$ 3x + 7 = 19 7y + 4 = -11 $\frac{b}{7} + 6 = 10$	$y =$ $p =$ $a =$ $x =$ $5 \qquad y =$ $b =$	-20 19 350 4 $-\frac{19}{7}$ 4
	d) e) f) g) h)	A number divided by 5 g Three times a number a Seven times a number a The sum of one-third of Three-quarters of a num	8 is 11. gives 10 times 7 dded to 7 gives dded to 4 gives a number and hber gives 5.	7. 5 19. 5 —15. 6 gives 10.	b) c) d) e) f) g)	$-7y = 140$ $p - 8 = 11$ $\frac{a}{5} = 10 \times 7$ $3x + 7 = 19$ $7y + 4 = -12$ $\frac{b}{3} + 6 = 10$ $\frac{3x}{3x} = 5$	$y =$ $p =$ $a =$ $x =$ $5 \qquad y =$ $b =$ $x =$	$ \begin{array}{r} -20 \\ 19 \\ 350 \\ 4 \\ -\frac{19}{7} \\ 4 \\ \underline{120} \\ \end{array} $

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PRACTICE IN WORKING WITH LINEAR EQUATIONS



Worksheet 3.14: Algebraic equations

This worksheet provides practice in solving equations using inverses. The equations have variables on one side or on both sides. There are examples with brackets and fractions too.

Questions

- 1) Solve following equations using inverses.
 - a) 5h + 7 = 33
 - b) -2(z-11) = 7z 41
 - c) $\frac{1}{3}(b+\frac{9}{2}) = \frac{18}{2}$
 - d) $x + 2(1 3x) = 3 \times 9$
 - e) k(7-3) = 24
 - f) $\frac{1}{2}(6x-8) = 7x 25$

2) Check whether the given values are the solution of the equation.

	Equation	Value	Is it the solution: Yes/ No?	Reasons
a)	6-2a=0	a = -3		
b)	$\frac{1}{2}\left(m-\frac{1}{2}\right) = -5$	$a = \frac{21}{2}$		
c)	$m - 7m = -\frac{66}{11}$	m = -3		
d)	$\frac{2f}{3} - 9 = 11$	<i>f</i> = 19		

3) Write equations for the following statements. Solve the equations after forming them.

- a) Two times a number subtract 9 gives -1.
- b) Two times a number subtracted from 9 gives -1.
- c) The sum of a number and one-quarter gives 2.
- d) The product of a number and one-quarter gives -2.
- e) The difference between 2 and a number gives -21.
- f) The sum of a number and 5 is the same as the product of the number and 6.
- g) A number subtract 5 is the same as the product of the number and 6.
- h) Ten subtract a number is the same as the number decreased by 10.
- 4) Which of the equations have the same solution?

Try to do this without solving the equations. Then check your predictions by solving the equations.

a)
$$a - (5 - a) = 5$$

b)
$$-a + (a - 5) = -5$$

- c) 2a (5 a) = 10
- d) 2a 2(5 a) = 10
- e) a 5 + a 5 = 2(5 a)



Worksheet 3.14: Algebraic equations

Qu	Questions					Answers		
1)	Solve fo	llowing equations using inv	1)					
	a) $5h + 7 = 33$					a) $h = \frac{26}{2}$		
	b) $-2(z-11) = 7z - 41$					z = 7		
	c) $\frac{1}{3}$	$(b + \frac{9}{2}) = \frac{18}{2}$				(c) $b = \frac{45}{45}$		
	d) <i>x</i> -	$+2(1-3x) = 3 \times 9$				2 16		
	e) k((7-3) = 24			($x = -\frac{1}{5}$		
	f) $\frac{1}{2}$	6x - 8) = 7x - 25			(e) $k = 6$		
	2				1	$x = \frac{21}{4}$		
2)	Check w	vhether the given values are	e the solution of the	e equation.				
		Equation	Value	Is it the solution: Yes	/ No?	Reaso	ns	
	a)	6 - 2a = 0	a = -3	No	,		}	
	b)	$1 \begin{pmatrix} m & 1 \end{pmatrix} = 5$	$a - \frac{21}{a}$	No			19	
	~)	$\frac{1}{2}(m-\frac{1}{2})=-5$	$u = \frac{1}{2}$			m = -	2	
	c)	$m - 7m = -\frac{66}{11}$	m = -3	No		m = 1	1	
	d)	$\frac{2f}{3} - 9 = 11$	<i>f</i> = 19	Yes		2f - 27 =	= 11	
		5			2f = 38			
					f = 19			
3)	Write e	quations for the following s	tatements. Solve th	ne equations after	3)			
	forming	them.				Equation	Solution	
	a) Tw	o times a number subtract	9 gives −1.		a)	2x - 9 = -1	x = 4	
	b) Tw	o times a number subtracte	ed from 9 gives -1		b)	9 - 2x = -1	<i>x</i> = 5	
	c) Th	e sum of a number and one	-quarter gives 2.		c)	$v + \frac{1}{2} = 2$	$v = \frac{7}{2}$	
	d) Th	e product of a number and	one-quarter gives -	-2.	-1)	<u> </u>	^y 4	
	e) The	e difference between 2 and	a number gives —	-21.	a)	$a \times \frac{1}{4} = -2$	a = -8	
	f) In	e sum of a number and 5 is	the same as the pr	oduct of the number	e)	2 - x = -21 or	<i>x</i> = 23 or	
	an a) Ar	u o. Numbor subtract 5 is the sar	ma as the product (of the number and 6		x - 2 = -21	x = -19	
	b) Te	n subtract a number is the s	ame as the numbe	or decreased by 10	f)	x + 5 = 6x	x = 1	
	11) 10				g)	x - 5 = 6x	x = -1	
					h)	10 - x = x - 10	<i>x</i> = 10	
4)	Which o	of the equations have the sa	me solution?		4)	a; b; d; e have the san	ne solution	
	Try to do this without solving the equations. Then check your predictions by					a) $a = 5$		
	solving the equations.					a = 5		
	a) a -	-(5-a) = 5				a = 15		
	(1) = 0	(1 + 2(u - 3)) = -3 + $(5 - a) - 20$				a = 5		
	d) 2 <i>a</i>	-2(5-a) = 10				-,		
	e) a -	-5 + a - 5 = 2(5 - a)						
	$c_{j} = 0$							