wits maths connect
supporting secondary maths

#super powers

PRACTICE IN WORKING WITH POWERS & EXPONENTS

VERSION 1.0

Grade 8 and 9

What is the **same**?
What is **different**?

$2^5 \qquad 5^2$

TRUE or FALSE?

$$ab = \frac{b}{a^{-1}}$$

WHY?

$$p + 0 = p$$

$$p \times 0 = 0$$

BUT

$$p^0 = 1$$

FILL IN THE GAPS

$$\square^2 \cdot \square^{\square} = 2^{10}$$

$$5^2 \times \square^{\square} = 25^2$$

$$\square^2 \times \square^{\square} = 40^2$$

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Science and Innovation
REPUBLIC OF SOUTH AFRICA

#super powers : Practice in working with powers & exponents

These materials were produced by the Wits Maths Connect Secondary (WMCS) project at the University of the Witwatersrand.
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#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS AND EXPONENTS

VERSION 1.0

About this booklet

The 36 worksheets in this booklet provide practice in working with exponents and powers for Grade 8 and Grade 9 levels. Answers are provided for each question.

The pack is called #**super**powers because mathematical superpowers are important skills for learning mathematics at high school. None of the well-known superheroes has mathematical superpowers! But they practice their superpowers in the same way that learners should practice their mathematical skills to develop powerful knowledge of mathematics.
We assume learners have been taught the content of exponents so that they can use this pack to practise working with powers and exponents. We provide a 10-page summary of the basics of working with powers and exponents, including the laws and definitions. We also include some ideas on what makes exponents confusing.
Our research in South African schools shows that learners tend to apply exponential laws where the laws are not appropriate. For example, many give the following incorrect answer: $3x + 2x = 5x^2$. A possible reason for such errors is that learners are taught laws of exponents before they have mastered the basics of algebra, and before they have had opportunity to derive the laws themselves. If learners are given the opportunity to expand powers and cancel factors, they may well be able to discover the laws for themselves. For example, expanding $p^4.p^2$ to $p.p.p.p \times p.p$, shows clearly that the answer is p^6. This is law 1: multiplying powers with the same base.

The worksheets are arranged in 6 sections, each with a Grade 8 and a Grade 9 cluster. The Grade 8 worksheets are numbered 1.**1**a, b, c; 2.**1**a, b, c; 3.**1**a, b, c etc. They involve only positive exponents. Grade 9 worksheets include negative exponents and are numbered 1.**2**a, b, c; 2.**2**a, b, c etc. The worksheets get progressively more difficult as you move from *a* to *b* to *c*. For Grade 9s, it may be helpful to begin with the Grade 8 worksheets and then continue immediately onto the relevant Grade 9 materials in each cluster. Grade 8s should only do the first 3 (or 4) worksheets for each law. We placed the worksheets on key definitions ($a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$) at the front of the pack. However, we do not recommend that learners begin with these worksheets. These definitions arise from law 2 so learners should engage with these worksheets while practising law 2. The definition of a^{-n} is not required in Grade 8.

Section	**No. of worksheets**		**Content**
	Grade 8	**Grade 9**	
Definitions	2	2	Revising the origins of 2 definitions: $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$
Law 1	3	3	Applying the law for multiplying powers, $a^m \times a^n = a^{m+n}$
Law 2	3	3	Applying the law for dividing powers, $a^m \div a^n = a^{m-n}$
Law 3	3	3	Applying the law for raising a power to a further power, $(a^m)^n = a^{m \times n}$
Laws 4 & 5	3	3	Applying the laws for dealing with products and quotients raised to powers, $(a \times t)^n = a^n t^n$ and $\left(\frac{a}{t}\right)^n = \frac{a^n}{t^n}$
Mixed examples	4	4	Collections of examples dealing with all 5 laws and the definitions

❖ Learners should pay attention to the numbering and layout of questions. Sometimes the numbering goes down the first column and then across; sometimes numbering goes across several columns and then down to the next row. This is deliberate – to encourage learners to pay attention to question numbering.

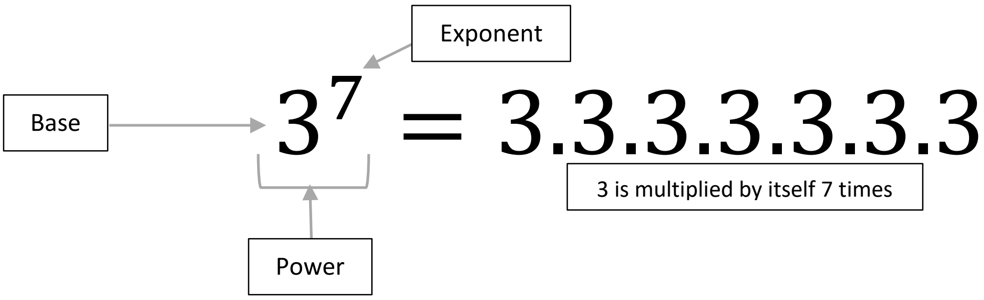
#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

NOTES ON POWERS AND EXPONENTS

In these notes we explain important concepts, terminology, laws and definitions that relate to powers and exponents. We provide lots of examples to illustrate the laws and definitions. Some examples have numeric bases (e.g. 3^5) and many involve algebraic terms and expressions (e.g. a^6; x^2y^3; $4mn^2$). We only work with numeric exponents in these materials so you won't find examples like 2^x. We have written these notes in simple language for Grade 8 and 9 learners. At the end of the notes we discuss some instances where exponential notation can be confusing.

QUICK REFERENCE



Five exponential laws

1.	$a^m \times a^n = a^{m+n}$	$a^{m+n} = a^m \times a^n$
2.	$a^m \div a^n = a^{m-n}$	$a^{m-n} = a^m \div a^n$
3.	$(a^m)^n = a^{m\times n}$	$a^{m\times n} = (a^m)^n$
4.	$(a \times t)^n = a^n t^n$	$a^n t^n = (a \times t)^n$
5.	$\left(\frac{a}{t}\right)^n = \frac{a^n}{t^n}$	$\frac{a^n}{t^n} = \left(\frac{a}{t}\right)^n$

Four definitions

1.	$a^n = a.a.a.a.a \dots a.a \quad n$ times	
2.	$a^0 = 1$	
3.	$a^{-m} = \frac{1}{a^m}$	$\frac{1}{a^m} = a^{-m}$
4.	$\frac{1}{a^{-m}} = a^m$	$a^m = \frac{1}{a^{-m}}$

Notes

- We have numbered the laws 1 – 5, and the definitions 1 – 4 There is nothing special about the numbering of the laws. Rather focus on the name/purpose of the law/definition than on the number. Other textbooks may use different numbers.
- Tips
  - Learn to say the laws using words – don't focus only on the algebraic version of the laws.
  - If you are having difficulty in remembering or applying the laws, go back to basics, i.e. expand each power and then cancel like factors. This will help you to make sense of the laws.
  - When you first start working with negative exponents (in Grade 9), convert negative exponents to positive exponents by using the definitions (#2 or #3). Then work with the positive exponents.
- In some textbooks, the authors use the terms *index* and *indices* (*indices* is the plural of *index*), when referring to exponents. We do not use those terms in these materials.
- Some textbooks will emphasise the restrictions on bases and exponents even in Grades 8 and 9. We have not done this except for the restriction that the base cannot be zero in the definitions.
- Assume that all algebraic examples are defined and that no denominators are zero.

1) What are exponents and powers?

Exponents and powers involve *repeated multiplication*. Exponential notation provides an efficient mathematical notation for describing repeated multiplication. Exponential laws provide shortcuts for working with powers and exponents. Exponential growth describes the growth of things that change by doubling, tripling, halving, etc. like compound interest and the growth of bacteria.

2) Exponential notation

Look at these 2 examples:

$$2 \times 2 \times 2 \times 2 \times 2$$
$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$$

Count the number of 2s and the number of 3s

Do you see that it is difficult to read an expression where a number is multiplied by itself many times? It also takes up a lot of space. To deal with both problems, mathematicians invented *exponential notation*.

Exponential notation involves a *base* and an *exponent*.
There are two examples alongside.
In the first example, the base is 2 and the exponent is 5.
In the second example, the base is 3 and the exponent is 15.
We write the exponent as a superscript.
The base and exponent together form a *power*.

We say “2 to the power of 5” or “2 to the exponent 5” or “2 to the 5” which means we are multiplying 2 by itself 5 times and we say “3 to the power of 15” or “3 to the exponent 15” or “3 to the 15” which means we are multiplying 3 by itself 15 times.

$$2^5$$

The base “holds up” the exponent, so 2 holds up 5, and 3 holds up 15

$$3^{15}$$

Multiplication can be thought of as *repeated addition* → $2 + 2 + 2 + 2 + 2 = 5 \times 2$ ← 2 is added 5 times or we can speak of *5 groups of 2*

Exponents can be thought of as *repeated multiplication* → $2 \times 2 \times 2 \times 2 \times 2 = 2^5$ ← 2 is multiplied by itself 5 times

When we use exponential notation in algebra (using letter bases), it will look like this: a^4 which means $a \times a \times a \times a$.
We say “a to the four” or “base a to the exponent four” or “a to the power of four”.

We can also work “backwards” so p^3 means $p \times p \times p$ and $t^7 = t \times t \times t \times t \times t \times t \times t$
The exponent tells us how many times the base is multiplied by itself.

Conventions (or accepted ways) in exponential notation

- We don’t write the exponent if the exponent is 1.
  e.g. m^1 is written as m and $ab^2 = a \times b \times b$

- When working with exponents, we can represent the operation of multiplication using the multiplication sign, a dot or brackets.
  - Using the multiplication sign: $a \times a \times a = a^3$
  - A dot (.) can replace the × sign: $a.a.a = a^3$ or $5 \times 5 \times 5 \times 5 = 5.5.5.5 = 5^4$
  - Brackets can replace the × sign: $(g)(g)(g)(g) = g^4$ also $5(4a) = 20a$
  - Brackets can also be used for grouping: $3.k.4.k.k = (3.4)(k.k.k) = 12k^3$

When you see a dot between two numbers, it means the numbers are being multiplied. So 3.4 is the same as 3×4. In SA schools, we use the decimal comma for decimal numbers. So 3,4 represents the decimal number “three comma four”.

- Remember that brackets don’t always indicate multiplication e.g. $9 - (2 + 3)$ tells us to add 2 and 3 before we subtract the answer (5) from 9. There is no multiplication required.

3) What kinds of numbers can we use for bases?

According to the definition, we can use any value as a base, except 0. So bases can be natural numbers, integers and even fractions. Here are some examples:

$$4^3 = 4.4.4 \qquad (-3)^2 = (-3)(-3)$$

$$\left(\frac{1}{2}\right)^5 = \frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2} \qquad \left(-\frac{2}{3}\right)^7 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)$$

When the base is negative, it is important to write the base in brackets.

4) What kinds of numbers can we use for exponents?

In Grade 8 we use only whole numbers for exponents, e.g. 2^3 and p^9

In Grade 9 you are introduced to negative exponents e.g. 2^{-3} and p^{-9}, which can be written as $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ and $p^{-9} = \frac{1}{p^9}$. We will explain this later. (See point 9d)

As you can see, a negative exponent doesn't mean the power will be a negative number.

In Grade 10 you will start to use fractions for the exponent. e.g. $p^{\frac{1}{2}}$ and $m^{\frac{4}{3}}$ and $6^{-\frac{2}{5}}$

5) A note on coefficients of powers

Coefficients are numbers that are multiplied by terms. Sometimes the terms contain algebraic symbols, e.g. $7x$. Sometimes they contain powers with numbers, e.g. 7.2^3. In both cases, 7 is the coefficient.

The expression $3x^2 + 5a - 10a^3$ has three terms which are listed below:

- $3x^2$: the power is x^2, the coefficient of x^2 is 3
- $5a$: the power is a (it has an exponent of 1), the coefficient of a is 5
- $-10a^3$: the power is a^3, the coefficient is -10.

If no number is written in front of a term, e.g. a^4, then the coefficient is positive 1, i.e. $a^4 = 1.a^4$

We need to be careful when we have coefficients and powers with negative bases.

Example 1: 4 multiplied by -5 squared: $4(-5)^2 = 4(-5)^2 = 4(-5)(-5) = 100$

Example 2: -4 multiplied by -5 squared: $-4(-5)^2 = (-4)(-5)(-5) = -100$

If you are given -5^2, then you should assume the base is positive and there is a coefficient of -1.
i.e. $-5^2 = (-1)\times 5^2 = (-1)(5)(5) = -25$

If you write -5^2 but you mean that the base is -5, then you must write the base in brackets,
i.e. $(-5)^2 = (-5)(-5) = 25$

6) Why are exponents useful in maths?

Exponential notation (i.e. powers) is useful to write very large and very small numbers in a notation called scientific notation.

a) *Writing very large number using exponents*

The mass of the earth is estimated at 5,97 $\times 10^{24}$ kg.

We know that 10^{24} means 10 multiplied by itself 24 times, so we have:

$5{,}97 \times 10$

We could also write the number as 597×10^{22} which is 5 970 000 000 000 000 000 000 000

This is still a very long number and we have to count the number of zeroes to know the size of the number. But if we use exponential notation, we can write the number in a shorter way which is also easier to read.

The average distance from the earth to the sun is estimated at 150 million km.
We can write 150 million as 150 000 000 km.
Using exponents we can write 150×10^6
We can also write it as $15 \times 10 \times 10^6 = 15 \times 10^7$
We can even write it as $1{,}5 \times 10 \times 10 \times 10^6 = 1{,}5 \times 10^8$ km.
This is called scientific notation (when there is one digit that is not zero before the decimal comma).

Reference: www.universetoday.com

b) *Writing very small numbers using negative exponents*
The mass of a dust particle is estimated at $0{,}000\ 000\ 000\ 753$ kg.
We can write this as a common fraction: $\frac{753}{1\ 000\ 000\ 000\ 000}$
Using exponents, we get: $\frac{753}{10^{12}} = 753 \times 10^{-12}$
In scientific notation the mass of a dust particle is: $7{,}53 \times 10^{-10}$

Reference: https://brainly.com

c) *Using exponents to represent growth*
A species of bacteria doubles every ten minutes. How many bacteria will there be after 60 minutes? How many bacteria will there be after 1 day?

After 0 minutes:	1 bacterium
After 1 period of 10 minutes:	$1 \times 2 = 2$ bacteria (2^1)
After 2 periods of 10 minutes:	$2 \times 2 = 4$ bacteria (2^2)
After 3 periods of 10 minutes:	$2 \times 2 \times 2 = 8$ bacteria (2^3)
After 4 periods of 10 minutes:	$2 \times 2 \times 2 \times 2 = 16$ bacteria … do you see a pattern?

So, after 6 periods of 10 minutes i.e. in 1 hour, there will be $2^6 = 64$ bacteria
After 1 day there will be 24×6 periods of doubling: $2^{24\times6} = 2^{144}$ bacteria (approximately $2{,}23 \times 10^{43}$)

d) Check out this YouTube video on folding paper. They claim you need to fold a piece of paper less than 50 times to cover the distance between the earth and the moon!! https://www.youtube.com/watch?v=AmFMJC45f1Q

7) Like and unlike terms in algebra

In algebra there are two interesting words: *like* and *unlike*. Both words are familiar on social media but their meanings in maths are different to their meanings on social media!!
In maths we use them when we refer to terms. We speak of like terms and unlike terms.

Like terms have the same (i.e. like) variables with the same (i.e. like) exponents for the variables.
Unlike terms have different variables or different exponents even if they have the same variables. Here are some examples:

Like terms	Notes
$2k + 3k$	Same variable k, same exponent 1
$5a - a$	Same variable a, same exponent 1
$3x^2 - x^2$	Same variable x, same exponent 2
$5a^2b + 2ba^2$	Same variables and exponents. Order of letters doesn't matter since multiplication is commutative

Unlike terms	Notes
$5a - 7b$	Two different variables
$3 + 3k$	Term with variable, term with constant
$3x + 7x^2$	Same variable but different exponents
$k^3 - x^3$	Same exponent but different variables
$7k - 7x$	Same coefficient but different variables

8) Operating on like and unlike terms

a) Adding and subtracting terms

We can add and subtract like terms. We cannot add and subtract unlike terms.

Examples of expressions that can be simplified by adding or subtracting because they contain like terms:	Examples of expressions that cannot be simplified by adding or subtracting because there are no like terms:	
$2a + 3a = 5a$ $2a^2 + a^2 = 3a^2$ $5k^3 - 3k^3 = 2k^3$ $a^2b + a^2b = 2a^2b$	$2a + 3b$ $2a - 2b$ $2a - 2$	$a + 4$ $3a^2 - 3a + 3$ $5a - 2ab$

In these materials you will learn about the exponential laws which provide short cuts for multiplying and dividing powers.

b) Multiplying terms

We can multiply like and unlike terms.

Here are 2 examples:

i) $p^3 \times p = (p \times p \times p)(p) = p^4$

ii) $3a^4b^2 \times 4a^2b$

$= 12(a \times a \times a \times a \times a \times a \times b \times b \times b)$

$= 12a^6b^3$

c) Dividing terms

We can divide like and unlike terms.

Here are 2 examples:

i) $p^5 \div p^2 = \frac{p \times p \times p \times \cancel{p} \times \cancel{p}}{\cancel{p} \times \cancel{p}} = p^3$

ii) $a^4b^3 \div a^2b = \frac{\cancel{a} \times \cancel{a} \times a \times a \times \cancel{b} \times b \times b}{\cancel{a} \times \cancel{a} \times \cancel{b}} = a^2b^2$

9) Operating on powers

a) *Adding and subtracting powers with bases that have variables*

In algebra we can only add and subtract powers if they are like terms, i.e. same variables, same exponents

e.g.	$p^2 + 2p^2 = 1p^2 + 2p^2 = 3p^2$	The like terms all have p^2
	$3.c^3 - 2.c^3 + 3d = 3c^3 - 2c^3 + 3d = c^3 + 3d$	The like terms that have c^3 can be added
	$a^2b + 7a^2b = 8a^2b$	The like terms all have a^2b

We cannot simplify the following because they are not like terms:

$a^2 + a$	Same bases but different exponents
$5a^2b - 3ab^2$	Same variables but different exponents
$c^3 + 3d$	Different variables

b) *Adding and subtracting powers when bases and exponents are numbers*

There are <u>two ways</u> to add and subtract powers when bases are numbers:

i) by adding/subtracting like terms

$5^2 + 2.5^2$

$= (1)5^2 + 2.5^2$

$= 3.5^2 = 3.25 = 75$

The like terms contain 5^2 and the first term can be written as $(1)5^2$

$3.4^3 - 2.4^3$

$= \mathbf{1.4^3 = 4^3 = 64}$

Like terms contain 4^3

ii) by simplifying each term first

$5^2 + 2.5^2$

$= 25 + 2 \times 25 = 25 + 50 = 75$

$3.4^3 - 2.4^3 = 3(64) - 2(64)$

$= 192 - 128 = 64$

<u>Notes for representing numeric powers</u>

- If you are asked to give answers in exponential form, write answers like this: $\mathbf{3.5^2}$; $\mathbf{4^3}$
- If you are asked to *simplify* or *evaluate*: $\mathbf{4^3 = 64}$ or $\mathbf{3.5^2 = 75}$
- If the power has a large exponent, e.g. $\mathbf{3^{15}}$, it is not necessary to simplify further.

c) *Multiplying powers*

Law 1: $\boldsymbol{a^m \times a^n = a^{m+n}}$

When multiplying powers *with the same bases*, we add the exponents

Example	Simplify by expanding	Using the law: $\boldsymbol{a^m \times a^n = a^{m+n}}$
$4^2 \times 4^3$	$= (4.4)(4.4.4) = 16 \times 64 = 1\,024$	$= 4^{2+3} = 4^5$ or $1\,024$
$a^2 \times a^3$	$= (a.a)(a.a.a) = a^5$	$= a^{2+3} = a^5$
$3a^2 \times 4a^3$	$= (3.4)(a.a)(a.a.a) = 12a^5$	$= 3 \times 4 \times a^{2+3} = 12a^5$

We can also think about the law "backwards", i.e. $\boldsymbol{a^{m+n} = a^m \times a^n}$

So, from the examples in the table: $4^5 = 4^{2+3} = 4^2 \times 4^3$ and $a^5 = a^{2+3} = a^2 \times a^3$

Of course, we could also "break up" a^5 as: $a^{4+1} = a^4.a^1$

Note

- $a^2 \times b^3 \neq ab^{2+3}$ i.e. $a^2 \times b^3 \neq ab^5$ and $a^2 \times b^3 \neq (ab)^5$
  We can see this by expanding: $a^2 \times b^3 = a.a.b.b.b = a^2b^3$
  but $ab^5 = a.b.b.b.b.b$ and $(ab)^5 = ab \times ab \times ab \times ab \times ab$ (see Law 4)
- When working with numerical bases, we do not multiply the bases themselves, i.e. $4^2 \times 4^3 \neq 16^5$
  We can see this by evaluating each part: $4^2 \times 4^3 = 16 \times 64 = 1024$ but $16^5 = 1\,048\,576$

d) *Dividing powers*

Law 2: $\boldsymbol{a^m \div a^n = a^{m-n}}$

When dividing powers *with the same bases*, we subtract the exponents

Example	Simplify by expanding	Using the law: $\boldsymbol{a^m \div a^n = a^{m-n}}$
$4^5 \div 4^2$	$= \frac{\cancel{4.4}.4.4.4}{\cancel{4.4}} = \frac{4^3}{1} = 4^3$	$= 4^{5-2} = 4^3 = 64$
$\mathrm{a}^6 \div \mathrm{a}^2$	$= \frac{\cancel{a.a}.a.a.a.a}{\cancel{a.a}} = \frac{a^4}{1} = a^4$	$= \mathrm{a}^{6-2} = \mathrm{a}^4$
$\frac{10a^5}{4a^3}$	$= \frac{10.a.a.\cancel{a.a.a}}{4.\cancel{a.a.a}} = \frac{5a^2}{2}$	$= \frac{10a^5}{4a^3} = \frac{5}{2}a^{5-3} = \frac{5a^2}{2}$

We can also think about the law "backwards", i.e. $\boldsymbol{a^{m-n} = a^m \div a^n}$

So, from the examples in the table: $4 = 4^{3-2} = \frac{4^3}{4^2}$ and $a^4 = a^{6-2} = a^6 \div a^2$

Of course, we could also "break up" a^4 as: $a^{5-1} = \frac{a^5}{a}$

Note

- When we apply the law for dividing powers, we do not "cancel" the bases, e.g. $\frac{4^5}{4^2} \neq 1^{5-2}$
  We can show this by expanding all powers. If the bases are numeric, we can expand or evaluate each part: $\frac{4^5}{4^2} = \frac{1024}{16} = 64$ but $1^{5-2} = 1^3 = 1$. So, since 1 is not equal to 64, we conclude that $\frac{4^5}{4^2} \neq 1^{5-2}$.
  Similarly, $\frac{a^6}{a^2} \neq 1^{6-2}$ as we saw from the expansion above: $\frac{\cancel{a.a}.a.a.a.a}{\cancel{a.a}} = a^4$ or $a^{6-2} = a^4$.
- We can use laws 1 and 2 together, for example:
  Simplify $\frac{p^6.p^2.p^8}{p^3}$ using laws 1 and 2 and check using expansion.
  Using the laws: $\frac{p^6.p^2.p^8}{p^3.p} = p^{6+2+8-3-1} = p^{12}$ Using expansion: $\frac{(p.p.p.\cancel{p.p.p})(p.p)(\cancel{p}.p.p.p.p.p.p.p)}{(\cancel{p.p.p}).\cancel{p}} = p^{12}$

A special case that leads to a definition when the exponent is zero

A very important special case arises when we divide powers with the same base and same exponent, e.g. $\frac{a^4}{a^4}$
We can also simplify it using law 2: $\frac{a^4}{a^4} = a^{4-4} = a^0$
We can also simplify it, using expansion and cancelling of like factors: $\frac{a^4}{a^4} = \frac{a \times a \times a \times a}{a \times a \times a \times a} = \frac{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}}{\cancel{a} \times \cancel{a} \times \cancel{a} \times \cancel{a}} = 1$

So now we have 2 different answers to the same problem! The mathematicians who first discovered this concluded that a^0 *must be the same as* 1 and so they created a definition which said that: *any base to the power of zero is equal to 1.* However, they soon realised that there might be problems if the base is zero. So they refined the definition as follows:

Any base (except zero) to the power of zero, is equal to 1
Using mathematical symbols we write: $\boldsymbol{a^0 = 1 \quad (a \neq 0)}$

This means $3^0 = 1$, $(-8)^0 = 1$, $x^0 = 1$

Note

When we defined exponential notation, e.g. p^5, we used repeated multiplication. We said the exponent tells us how many times p is multiplied by itself. But when we have an exponent of zero, e.g. p^0, it does not make sense to say that p is multiplied by itself zero times. We cannot use the idea of repeated multiplication for an exponent of zero. We must just accept the definition that $p^0 = 1$.

What happens when the exponent in the denominator is larger than the exponent in the numerator?

In Grade 8, learners work mainly with examples where the exponent of the numerator is larger than the exponent of the denominator, e.g. $\frac{x^4}{x^3}$ and $\frac{2^8}{2^5}$. Although in these worksheets there are some Grade 8 examples where the exponent of the denominator is larger than the exponent of the numerator, e.g. $\frac{x^3}{x^4}$ and $\frac{2^5}{2^8}$. In such cases, we expand and cancel like factors e.g. $\frac{x^3}{x^4} = \frac{\cancel{x.x.x}}{\cancel{x.x.x}.x} = \frac{1}{x}$.

In Grade 9, you are expected to apply the law for dividing powers.

If we expand and cancel like factors like we did in Grade 8: $\frac{x^3}{x^4} = \frac{x.x.x}{x.x.x.x} = \frac{1}{x}$ and $\frac{2^5}{2^8} = \frac{2.2.2.2.2}{2.2.2.2.2.2.2.2} = \frac{1}{2^3}$
But, if we apply the law for dividing powers with same bases: $\frac{x^3}{x^4} = x^{3-4} = x^{-1}$ and $\frac{2^5}{2^8} = 2^{5-8} = 2^{-3}$

Once again, we have 2 different answers to the same problems! The mathematicians who first discovered this decided to create another new definition which goes like this: $x^{-1} = \frac{1}{x}$ and $2^{-3} = \frac{1}{2^3}$
We can also write it "backwards": $\frac{1}{x} = x^{-1}$ and $\frac{1}{2^3} = 2^{-3}$
In words we say: *Any base with a negative exponent is the same as 1 over that base with a positive exponent.* Of course, the base cannot be zero because $\frac{1}{0}$ is undefined.

Any base (except zero) to the power of negative $\boldsymbol{n}$, *is the same as*
1 over the base to the power of positive $\boldsymbol{n}$.
Using mathematical symbols we write: $\boldsymbol{a^{-n} = \frac{1}{a^n} \quad (a \neq 0)}$

This extends to another similar definition: $\frac{1}{a^{-n}} = a^n$
We start by substituting $a^{-n} = \frac{1}{a^n}$ in the denominator: $\frac{1}{a^{-n}} = \frac{1}{\frac{1}{a^n}} = 1 \div \frac{1}{a^n} = 1 \times \frac{a^n}{1} = a^n$

Question	Simplify by expanding	Using the definition	
$\frac{3^4}{3^6}$	$=\frac{3.3.3.3}{3.3.3.3.3.3}=\frac{1}{3.3}=\frac{1}{3^2}=\frac{1}{9}$	$=3^4\times\frac{1}{3^6}=3^4.3^{-6}=3^{4-6}=3^{-2}=\frac{1}{9}$	We applied: $\frac{1}{a^m}=a^{-m}$ and $a^{-m}=\frac{1}{a^m}$
$\frac{a^3}{a^5}$	$=\frac{\cancel{a.a.a}}{a.a.\cancel{a.a.a}}=\frac{1}{a^2}$	$=a^3\times\frac{1}{a^5}=a^3\times a^{-5}=a^{3-5}=a^{-2}=\frac{1}{a^{\,2}}$	
$a^2\times ab^{-4}$	$=a^2\times\frac{a}{b^4}=\frac{a.a.a}{b.b.b.b}=\frac{a^3}{b^4}$	$=a^{2+1}.\frac{1}{b^4}=\frac{a^3}{b^4}$	
$a^2.a^{-5}$	$=\frac{a^2}{a^5}=\frac{\cancel{a.a}}{\cancel{a.a}.a.a.a}=\frac{1}{a^3}$	$=a^{2-5}=a^{-3}$	
$(-5)^{-2}$	$=\frac{1}{(-5)^2}=\frac{1}{(-5)(-5)}=\frac{1}{25}$	$=\frac{1}{(-5)^2}=\frac{1}{25}$	$m^{-n}=\frac{1}{m^n}$

e) *Raising a power to a further power*

Law 3: $(\boldsymbol{a^m})^{\boldsymbol{n}}=\boldsymbol{a^{m\times n}}$

When raising a power to a further power, we multiply the exponents

Example	Simplify by expanding and applying law 1	Using the law: $(\boldsymbol{a^m})^{\boldsymbol{n}}=\boldsymbol{a^{m\times n}}$
$(3^2)^4$	$=3^2\times3^2\times3^2\times3^2=3^{2+2+2+2}=3^8=6\,561$ or $3.3.3.3.3.3.3.3=3^8=6\,561$	$=(3^2)^4=3^{2\times4}=3^8$ or $6\,561$
$(a^3)^2$	$=a^3.a^3=a^{3+3}=a^6$ or $(a.a.a).(a.a.a)=a^6$	$=a^{3\times2}=a^6$
$(4x^3)^3$	$=4x^3.4x^3.4x^3=64x^{3+3+3}=64x^9$ or $(4.x.x.x).(4.x.x.x).(4.x.x.x)=64x^9$	$=4^3.x^{3\times3}=64x^9$
$4(x^3)^3$	$=4(x^3)^3=4.x^3.x^3.x^3=4x^{3+3+3}=4x^9$ or $4(x.x.x).(x.x.x).(x.x.x)=4x^9$	$=4.x^{3\times3}=4x^9$

We can also think about the law "backwards", i.e. $\boldsymbol{a^{m\times n}}=(\boldsymbol{a^m})^{\boldsymbol{n}}$

So, from the examples in the table:
$3^8=3^{2\times4}=(3^2)^4$ or $3^8=3^{4\times2}=(3^4)^2$
and $a^6=a^{2\times3}=(a^2)^3$ or $a^6=a^{3\times2}=(a^3)^2$

Note

The law for raising a power to a further power is a short-cut. We can show why it works by:

- expanding the "outer" power and then applying law 1. e.g. $(a^3)^2=a^3.a^3=a^{3+3}$ OR
- we can expand the "inner" power and then count the number of times the base is multiplied by itself. e.g. $(a.a.a).(a.a.a)=a^6$

f) *Working with exponents when the base contains a product*

Law 4: $(a\times t)^n=a^nt^n$

When the base of a power consists of a product, we apply the exponent to each factor of the product.

Example	Simplify by expanding	Using the law: $(\boldsymbol{a}\times\boldsymbol{t})^{\boldsymbol{n}}=\boldsymbol{a^nt^n}$
$(5\times3)^2$	$=(5\times3)\times(5\times3)=15\times15=225$ or $=5.5\times3.3=25\times9=225$	$=5^2\times3^2=25\times9=225$
$(m\times n)^4$	$=(m\times n)\times(m\times n)\times(m\times n)\times(m\times n)$ $=m.m.m.m\times n.n.n.n=m^4n^4$	$=m^4.n^4$
$(5a)^3$	$=(5a)(5a)(5a)=5.5.5\times a.a.a=125a^3$	$=5^3.a^3=125a^3$

We can also think about the law "backwards", i.e. $a^nt^n=(a\times t)^n$

So, from the examples in the table: $5^2.3^2=(5.3)^2$ and $m^4.n^4=(mn)^4$

Note

- Consider the example: $(4ab)^3$
  The base consists of a product of factors. $4ab$ is the base and its factors are: $4, a$ and b. The exponent 3 can be applied to each factor so we get $4^3a^3b^3$.
- This law only applies when the base is a product. It does not apply if the base consists of a sum (or difference), e.g. $(4+3)^2 \neq 4^2+3^2$. In a similar way $(a-b)^2 \neq a^2-b^2$
- We apply law 4 when we multiply powers with <u>different</u> bases but the <u>same</u> exponents. Note that we do not add exponents, e.g. $a^3b^3 \neq (ab)^6$
- We must use brackets to indicate that the exponent applies to all factors in the base, , e.g. $x^4y^4=(xy)^4$
  We cannot write the answer as xy^4 because that represents $x.y.y.y.y$, not $x.x.x.x \times y.y.y.y$

g) *Working with exponents when the base is a fraction*

Law 5: $\boldsymbol{\left(\frac{a}{t}\right)^n = \frac{a^n}{t^n}}$

When the base of a power is a fraction, we apply the exponent to the numerator and the denominator.

Example	**Simplify by expanding**	**Using the law: $\left(\frac{a}{t}\right)^n = \frac{a^n}{t^n}$**
$\left(\frac{5}{3}\right)^2$	$=\left(\frac{5}{3}\right)\left(\frac{5}{3}\right)=\frac{5.5}{3.3}=\frac{5^2}{3^2}=\frac{25}{9}$	$=\frac{5^2}{3^2}=\frac{25}{9}$
$\left(\frac{k^3}{m}\right)^2$	$=\left(\frac{k^3}{m}\right)\left(\frac{k^3}{m}\right)=\frac{k^3.k^3}{m^2}=\frac{(k.k.k)(k.k.k)}{m.m}=\frac{k^6}{m^2}$	$=\frac{k^{3\times2}}{m^2}=\frac{k^6}{m^2}$
$\left(\frac{5a}{b}\right)^3$	$=\left(\frac{5a}{b}\right)\left(\frac{5a}{b}\right)\left(\frac{5a}{b}\right)=\frac{5.5.5\times a.a.a}{b.b.b}=\frac{125a^3}{b^3}$	$=\frac{(5a)^3}{b^3}$ or $\frac{125a^3}{b^3}$

We can also think about the law "backwards", i.e. $\boldsymbol{\frac{a^n}{t^n}=\left(\frac{a}{t}\right)^n}$

So, from the examples in the table: $\frac{5^2}{3^2}=\left(\frac{5}{3}\right)^2$ and $\frac{k^6}{m^2}=\left(\frac{k^3}{m}\right)^2$

Note

- Consider the example: $\left(\frac{4a}{b}\right)^3$
  It consists of a base which contains a fraction. Also, the numerator contains a product of factors. The base is $\frac{4a}{b}$. The factors of the numerator are 4 and a. The exponent 3 can be applied to each factor in the numerator and to the denominator, so we get $\frac{4^3a^3}{b^3}$.
- When we apply the law backwards, we create a single power where the base is a fraction.

h) *Examples of all laws with negative exponents*

Law	**Numeric examples**	**Algebraic examples**
1	$4^2\times4^{-3}=4^{2-3}=4^{-1}=\frac{1}{4}$	$a^2\times a^{-5}=a^{2-5}=a^{-3}=\frac{1}{a^3}$
2	$4^{-5}\div4^2=\frac{4^{-5}}{4^2}=4^{-5-2}=4^{-7}=\frac{1}{4^7}$	$a^6\div a^{-2}=\frac{a^6}{a^{-2}}=a^{6+2}=a^8$
3	$(3^{-2})^4=3^{-8}=\frac{1}{3^8}$ $5(3^{-2})^4=5.3^{-8}=\frac{5}{3^8}$	$(a^3)^{-2}=a^{3\times(-2)}=a^{-6}=\frac{1}{a^6}$ $5(a^3)^{-2}=5a^{3\times(-2)}=5a^{-6}=\frac{5}{a^6}$
4	$(5\times3)^{-2}=5^{-2}.3^{-2}=\frac{1}{5^23^2}$ or $(5\times3)^{-2}=(15)^{-2}=\frac{1}{15^2}$	$(m\times n)^{-4}=m^{-4}.n^{-4}=\frac{1}{m^4n^4}$
5	$\left(\frac{5}{3}\right)^{-2}=\frac{5^{-2}}{3^{-2}}=5^{-2}.\frac{1}{3^{-2}}=\frac{1}{5^2}.3^2=\frac{3^2}{5^2}=\frac{9}{25}$	$\left(\frac{5a}{b}\right)^{-3}=\frac{(5a)^{-3}}{b^{-3}}=\frac{b^3}{5^3a^3}$

10) What makes exponents and powers confusing?

There are several aspects that make exponents and powers confusing. In this final section we focus on 5 issues: language, notation, definitions, application of the laws and impact of negative numbers.

a) *Language*

We use the word *power* in different ways when referring to exponents. Strictly speaking, *power* refers to the base <u>and</u> the exponent, e.g. 3^4 or x^5. But we sometimes say "three to the power of 4" when 4 is actually the *exponent* and the power refers to 3^4. Similarly, we say "x to the power of 5" when 5 is the exponent.

b) *Notation*

i) Choice of letters

The choice of letters in the laws and definitions may be confusing. Consider: $a^m.a^n = a^{m+n}$ and $m^t.n^t = (mn)^t$. Here m and n are defined as exponents in the first example but they are bases in the second example. It is important to *pay attention to what the letters represent* rather than focusing on the position of the letters. We can choose any letters for the base and the exponent.

ii) Dealing with negative bases

Note that -2^5 means $(-1)(2^5)$. If we want to represent "negative two raised to the exponent 5", then we must use brackets as follows: $(-2)^5$

iii) Making sense of the minus symbol

Exponential notation can be confusing, particularly with the minus sign. When we see -7 we treat the minus symbol as the *sign* of 7. When we see the algebraic expression, $x - 7$, we treat the minus symbol as *subtraction*. But, in exponents, the minus symbol has a slightly different meaning. When we see 2^{-3} the minus symbol does not influence the sign of the number. It tells us to apply the definition for negative powers: $2^{-3} = \frac{1}{2^3} = 8.$ When we see examples with negative bases and negative exponents, e.g. $(-2)^{-3}$, it can be tempting to think we are multiplying two negative numbers but this is not the case. Once again, we apply the definition for negative powers: $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{(-2)(-2)(-2)} = -\frac{1}{8}$.

However, there are indeed times when we treat the negative exponent as a negative number and/or as subtraction: e.g. $m^5.m^{-3} = m^{5+(-3)} = m^2$ and $(m^{-2})^5 = m^{-2\times 5} = m^{-10}$

c) *Definitions*

The definitions can be confusing. When you first learn exponential notation, you are taught that x^4 means $x.x.x.x$, i.e. x multiplied by itself 4 times. This is repeated multiplication. But when we define $x^0 = 1$, we must ignore the idea of repeated multiplication – we do not say that x is multiplied by itself zero times. Similarly, when we define $x^{-5} = \frac{1}{x^5}$, we do not say that x is multiplied by itself negative 5 times. Again, we must ignore the idea of repeated multiplication and just accept the mathematical definition.

d) *Laws*

Teachers often say that learners use the exponential laws incorrectly. Often this happens because we don't pay attention to all aspects of the law.

i) Law 1 is often applied incorrectly because we say: "when you multiply bases, you add exponents". This can lead to $a^3.b^2 = ab^5$ which is incorrect because law 1 only applies to powers with the <u>same</u> base.

ii) Law 4 might cause confusion about same bases because of examples like $a^3.b^3 = (ab)^3$. Here we write the bases as a product but <u>we do not add the exponents</u>.

iii) Often we confuse law 1 and law 3: $a^4 \times a^3 = a^{4+3} = a^7$ but $(a^4)^3 = a^{4\times 3} = a^{12}$. The difference can be seen clearly if we expand each factor and focus on the repeated multiplication.

iv) Numeric bases cause additional problems: We say: "when you multiply powers with the same bases …" e.g. $2^3 \times 2^6$, but we don't actually multiply the bases to get the answer. The answer is 2^9, not 4^9.

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Definition of a^0: Worksheet 1

In this worksheet we help you to make sense of the mathematical definition: $\boldsymbol{a^0 = 1}$
We can say "any base to the power of 0 is equal to 1". e.g. $3^0 = 1$, $50^0 = 1$, $b^0 = 1$ etc.
There is one restriction on this definition: the base may not equal zero.
So the full definition is: "any base (except 0) to the power of 0 is equal to 1"
In the notes we explained why this is a definition and not a law.

You will work through 4 sets of examples which will help you to see why we can conclude that "any base (except 0) to the exponent of 0 is equal to 1".

Questions

1) This pattern has the rule "divide the question and answer by 2 at every step".
e.g. $2^4 \div 2 = 2^3$ and $16 \div 2 = 8$, so $2^3 = 8$.
Fill in the blanks.

$2^4 = 16$	$2^3 = 8$	$2^2 = \square$	$\square = 2$	$2^0 = \square$

2) Look at the four questions below. Each question has a numerator of 3^4.

i) $\frac{3^4}{3^1}$ ii) $\frac{3^4}{3^2}$ iii) $\frac{3^4}{3^3}$ iv) $\frac{3^4}{3^4}$

a) What do you notice about the powers in each denominator?
b) Simplify each question by expanding and cancelling.
c) Simplify each question using the law for dividing exponents: $a^m \div a^n = a^{m-n}$.

3) Look at the four questions below:

i) $\frac{p^6}{p^3}$ ii) $\frac{p^6}{p^4}$ iii) $\frac{p^6}{p^5}$ iv) $\frac{p^6}{p^6}$

a) What is the same in each question?
b) What is different in each question?
c) Simplify each question by expanding and cancelling
d) Simplify each question using the law for dividing exponents: $a^m \div a^n = a^{m-n}$.
e) Use your answers to Q3c and Q3d to write the answer to Q3iv) in two ways.

4) Look at the whole of Q2 and the whole of Q3:

a) What is the same about them?
b) What is different?
c) Repeat Q3. Start with $\frac{x^7}{x^2}$ then $\frac{x^7}{x^3}$ and so on, to create the same pattern as in Q3.

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Definition of a^0: Worksheet 1

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1) This pattern has the rule “divide the question and answer by 2 at every step”.
e.g. $2^4 \div 2 = 2^3$ and $16 \div 2 = 8$, so $2^3 = 8$. Fill in the blanks.
<table><tr><td>$2^4 = 16$</td><td>$2^3 = 8$</td><td>$2^2 = \square$</td><td>$\square^{\square} = 2$</td><td>$2^0 = \square$</td></tr></table></td><td>1)
$2^2 = \mathbf{4}$; $\mathbf{2^1} = 2$; $2^0 = \mathbf{1}$</td></tr>
<tr><td>2) Look at the four questions below. Each question has a numerator of 3^4.
i) $\frac{3^4}{3^1}$ ii) $\frac{3^4}{3^2}$ iii) $\frac{3^4}{3^3}$ iv) $\frac{3^4}{3^4}$
Questions and answers to a) and d)
a) What do you notice about the powers in each denominator?
<u>Answer</u>: Powers in the denominators increase by 3^1 each time.
d) Use your answers to Q2b and Q2c to write the answer to Q2iv) in two ways.
<u>Answer</u>: $\frac{3^4}{3^4} = 1$ or $\frac{3^4}{3^4} = 3^0$</td><td>2) Questions and answers to b) and c)
b) Simplify each question by expanding and cancelling. <u>Answer</u>:
i) $\frac{\cancel{3}.3.3.3}{\cancel{3}} = \frac{3.3.3}{1} = 3^3$ ii) $\frac{\cancel{3}.\cancel{3}.3.3}{\cancel{3}.\cancel{3}} = \frac{3.3}{1} = 3^2$
iii) $\frac{\cancel{3}.\cancel{3}.\cancel{3}.3}{\cancel{3}.\cancel{3}.\cancel{3}} = \frac{3}{1} = 3^1$ iv) $\frac{\cancel{3}.\cancel{3}.\cancel{3}.\cancel{3}}{\cancel{3}.\cancel{3}.\cancel{3}.\cancel{3}} = \frac{1}{1} = 1$
c) Simplify each question using the law for dividing exponents:$a^m \div a^n = a^{m-n}$.
<u>Answer</u>:
i) $3^{4-1} = 3^3$ ii) $3^{4-2} = 3^2$ iii) $3^{4-3} = 3^1$ iv) $3^{4-4} = 3^0$</td></tr>
<tr><td>3) Look at the four questions below:
i) $\frac{p^6}{p^3}$ ii) $\frac{p^6}{p^4}$ iii) $\frac{p^6}{p^5}$ iv) $\frac{p^6}{p^6}$
Questions and answers to a), b) and e)
a) What is the same in each question?
<u>Answer</u>: All are powers of p, and all have a numerator of p^6.
b) What is different in each question?
<u>Answer</u>: The exponents in the denominators increase by 1 from 3 to 6.
e) Use your answers to Q3c and Q3d to write the answer to Q3iv) in two ways.
<u>Answer</u>: $\frac{p^6}{p^6} = 1$ or $\frac{p^6}{p^6} = p^0$</td><td>3) Questions and answers to c) and d)
c) Simplify each question by expanding and cancelling. <u>Answer</u>:
i) $\frac{\cancel{p}.\cancel{p}.\cancel{p}.p.p.p}{\cancel{p}.\cancel{p}.\cancel{p}} = \frac{p.p.p}{1} = p^3$ ii) $\frac{\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}.p.p}{\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}} = \frac{p.p}{1} = p^2$
iii) $\frac{\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}.p}{\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}} = p^1$ iv) $\frac{\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}}{\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}.\cancel{p}} = \frac{1}{1} = 1$
d) Simplify each question using the law for dividing exponents: $a^m \div a^n = a^{m-n}$.
<u>Answer</u>:
i) $p^{6-3} = p^3$ ii) $p^{6-4} = p^2$ iii) $p^{6-5} = p^1$ iv) $p^{6-6} = p^0$</td></tr>
<tr><td>4) Look at the whole of Q2 and the whole of Q3:
a) What is the same about them?
b) What is different?
c) Repeat Q3. Start with $\frac{x^7}{x^2}$ then $\frac{x^7}{x^3}$ and so on, to create the same pattern as in Q3.</td><td>4)
a) Constant numerator; in denominators, exponents increase by 1 each time.
b) Q2 has number bases, Q3 has variable bases.
c) Final pattern: $\frac{x^7}{x^2} = x^5; \frac{x^7}{x^3} = x^4; \frac{x^7}{x^4} = x^3; \frac{x^7}{x^5} = x^2; \frac{x^7}{x^6} = x^1; \frac{x^7}{x^7} = 1 = x^0$</td></tr>
</table>

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Definition of a^0: Worksheet 2

In Worksheet 1 we discovered that if a power is the same in the numerator and denominator e.g. $\frac{2^4}{2^4}$; $\frac{p^6}{p^6}$, then:

- the result is 1. e.g. $\frac{2^4}{2^4} = 1; \frac{p^6}{p^6} = 1$ and
- the result can also be written as: the base to the power of 0 (provided the base is not 0)
  e.g. $\frac{2^4}{2^4} = 2^{4-4} = 2^0$; $\frac{p^6}{p^6} = p^{6-6} = p^0$

So we can say $1 = a^0$ or $a^0 = 1$ provided that $a \neq 0$.

This worksheet provides more practice to help you understand even better how to use this definition in questions dealing with exponents and powers.

Questions

1) Fill in the blanks to show that $m^0 = 1$ and $3^0 = 1$:
   a) $\frac{m^8}{m^{\square}} = m^{\square-8} = m^{\square}$ and $\frac{m^8}{m^{\square}} = 1$ so $m^{\square} = 1$
   b) $\frac{3^5}{\square} = \square^0$ and $\frac{3^5}{\square} = 1$ so $1 = \square^0$

2) Use the definition of $a^0 = 1$ (i.e. any number to the power of zero equals 1) to simplify the following:
   a) 7^0 b) $(5)^0$ c) $(-5)^0$ d) y^0 e) $x.y^0$ f) $p^4.p^0$

3) Use the law for dividing powers, $a^m \div a^n = a^{m-n}$ and the definition of $a^0 = 1$ to say whether these statements are TRUE or FALSE. If FALSE, correct the part on the right of the equals sign.
   a) $\frac{k^9}{k^9} = k$ b) $\frac{k^9}{k^9} = k^0$ c) $\frac{k^9}{k^9} = 0$
   d) $\frac{k^9}{k^9} = 1$ e) $\frac{5^2}{5^2} = 1^0$ f) $\frac{0}{5^0} = 1$

4) Simplify the following. **Note:** The simplest form of a^0 is 1.
   a) $\frac{5^8}{5^8}$ b) $\frac{5^8}{5^2 \times 5^6}$ c) $\frac{2^5.2^3.2^4}{2^{12}}$
   d) $\frac{2^{12}}{2^2.2^{10}}$ e) $\frac{x^5.y^0}{x^5}$ f) $\frac{3x^5.y^0}{x^0}$

5) Simplify.
   a) $\frac{w^0x^2.x^5}{x^7}$ b) $\frac{n^{0+3}}{n^3}$ c) $\frac{k.k^{-1}.m^{16}}{16.m^{16}}$
   d) $\frac{3^3.5^0}{3^3.5^{2+2}}$ e) $x^0 \div x^{12}$ f) $5.a^0 + 5.a^0$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Definition of a^0: Worksheet 2

Answers

Questions	**Answers**
1) Fill in the blanks to show that $m^0 = 1$ and $3^0 = 1$: a) $\frac{m^8}{m^\square} = m^{\square-8} = m^\square$ and $\frac{m^8}{m^\square} = 1$ so $m^\square = 1$ b) $\frac{3^5}{\square} = \square^0$ and $\frac{3^5}{\square} = 1$ so $1 = \square^0$	1) a) $\frac{m^8}{m^{\mathbf{8}}} = m^{\mathbf{8}-8} = m^{\mathbf{0}}$ and $\frac{m^8}{m^{\mathbf{8}}} = 1$ so $m^{\mathbf{0}} = 1$ b) $\frac{3^5}{\mathbf{3^5}} = \mathbf{3}^0$ and $\frac{3^5}{\mathbf{3^5}} = 1$ so $1 = \mathbf{3}^0$
2) Use the definition of $a^0 = 1$ (i.e. any number to the power of zero equals 1) to simplify the following: a) 7^0 b) $(5)^0$ c) $(-5)^0$ d) y^0 e) $x.y^0$ f) $p^4.p^0$	2) a) 1 b) 1 c) 1 d) 1 e) x f) p
3) Use the law for dividing powers, $a^m \div a^n = a^{m-n}$ and the definition of $a^0 = 1$ to say whether these statements are TRUE or FALSE. If FALSE, correct the part on the right of the equals sign. a) $\frac{k^9}{k^9} = k$ b) $\frac{k^9}{k^9} = k^0$ c) $\frac{k^9}{k^9} = 0$ d) $\frac{k^9}{k^9} = 1$ e) $\frac{5^2}{5^2} = 1^0$ f) $\frac{0}{5^0} = 1$	3) a) FALSE, k^0 b) TRUE c) FALSE, k^0 or 1 d) TRUE e) FALSE, 5^0 f) FALSE, 0
4) Simplify the following. **Note:** The simplest form of a^0 is 1. a) $\frac{5^8}{5^8}$ b) $\frac{5^8}{5^2 \times 5^6}$ c) $\frac{2^5.2^3.2^4}{2^{12}}$ d) $\frac{2^{12}}{2^2.2^{10}}$ e) $\frac{x^5.y^0}{x^5}$ f) $\frac{3x^5.y^0}{x^0}$	4) a) $5^0 = 1$ b) $5^0 = 1$ c) $12^0 = 1$ d) $12^0 = 1$ e) $x^0.y^0 = 1$ f) $3x^5$
5) Simplify. a) $\frac{w^0x^2.x^5}{x^7}$ b) $\frac{n^{0+3}}{n^3}$ c) $\frac{k.k^{-1}.m^{16}}{16.m^{16}}$ d) $\frac{3^3.5^0}{3^3.5^{2+2}}$ e) $x^0 \div x^{12}$ f) $5.a^0 + 5.a^0$	5) a) 1 b) 1 c) $\frac{k^0.m^0}{16} = \frac{1}{16}$ d) $\frac{1}{5^4}$ e) $\frac{1}{x^{12}}$ f) 10

Definition of a^{-n}: Worksheet 1

In this worksheet we will help you make sense of the mathematical definition: $\boldsymbol{a^{-n} = \frac{1}{a^n}}$

We will also look at this version: $\boldsymbol{\frac{1}{a^{-n}} = a^n}$.

You will work through 3 sets of examples which will help you to see why we can conclude that "a to the power of negative n is the same as 1 divided by a to the positive n" and "1 divided by a to the negative n is the same as a to the power of positive n". Yes, it's quite difficult to write it in words but much easier to use symbols! Remember that we can use any letters to show this definition, e.g. $m^{-t} = \frac{1}{m^t}$ or $x^{-u} = \frac{1}{x^u}$ or $\frac{1}{p^{-a}} = p^a$

Questions

1) Look at the four questions below, each question has a numerator of 5^3:

   i) $\frac{5^3}{5^2}$ ii) $\frac{5^3}{5^3}$ iii) $\frac{5^3}{5^4}$ iv) $\frac{5^3}{5^5}$

   a) Focus on the denominators for each question. What do you notice?
   b) Simplify each question by expanding and cancelling.
   c) Simplify each question using the rule for diving powers: $a^m \div a^n = a^{m-n}$.
   d) Use your answers to Q1b and Q1c to write the answers to Q1iii) and Q1iv) in two ways.
   e) Predict the answer to $\frac{5^6}{5^9}$.

2) Look at the four questions below:

   i) $\frac{p^6}{p^6}$ ii) $\frac{p^6}{p^7}$ iii) $\frac{p^6}{p^8}$ iv) $\frac{p^6}{p^9}$

   a) What is the same in each question?
   b) What is different in each question?
   c) Simplify each question by expanding and cancelling.
   d) Simplify each question using the rule for dividing powers: $a^m \div a^n = a^{m-n}$.
   e) Use your answers to Q2c and Q2d to write the answers to Q2i) to Q2iv) in two ways.

3)
   a) Use the law $a^m \times a^n = a^{m+n}$ to simplify $3^2 \times 3^4$.
   b) Use the law $a^m \div a^n = a^{m-n}$ to simplify $3^2 \div 3^4$.
   c) Work out the value of $3^2 \div 3^4$ using the values for 3^2 and 3^4.
   d) Use your answer to Q3b to write $\frac{1}{9}$ in another way.
   e) Can you show that $\frac{1}{a^{-n}} = a^n$?

      Here's a hint: write $1 \div a^{-n} = 1 \div \frac{1}{\square}$ …

Definition of a^{-n} Worksheet 1

Answers

Questions	Answers
1) Look at the four questions below, each question has a numerator of 5^4: i) $\frac{5^3}{5^2}$ ii) $\frac{5^3}{5^3}$ iii) $\frac{5^3}{5^4}$ iv) $\frac{5^3}{5^5}$ **Questions and answers to a), d)and e)** a) Focus on the denominators for each question. What do you notice? *Answer*: The exponents in the denominators increase by 1 from 2 to 5. d) Use your answers to Q1b and Q1c to write the answers to Q1iii) and Q1iv) in two ways. *Answer:* Q1iii) $\frac{5^3}{5^4} = \frac{1}{5}$ or 5^{-1} Q1iv) $\frac{5^3}{5^5} = \frac{1}{5^2}$ or 5^{-2} e) Predict the answer to $\frac{5^6}{5^9}$. *Answer*: $\frac{5^6}{5^9} = \frac{1}{5^3}$ or 5^{-3}	1) **Questions and answers to b) and c)** b) Simplify each question by expanding and cancelling. *Answer*: i) $\frac{\cancel{5.5.5}}{\cancel{5.5}} = \frac{5}{1} = 5^1$ ii) $\frac{\cancel{5.5.5}}{\cancel{5.5.5}} = 1$ iii) $\frac{\cancel{5.5.5}}{\cancel{5.5.5}.5} = \frac{1}{5}$ iv) $\frac{\cancel{5.5.5}}{\cancel{5.5.5}.5.5} = \frac{1}{5^2}$ c) Simplify each question using the rule for diving powers: $a^m \div a^n = a^{m-n}$. *Answer*: i) $5^{3-2} = 5^1$ ii) $5^{3-3} = 5^0$ iii) $5^{3-4} = 5^{-1}$ iv) $5^{3-5} = 5^{-2}$
2) Look at the four questions below: i) $\frac{p^6}{p^6}$ ii) $\frac{p^6}{p^7}$ iii) $\frac{p^6}{p^8}$ iv) $\frac{p^6}{p^9}$ **Questions and answers to a), b) and e)** a) What is the same in each question? *Answer*: Each question involves powers of p,and has a numerator of p^6. b) What is different in each question? *Answer*: The exponents in the denominators increase by 1 from 6 to 9. e) Use your answers to Q2c and Q2d to write the answers to Q2i) to Q2iv) in two ways. *Answer*: i) $1 = p^0$ ii) $\frac{1}{p} = p^{-1}$ iii) $\frac{1}{p^2} = p^{-2}$ iv) $\frac{1}{p^3} = p^{-3}$	2) **Questions and answers to c) and d)** c) Simplify each question by expanding and cancelling. *Answer*: i) $\frac{\cancel{p.p.p.p.p.p}}{\cancel{p.p.p.p.p.p}} = \frac{1}{1} = 1$ ii) $\frac{\cancel{p.p.p.p.p.p}}{\cancel{p.p.p.p.p.p}.p} = \frac{1}{p}$ iii) $\frac{\cancel{p.p.p.p.p.p}}{\cancel{p.p.p.p.p.p}.p.p} = \frac{1}{p^2}$ iv) $\frac{\cancel{p.p.p.p.p.p}}{\cancel{p.p.p.p.p.p}.p.p.p} = \frac{1}{p^3}$ d) Simplify each question using the rule for dividing powers: $a^m \div a^n = a^{m-n}$. *Answer*: i) $p^{6-6} = p^0$ ii) $p^{6-7} = p^{-1}$ iii) $p^{6-8} = p^{-2}$ iv) $p^{6-9} = p^{-3}$
3) a) Use the law $a^m \times a^n = a^{m+n}$ to simplify $3^2 \times 3^4$. b) Use the law $a^m \div a^n = a^{m-n}$ to simplify $3^2 \div 3^4$. c) Work out the value of $3^2 \div 3^4$ using the values for 3^2 and 3^4. d) Use your answer to Q3b to write $\frac{1}{9}$ in another way. e) Can you show that $\frac{1}{a^{-n}} = a^n$? Here's a hint: write $1 \div a^{-n} = 1 \div \frac{1}{\square}$ ...	3) **Answers** a) $3^{2+4} = 3^6$ b) $3^{2-4} = 3^{-2}$ c) $3^2 \div 3^4 = 9 \div 81 = \frac{1}{9}$ d) $\frac{1}{9} = 3^{-2}$ e) $1 \div a^{-n} = 1 \div \frac{1}{a^n}$ $= 1 \times \frac{a^n}{1} = a^n$

Definition of a^{-n}: Worksheet 2

In Worksheet 1 you saw where the definitions: $\boldsymbol{a^{-n}} = \frac{\boldsymbol{1}}{\boldsymbol{a^n}}$ and $\frac{\boldsymbol{1}}{\boldsymbol{a^{-n}}} = \boldsymbol{a^n}$ come from.

This worksheet focuses on using $\boldsymbol{a^{-n}} = \frac{\boldsymbol{1}}{\boldsymbol{a^n}}$ and $\frac{\boldsymbol{1}}{\boldsymbol{a^{-n}}} = \boldsymbol{a^n}$ together with the laws of exponents and the definition for a^0.

Questions

1) Rewrite with positive exponents:

a) 7^{-1} b) 4^{-9} c) y^{-7} d) $\frac{1}{x^{-4}}$ e) $\frac{3}{z^{-5}}$

2) Use the laws for multiplying and dividing powers and the definitions to say whether these statements are TRUE or FALSE. If FALSE, correct the part on the right of the equals sign.

a) $m^{-9}.m^{9} = m$ b) $\frac{m^{-9}}{m^{9}} = m^0$ c) $\frac{m^{-9}}{m^{-2}.m^{-7}} = m^0$ d) $m^{-9}.m^{9} = 1$

3) Simplify. Write answers with positive exponents.

a) $m^{-5}.m^{-12}.m^{14}$ b) $x^{-8}y^{0} \times x^{6}y^{-6}$

c) $a^7b^{-5}c^{-1}.ab^5.abc^{-4}$ d) $p^4 \times \frac{2}{p^{-3}}$

4) Write power with positive exponents.

a) $(6^{-2})^3$ b) $(3^{-3})^2$ c) $(d^{-5})^3$ d) $\frac{1}{(d^{-2})^3}$

5) Group the powers that give the same answer.

a) 4^{-5} b) $(4^{-2})^3$ c) $4^{-2} \times 4^{-3}$

d) $(m^{-4})^2$ e) $(m^6)^{-1}$ f) $\left(\frac{1}{m}\right)^6$

6) Negative bases and negative coefficients. Fill in the blanks.

a) $(-2)^2 = (-2).\square = \square^{\square}$ b) $-(-2)^2 = (-1)(-2).\square = \square^{\square}$

c) $(-2)^3 = (-2).\square.\square = \square^{\square}$ d) $-(-2)^3 = (-1)(-2).\square.\square = \square^{\square}$

7) Simplify. Write answers with positive exponents.

a) $-\left(\frac{3}{5}\right)^{-2}$ b) $\left(\frac{-3}{5}\right)^{-2}$ c) $-\left(\frac{-3}{5^{-4}}\right)^{-2}$

d) $(a^6)^{-3} \times -(a^{-3})^6$ e) $5(b)^{-2} \times (-2b)^{-2}$

Definition of a^{-n} Worksheet 2

Answers

Questions	**Answers**
1) Rewrite with positive exponents: a) 7^{-1} b) 4^{-9} c) y^{-7} d) $\frac{1}{x^{-4}}$ e) $\frac{3}{z^{-5}}$	1) a) $\frac{1}{7}$ b) $\frac{1}{4^9}$ c) $\frac{1}{y^7}$ d) x^4 e) $3z^5$
2) Use the laws for multiplying and dividing powers and the definitions to say whether these statements are TRUE or FALSE. If FALSE, correct the part on the right of the equals sign. a) $m^{-9}.m^9 = m$ b) $\frac{m^{-9}}{m^9} = m^0$ c) $\frac{m^{-9}}{m^{-2}\cdot m^{-7}} = 0$ d) $m^{-9}.m^9 = 1$	2) a) FALSE, m^0 or 1 b) FALSE, $\frac{1}{m^{18}}$ c) FALSE, m^0 or 1 d) TRUE
3) Simplify. Write answers with positive exponents. a) $m^{-5}.m^{-12}.m^{14}$ b) $x^{-8}y^0 \times x^6y^{-6}$ c) $a^7b^{-5}c^{-1}.ab^5.abc^{-4}$ d) $p^4 \times \frac{2}{p^{-3}}$	3) a) $\frac{1}{m^3}$ b) $\frac{1}{x^2y^6}$ c) $\frac{a^9b}{c^5}$ d) $2p^7$
4) Write powers with positive exponents a) $(6^{-2})^3$ b) $(3^{-3})^2$ c) $(d^{-5})^3$ d) $\frac{1}{(d^{-2})^3}$	4) a) $\frac{1}{6^6}$ b) $\frac{1}{3^6}$ c) $\frac{1}{d^{15}}$ d) $\frac{1}{d^{-6}} = d^6$
5) Group the powers that give the same answer. a) 4^{-5} b) $(4^{-2})^3$ c) $4^{-2} \times 4^{-3}$ d) $(m^{-4})^2$ e) $(m^6)^{-1}$ f) $\left(\frac{1}{m}\right)^6$	5) a) and c): 4^{-5}; e) and f): $\frac{1}{m^6}$
6) Negative bases and negative coefficients. Fill in the blanks. a) $(-2)^2 = (-2).\square = \square^{\square}$ b) $-(-2)^2 = (-1)(-2).\square = \square^{\square}$ c) $(-2)^3 = (-2).\square.\square = \square^{\square}$ d) $-(-2)^3 = (-1)(-2).\square.\square = \square^{\square}$	6) a) $(-2)^2$ $= (-2).(\mathbf{-2}) = \mathbf{2^4}$ b) $-(-2)^2$ $= (-1)(-2).(\mathbf{-2}) = \mathbf{-2^4}$ c) $(-2)^3$ $= (-2).(\mathbf{-2}).(\mathbf{-2}) = \mathbf{-2^3}$ d) $-(-2)^3$ $= (-1)(-2).(\mathbf{-2}).(\mathbf{-2})$ $= \mathbf{2^3}$
7) Simplify. Write answers with positive exponents. a) $-\left(\frac{3}{5}\right)^{-2}$ b) $\left(\frac{-3}{5}\right)^{-2}$ c) $-\left(\frac{-3}{5^{-4}}\right)^{-2}$ d) $(a^6)^{-3} \times -(a^{-3})^6$ e) $5(b)^{-2} \times (-2b)^{-2}$	7) a) $-\frac{5^2}{3^2}$ b) $\frac{5^2}{3^2}$ c) $-\frac{5^8}{3^2}$ d) $-a^{-36} = -\frac{1}{a^{36}}$ e) $\frac{5}{b^2} \times \frac{1}{(-2b)^2} = \frac{5}{4b^4}$

#superpowers

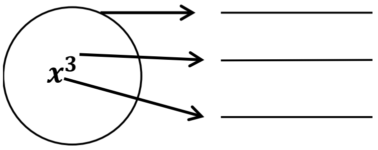
PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 1.1 a

This worksheet focuses on multiplying powers with 1 as the coefficient and a maximum of two variables.

Questions

1) Look at x^3. Give names for x^3; 3 and x.



2) Expand: x^5

3) Write as a power:
   a) $a \times a \times a \times a$
   b) $a \times b \times b \times a$

4) Fill in the blanks:
   a) $m^6 \times m^3 = m^{\square}$
   b) $m^3 \times m^6 = m^{\square}$
   c) $r^3 \times r^4 = \square^7$
   d) $h^3 \times \square^7 = h^{10}$
   e) $d^6 \times d^6 = \square$
   f) $a^5 \times \square = a^8$
   g) $\square \times z^4 = z^8$
   h) $x.x^2 \times x^3 = \square$

Notice that sometimes the numbering goes down, then across. Sometimes it, goes across and then down to the next row.

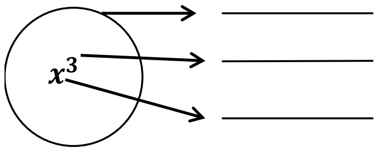
5) Write the answers as powers:
   a) $a^3b^7 \times a^2b^4$
   b) $j^6k^2 \times j^5 \times k^4 \times j^3k^5$
   c) $a^4b \times a^3b^2$
   d) $mn \times m \times n$
   e) $ab^4 \times a^8b \times c^4$

6)
   a) Choose the correct answer for $x^2.x^3 + x^6$:
      i) x^{11}
      ii) x^{12}
      iii) $x^5 + x^6$
      iv) $2x^{11}$
   b) Check that your answer for Q6a is correct by substituting $x = 2$
      i.e. $x^2.x^3 + x^6 = (2)^2.(2)^3 + (2)^6 = \square$
   c) Choose one of the incorrect answers from Q6a and say why the answer is wrong.

7) Simplify:
   a) $x^5.x^2.x^4$
   b) $c^2d^7 \times c^5d^2$
   c) $m^2n \times m^3n^2p \times m \times n$
   d) $a^3 + a^3.a^3$
   e) $xyz.xyz^4$

Worksheet 1.1 a

Answers

Questions	**Answers**
1) Look at x^3. Give names for x^3; 3 and x.  2) Expand x^5 3) Write as a power: a) $a \times a \times a \times a$ b) $a \times b \times b \times a$	1) x^3 : power, 3 : exponent, x : base 2) $x \times x \times x \times x \times x$ 3) a) a^4 b) a^2b^2
4) Fill in the blanks: a) $m^6 \times m^3 = m^{\square}$ b) $m^3 \times m^6 = m^{\square}$ c) $r^3 \times r^4 = \square^7$ d) $h^3 \times \square^7 = h^{10}$ e) $d^6 \times d^6 = \square$ f) $a^5 \times \square = a^8$ g) $\square \times z^4 = z^8$ h) $x.x^2 \times x^3 = \square$	4) a) $m^6 \times m^3 = m^{\boldsymbol{9}}$ b) $m^3 \times m^6 = m^{\boldsymbol{9}}$ c) $r^3 \times r^4 = \boldsymbol{r}^7$ d) $h^3 \times \boldsymbol{h}^7 = h^{10}$ e) $d^6 \times d^6 = \boldsymbol{d^{12}}$ f) $a^5 \times \boldsymbol{a^3} = a^8$ g) $\boldsymbol{z^4} \times z^4 = z^8$ h) $x.x^2 \times x^3 = \boldsymbol{x^6}$
5) Write the answers as powers: a) $a^3b^7 \times a^2b^4$ b) $j^6k^2 \times j^5 \times k^4 \times j^3k^5$ c) $a^4b \times a^3b^2$ d) $mn \times m \times n$ e) $ab^4 \times a^8b \times c^4$	5) a) a^5b^{11} b) $j^{14}k^{11}$ c) a^7b^3 d) m^2n^2 e) $a^9b^5c^4$
6) a) Choose the correct answer for $x^2.x^3 + x^6$: i) x^{11} ii) x^{12} iii) $x^5 + x^6$ iv) $2x^{11}$ b) Check that your answer for Q6a is correct by substituting $x = 2$ i.e. $x^2.x^3 + x^6 = (2)^2.(2)^3 + (2)^6 = \square$ c) Choose one of the incorrect answers from Q6a and say why the answer is wrong.	6) a) iii b) $(2)^2.(2)^3 + (2)^6 = 4 \times 8 + 64 = 96$ c) i) $(2)^{11} = 2048$ ii) $(2)^{12} = 4096$ iii) $(2)^5 + (2)^6 = 96$ iv) $2(2)^{11} = 2(2048) = 4096$ d) i) Added exponents, ignored addition of bases ii) Multiplied exponents of first 2 factors, ignored addition of bases iv) Added exponents, added unlike terms (to get coefficient of 2)
7) Simplify: a) $x^5.x^2.x^4$ b) $c^2d^7 \times c^5d^2$ c) $m^2n \times m^3n^2p \times m \times n$ d) $a^3 + a^3.a^3$ e) $xyz.xyz^4$	7) a) x^{11} b) c^7d^9 c) m^6n^4p d) $a^3 + a^6$ e) $x^2y^2z^5$

Worksheet 1.1 b

This worksheet focuses on multiplying powers with variable and numerical bases, some of which are not prime numbers.

Questions

1) Are these statements TRUE or FALSE?
   - a) $a^2 \times a^3 = a^5$
   - b) $x^4 \times xy^3 = x^4y^3$
   - c) $ab \times ab = ab^2$
   - d) $3^4 \times 3^2 = 3^6$
   - e) $3^4 \times 3^2 = 9^6$
   - f) $3^4 \times 3^2 = 3^8$

2) Write down the first six prime numbers. We have started the list. 2; 3 ___; ___; ___; ___

3) Fill in the blanks.
   - a) $x^6 \times x^5 = x^{\square}$
   - b) $2^6 \times 2^5 = 2^{\square}$
   - c) $p^{10} \times p^2 = \square^{12}$
   - d) $13^{10} \times \square^5 = 13^{15}$
   - e) $a^8 \times a^8 = \square$
   - f) $7^6 \times \square = 7^{12}$
   - g) $\square \times 2^4 = 2^5$
   - h) $m.m^9 \times m^4 = \square$

4) Match the statement in column A with its partner in column B:

Column A	**Column B**
a) a^3	i) $a + 3$
b) $3a$	ii) 5^{73}
c) $3 + a$	iii) 5^{72}
d) $5^{72} \times 5$	iv) $a \times a \times a$
	v) $a + a + a$

5)
   - a) Write $2 \times 3 \times 2 \times 3$ as powers in as many ways as you can. Bases do not have to all be prime numbers.
   - b) What are the different ways of writing $3 \times b \times 3 \times b$ as a power?

4) Simplify and write your answers as powers with prime number bases: e.g. $8 = 2 \times 2 \times 2 = 2^3$
   - a) $7^4 \times 5^4 \times 7^8 \times 5^8$
   - b) $8 \times 2^7 \times 2^5$
   - c) $9 \times 4 \times 3^8 \times 3 \times 2$

6) Simplify and write as powers with prime bases:
   - a) $abc^3 \times ab^3c \times a^3bc$
   - b) $m^2p^5 \times n^3 \times p^6 \times n^2m^3$
   - c) $9^4 \times 3^2 \times 3$
   - d) $x^8y^7 \times x^4 + y^{12}$
   - e) $5^4 \times 7^{31} \times 5^{13} \times 7$
   - f) $7^3 + 7^{11} \times 3^5 \times 7^5$
   - g) $x^9y^3 \times x^3y^7 \times x^2y^7$
   - h) $jk \times j^2k \times jk^2 \times j^2$

Worksheet 1.1 b

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1) Are these statements TRUE or FALSE?
a) $a^2\times a^3=a^5$ d) $3^4\times3^2=3^6$
b) $x^4\times xy^3=x^4y^3$ e) $3^4\times3^2=9^6$
c) $ab\times ab=ab^2$ f) $3^4\times3^2=3^8$</td><td>1)
a) True   d) True
b) False   e) False
c) False   f) False</td></tr>
<tr><td>2) Write down the first six prime numbers. We have started the list. 2; 3; ___; ___; ___; ___</td><td>2) 2; 3; 5; 7; 11; 13</td></tr>
<tr><td>3) Fill in the blanks.
a) $x^6\times x^5=x^{\square}$ d) $13^{10}\times\square^5=13^{15}$ g) $\square\times2^4=2^5$
b) $2^6\times2^5=2^{\square}$ e) $a^8\times a^8=\square$ h) $m.m^9\times m^4=\square$
c) $p^{10}\times p^2=\square$ f) $7^6\times\square=7^{12}$</td><td>3)
a) $x^6\times x^5=x^{\mathbf{11}}$ d) $13^{10}\times\mathbf{13}^5=13^{15}$ g) $\mathbf{2}\times2^4=2^5$
b) $2^6\times2^5=2^{\mathbf{11}}$ e) $a^8\times a^8=\mathbf{a^{16}}$ h) $m.m^9\times m^4=\mathbf{m^{14}}$
c) $p^{10}\times p^2=\mathbf{p}^{12}$ f) $7^6\times\mathbf{7^6}=7^{12}$</td></tr>
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<tr><td>a) a^3</td><td>i) $a+3$</td></tr>
<tr><td>b) $3a$</td><td>ii) 5^{73}</td></tr>
<tr><td>c) $3+a$</td><td>iii) 5^{72}</td></tr>
<tr><td>d) $5^{72}\times5$</td><td>iv) $a\times a\times a$</td></tr>
<tr><td></td><td>v) $a+a+a$</td></tr>
</table></td><td>4)
a) iv
b) v
c) i
d) ii</td></tr>
<tr><td>5)
a) Write $2\times3\times2\times3$ as powers in as many ways as you can. Bases do not have to all be prime numbers.
b) What are the different ways of writing $3\times b\times3\times b$ as a power?</td><td>5)
a) Many possibilities,
e.g. $2^2\times3^2$; 6^2; $(2\times3)^2$; 36^1; $12^1\times3^1$; $4^1\times3^2$
b) 3^2b^2; $9b^2$; $(3b)^2$</td></tr>
<tr><td>6) Simplify and write your answers as powers with prime number bases: e.g. $8=2\times2\times2=2^3$
a) $7^4\times5^4\times7^8\times5^8$ b) $8\times2^7\times2^5$ c) $9\times4\times3^8\times3\times2$</td><td>6)
a) $7^{12}\times5^{12}$ b) $2^3\times2^{12}=2^{15}$
c) $3^2\times2^2\times3^9\times2=2^3\times3^{11}$</td></tr>
<tr><td>7) Simplify and write as powers with prime bases:
a) $abc^3\times ab^3c\times a^3bc$ e) $5^4\times7^{31}\times5^{13}\times7$
b) $m^2p^5\times n^3\times p^6\times n^2m^3$ f) $7^3+7^{11}\times3^5\times7^5$
c) $9^4\times3^2\times3$ g) $x^9y^3\times x^3y^7\times x^2y^7$
d) $x^8y^7\times x^4+y^{12}$ h) $jk\times j^2k\times jk^2\times j^2$</td><td>7)
a) $a^5b^5c^5$ e) $5^{17}\times7^{32}$
b) $m^5n^5p^{11}$ f) $7^3+7^{16}\times3^5$
c) 3^{11} g) $x^{14}y^{17}$
d) $x^{12}y^7+y^{12}$ h) j^6k^4</td></tr>
</table>

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 1.1 c

This worksheet focuses on multiplying powers with variable and numerical bases having positive or negative coefficients.

Questions

1)

a) Which of these is the same as $x^2 \times x^7$?

i) $x^{2\times 7}$ ii) x^{2+7} iii) $2x^{2\times 7}$

b) Which of these are NOT the same as $3 \times 5 \times 3 \times 5$?

i) $3^2 \times 5^2$ ii) 35^2 iii) 15^2 iv) $(3 \times 5)^2$

2) Fill in the blanks:

a) $2 \times a \times a = 2a^{\square}$

b) $3 \times x \times x \times x \times x = \square x^{\square}$

c) $b \times b \times 2 \times b \times b \times b = \square b^{\square}$

d) $3 \times a \times 3 \times a \times a = 3^{\square} a^{\square}$

e) $x \times x \times 5 \times 5 \times 5 \times 5 = \square$

3) Fill in the blanks:

a) $12 \times \square \times \square \times \square \times \square = 12a^4$

b) $2 \times \square \times a \times \square \times \square \times \square = 12a^4$

c) $\square^{\square} \times 3 \times a^4 = 12a^4$

d) $4a \times \square = 12a^4$

e) $\square \times 2a^2 = 12a^4$

4) Simplify:

a) $3m^2 \times 2p^4$

b) $5x^7 \times 3y^{12} \times 2x^{15}$

c) $4q^2s^8 \times qr \times 11qs^2$

d) $-15m^8\, n^2 \times 2mn \times n^{13}$

e) $-6a^9 \times 4b^9 \times (-2ab^3)$

5)

a) Simplify $4a^3 + 3a^3$

b) Simplify $4a^3 \times 3a^3$

c) Are the answers to Q5a and Q5b the same?

d) Justify your answer for Q5c by substituting:

i) $a = 1$

ii) $a = 2$

6) Simplify:

a) $2x \times 2x \times 2x$

b) $-3a^{12} \times a^6b^5$

c) $5m^3 \times (-4m^7n^9) \times (-mn)$

d) $-t \times -6t^7u^2 \times (-10t^5u)$

e) $\frac{1}{2}x^5 \times 8x^3$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 1.1 c

Answers

Questions	**Answers**
1) a) Which of these is the same as $x^2 \times x^7$? i) $x^{2\times 7}$ ii) x^{2+7} iii) $2x^{2\times 7}$ b) Which of these are NOT the same as $3 \times 5 \times 3 \times 5$? i) $3^2 \times 5^2$ ii) 35^2 iii) 15^2 iv) $(3 \times 5)^2$	1) a) iii b) only ii
2) Fill in the blanks: a) $2 \times a \times a = 2a^{\square}$ b) $3 \times x \times x \times x \times x = \square x^{\square}$ c) $b \times b \times 2 \times b \times b \times b = \square b^{\square}$ d) $3 \times a \times 3 \times a \times a = 3^{\square}a^{\square}$ e) $x \times x \times 5 \times 5 \times 5 \times 5 = \square$	2) a) $2a^{\mathbf{2}}$ b) $\mathbf{3}x^{\mathbf{4}}$ c) $\mathbf{2}b^{\mathbf{5}}$ d) $3^{\mathbf{2}}a^{\mathbf{3}}$ e) $\mathbf{5^4 x^2}$ or $\mathbf{625x^2}$
3) Fill in the blanks: a) $12 \times \square \times \square \times \square \times \square = 12a^4$ b) $2 \times \square \times a \times \square \times \square \times \square = 12a^4$ c) $\square^{\square} \times 3 \times a^4 = 12a^4$ d) $4a \times \square = 12a^4$ e) $\square \times 2a^2 = 12a^4$	3) a) $12 \times \boldsymbol{a} \times \boldsymbol{a} \times \boldsymbol{a} \times \boldsymbol{a} = 12a^4$ b) $2 \times \mathbf{6} \times a \times \boldsymbol{a} \times \boldsymbol{a} \times \boldsymbol{a} = 12a^4$ c) $\mathbf{2^2} \times 3 \times a^4 = 12a^4$ d) $4a \times \boldsymbol{3a^3} = 12a^4$ e) $\boldsymbol{6a^2} \times 2a^2 = 12a^4$
4) Simplify: a) $3m^2 \times 2p^4$ b) $5x^7 \times 3y^{12} \times 2x^{15}$ c) $4q^2s^8 \times qr \times 11qs^2$ d) $-15m^8\,n^2 \times 2mn \times n^{13}$ e) $-6a^9 \times 4b^9 \times (-2ab^3)$	4) a) $6m^2p^4$ b) $30x^{22}y^{12}$ c) $44q^4rs^{10}$ d) $-30m^9n^{16}$ e) $48a^{10}b^{12}$
5) a) Simplify $4a^3 + 3a^3$ b) Simplify $4a^3 \times 3a^3$ c) Are the answers to Q5a and Q5b the same? d) Justify your answer for Q5c by substituting: ii) $a = 1$ iii) $a = 2$	5) a) $7a^3$ b) $12a^6$ c) No d) For $a = 1$ a) $4(1)^3 + 3(1)^3 7(1)^3 = 7$ b) $4(1)^3 \times 3(1)^3$ $= 12(1)^6 = 12$ For $a = 2$ a) $4(2)^3 + 3(2)^3 = 7(2)^3 = 56$ b) $4(2)^3 \times 3(2)^3$ $= 12(2)^6 = 768$ Therefore answer to Q5a ≠ Q5b
6) Simplify: a) $2x \times 2x \times 2x$ b) $-3a^{12} \times a^6b^5$ c) $5m^3 \times (-4m^7n^9) \times (-mn)$ d) $-t \times -6t^7u^2 \times (-10t^5u)$ e) $\frac{1}{2}x^5 \times 8x^3$	6) a) $8x^3$ b) $-3a^{18}b^5$ c) $20m^{11}n^{10}$ d) $-60t^{13}u^3$ e) $4x^8$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 1.2 a

This worksheet focuses on multiplying powers having negative exponents, coefficients of 1 and a maximum of three variables.

Questions

1) Writing a^{-n} with a positive exponent. Fill in the blanks.

$\frac{1}{a^n} = \frac{a^0}{\square} = a^{0-n} = a^{\square}$
So, $a^{-n} = \frac{1}{a^{\square}}$

and

$\frac{1}{a^{-n}} = 1 \div \frac{1}{a^n} = 1 \times \frac{a^{\square}}{1} = a^{\square}$
So, $\frac{1}{a^{-n}} = a^{\square}$

2) Rewrite the following powers with positive exponents:
a) x^{-5} b) p^{-2} c) k^{-9} d) m^{-100}

3) Fill in the blanks:
a) $\frac{1}{m^6} = m^{\square}$ b) $\frac{1}{x^{\square}} = x^{-10}$ c) $r^7 \times r^{-8} = \square$
d) $h^{-13}.h^{\square} = h^{10}$ e) $d^{\square} \times d^{12} = d^{-3}$ f) $a^{-7} \times \square = a^{-9}$

4) Fill in the blanks:
a) $a^3 \times \square = a^{-3}$
b) $a^{-5} \times \square = a^{-3}$
c) $\square \times a = a^{-3}$
d) $\square \times \frac{1}{a} = a^{-3}$

5) What can you put in the boxes to make the statement $\square \times \square = a^{-3}$ true?
You must choose powers that have:
a) two negative exponents
b) zero as one of the exponents
c) at least one positive exponent

6) Fill in the blanks:
a) $x^6.x^{-6} = \square$ b) $\square \times \square = y^0 = 1$ c) $\frac{1}{m^{\square}} \times m^{\square} = 1$
d) $m^{\square}n^{\square} \times m^{\square}n^{\square} = 1$ e) $x^2y^{-7} \times x^{-2}y = \square$ f) $ab^3 \times \square = a^3$

7) Simplify:
a) $m^9.m^{-12}.m^{14}$
b) $x^{-6}y^6 \times x^6y^{-6}$
c) $a^7b^{-13}c^{-1}.ab^5.abc$
d) $p^4 \times \frac{1}{p^{-5}}$

Notice that sometimes the numbering goes down, then across. Sometimes it, goes across and then down to the next row.

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 1.2 a

Answers

Questions	Answers
1) Writing a^{-n} with a positive exponent. Fill in the blanks. $\frac{1}{a^n} = \frac{a^0}{\square} = a^{0-n} = a^{\square}$ and $\frac{1}{a^{-n}} = 1 \div \frac{1}{a^n} = 1 \times \frac{a^{\square}}{1} = a^{\square}$ So, $a^{-n} = \frac{1}{a^{\square}}$ and So, $\frac{1}{a^{-n}} = a^{\square}$	1) $\frac{1}{a^n} = \frac{a^0}{\boldsymbol{a^n}} = a^{0-n} = a^{\boldsymbol{-n}}$ and $\frac{1}{a^{-n}} = 1 \div \frac{1}{a^n} = 1 \times \frac{\boldsymbol{a^n}}{1} = a^{\boldsymbol{n}}$ So, $a^{-n} = \frac{1}{\boldsymbol{a^n}}$ and So, $\frac{1}{a^{-n}} = a^{\boldsymbol{n}}$
2) Rewrite the following powers with positive exponents: a) x^{-5} b) p^{-2} c) k^{-9} d) m^{-100}	2) a) $\frac{1}{x^5}$ b) $\frac{1}{p^2}$ c) $\frac{1}{k^9}$ d) $\frac{1}{m^{100}}$
3) Fill in the blanks: a) $\frac{1}{m^6} = m^{\square}$ b) $\frac{1}{x^{\square}} = x^{-10}$ c) $r^7 \times r^{-8} = \square$ d) $h^{-13}.h^{\square} = h^{10}$ e) $d^{\square} \times d^{12} = d^{-3}$ f) $a^{-7} \times \square = a^{-9}$	3) a) $\frac{1}{m^6} = m^{\boldsymbol{-6}}$ b) $\frac{1}{x^{\boldsymbol{10}}} = x^{-10}$ c) $r^7 \times r^{-8} = \boldsymbol{r^{-1}}$ or $\frac{\mathbf{1}}{\mathbf{r}}$ d) $h^{-13}.h^{\boldsymbol{23}} = h^{10}$ e) $d^{\boldsymbol{-15}} \times d^{12} = d^{-3}$ f) $a^{-7} \times \boldsymbol{a^{-2}} = a^{-9}$
4) Fill in the blanks: a) $a^3 \times \square = a^{-3}$ b) $a^{-5} \times \square = a^{-3}$ c) $\square \times a = a^{-3}$ d) $\square \times \frac{1}{a} = a^{-3}$ 5) What can you put in the boxes to make the statement $\square \times \square = a^{-3}$ true? You must choose powers that have: a) two negative exponents b) zero as one of the exponents c) at least one positive exponent	4) a) a^{-6} b) a^2 c) a^{-4} d) a^{-2} 5) Many possibilities, examples shown below a) $\boldsymbol{a^{-2}} \times \boldsymbol{a^{-1}} = a^{-3}$ b) $\boldsymbol{a^{0}} \times \boldsymbol{a^{-3}} = a^{-3}$ c) e.g. $\boldsymbol{a^{5}} \times \boldsymbol{a^{-8}} = a^{-3}$; $\boldsymbol{a^{1}} \times \boldsymbol{a^{-4}} = a^{-3}$
6) Fill in the blanks: a) $x^6.x^{-6} = \square$ b) $\square \times \square = y^0 = 1$ c) $\frac{1}{m^{\square}} \times m^{\square} = 1$ d) $m^{\square}n^{\square} \times m^{\square}n^{\square} = 1$ e) $x^2y^{-7} \times x^{-2}y = \square$ f) $ab^3 \times \square = a^3$	6) a) x^0 or 1 b) Exponents must be additive inverses, e.g. $y^{-3} \times y^3$ or $y^5 \times y^{-5}$ c) Exponents must be the same, e.g. $\frac{1}{m^2} \times \boldsymbol{m^2} = 1$ or $\frac{1}{m^{-3}} \times \boldsymbol{m^{-3}}$ d) Exponents must be additive inverses, e.g. $m^2n^{-3} \times m^{-2}n^3$ or $m^{-4}n^5 \times m^4n^{-5}$ e) $x^2y^{-7} \times x^{-2}y = \boldsymbol{y^{-6}}$ f) $ab^3 \times \boldsymbol{a^2b^{-3}} = a^3$
7) Simplify: a) $m^9.m^{-12}.m^{14}$ b) $x^{-6}y^6 \times x^6y^{-6}$ c) $a^7b^{-13}c^{-1}.ab^5.abc$ d) $p^4 \times \frac{1}{p^{-5}}$	7) a) m^{11} b) 1 or x^0y^0 c) $a^9b^{-7} or \frac{a^9}{b^7}$ d) p^9

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 1.2 b

This worksheet focuses multiplying powers with variable and numerical bases, negative exponents, and making bases prime numbers. Up to 3 variables are used.

Questions

1) Write TRUE or FALSE for each statement:
   a) $x^4 \times x^{-9} = x^{-5}$
   b) $x^4 \times x^{-9} = x^5$
   c) $m^{-10}.m^{10} = m$
   d) $m^{-10}.m^{10} = m^0$
   e) $m^{-10}.m^{10} = 0$
   f) $m^{-10}.m^{10} = 1$

2) Simplify and write all answers with positive exponents:
   a) $a^8 \times a^{-1}$ b) $(p)^{-13}.(p)^{12}$ c) $mn^{17} \times m^{-10}n$

3) Simplify and give your answers with positive exponents and prime bases:
   a) $5^8 \times 5^{-6}$ b) $8^{-14} \times 8^7$ c) $9^{-6} \times 9^{-15}$

4) Which of the following represents $2^{-3} \times 3^{-2}$?
   a) $(2 \times 3)^{-5}$ b) $\frac{1}{2^3} \times \frac{1}{3^2}$ c) $\frac{1}{2^3 \times 3^2}$ d) 6^{-5} e) $\frac{1}{8 \times 9}$

5) Each of the statements in the box below has an error.

A. $5^4 \times 5^2 = 25^6$
B. $ab.ab = ab^2$
C. $ab + ab = (ab)^2$
D. $7^{-3} \times 7^3 = 7$
E. $3^4 \times \frac{1}{3^{-3}} = 3^1$

   a) Explain the error in each statement.
   b) Change the part on the right side of the equal sign to make the statement true.

6) Simplify and give your answers with positive exponents and prime bases:
   a) $w^{-6}xy^{18}.x^{-7}.w^{-12}y$
   b) $(xyz)^0.x^{-5}y^4$
   c) $5^{-2} \times 5^{-4} \times 5^{-8}$
   d) $4^{-6} \times 2^6 \times 8$
   e) $x^{-12} + x^3.x^9$
   f) $km.km^{-16}$
   g) $(n^0)^2.n^{-5}$

Worksheet 1.2 b

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1) Write TRUE or FALSE for each statement:
a) $x^4 \times x^{-9} = x^{-5}$ b) $x^4 \times x^{-9} = x^5$ c) $m^{-10}.m^{10} = m$
d) $m^{-10}.m^{10} = m^0$ e) $m^{-10}.m^{10} = 0$ f) $m^{-10}.m^{10} = 1$</td><td>1)
a) True b) False c) False
d) True e) False f) True</td></tr>
<tr><td>2) Simplify and write all answers with positive exponents:
a) $a^8 \times a^{-1}$ b) $(p)^{-13}.(p)^{12}$ c) $mn^{17} \times m^{-10}n$</td><td>2)
a) a^7 b) $\frac{1}{p}$ c) $\frac{n^7}{m^9}$</td></tr>
<tr><td>3) Simplify and give your answers with positive exponents and prime bases:
a) $5^8 \times 5^{-6}$ b) $8^{-14} \times 8^7$ c) $9^{-6} \times 9^{-15}$</td><td>3)
a) 5^2 b) $8^{-7} = \frac{1}{8^7} = \frac{1}{(2^3)^7} = \frac{1}{2^{21}}$ c) $9^{-21} = \frac{1}{9^{21}} = \frac{1}{(3^2)^{21}} = \frac{1}{3^{42}}$</td></tr>
<tr><td>4) Which of the following represents $2^{-3} \times 3^{-2}$?
a) $(2 \times 3)^{-5}$ b) $\frac{1}{2^3} \times \frac{1}{3^2}$ c) $\frac{1}{2^3 \times 3^2}$ d) 6^{-5} e) $\frac{1}{8 \times 9}$</td><td>4) b, c and e. They all equal $\frac{1}{72}$</td></tr>
<tr><td>5) Each of the statements in the box below has an error.
A. $5^4 \times 5^2 = 25^6$
B. $ab.ab = ab^2$
C. $ab + ab = (ab)^2$
D. $7^{-3} \times 7^3 = 7$
E. $3^4 \times \frac{1}{3^{-3}} = 3^1$
a) Explain the error in each statement.
b) Change the part on the right-side of the equal sign to make the statement true.</td><td>5)
<table>
<tr><th></th><th>a) Explanation of error</th><th>b) Change made</th></tr>
<tr><td>A. $5^4 \times 5^2 = 25^6$</td><td>Multiplied bases</td><td>5^6</td></tr>
<tr><td>B. $ab.ab = ab^2$</td><td>Did not add exponents of a</td><td>a^2b^2</td></tr>
<tr><td>C. $ab + ab = (ab)^2$</td><td>Adding bases not multiplying</td><td>$2ab$</td></tr>
<tr><td>D. $7^{-3} \times 7^3 = 7$</td><td>Anything with an exponent of 0 is 1</td><td>$7^0 = 1$</td></tr>
<tr><td>E. $3^4 \times \frac{1}{3^{-3}} = 3^1$</td><td>Treated exponents as $4 - 3 = 1$</td><td>$3^4 \times 3^3 = 3^7$</td></tr>
</table></td></tr>
<tr><td>6) Simplify and give your answers with positive exponents and prime bases:
a) $w^{-6}xy^{18}.x^{-7}.w^{-12}y$ b) $(xyz)^0.x^{-5}y^4$ c) $5^{-2} \times 5^{-4} \times 5^{-8}$
d) $4^{-6} \times 2^6 \times 8$ e) $x^{-12} + x^3.x^9$ f) $km.km^{-16}$
g) $(n^0)^2.n^{-5}$</td><td>6)
a) $\frac{y^{19}}{x^6w^{18}}$ b) $\frac{y^4}{x^5}$ c) $5^{-14} = \frac{1}{5^{14}}$
d) $(2^2)^{-6} \times 2^6 \times 2^3$
$= 2^{-12} \times 2^9 = 2^{-3} = \frac{1}{2^3}$
e) $x^{-12} \times x^{12}$
$= x^0 = 1$
f) $k^2m^{-15} = \frac{k^2}{m^{15}}$
g) $n^0 \times n^{-5} = 1 \times \frac{1}{n^5} = \frac{1}{n^5}$</td></tr>
</table>

#super$^{\text{powers}}$
PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 1.2 c

This worksheet focuses on multiplying powers with negative and fraction coefficients, and negative and positive numerical exponents.

Questions

1) Match the statement in column A with the correct answer in column B.
If there is no matching statement in column B, you must provide the correct statement.

Column A	Column B
a) $2(5m)$	i) $\frac{1}{5m}$
b) $m(5+2)$	ii) $10m$
c) $(5m)^2$	iii) $\frac{5}{m}$
d) $5m^{-1}$	iv) $25m^2$
	v) $10+2m$

2) Fill in the blanks:
   a) $2 \times k^2 \times k^{-1} = 2k^{\square}$
   b) $n^2 \times 3 \times n^{-2} \times n \times n^0 = \square n^{\square}$
   c) $3 \times p \times 3^2 \times p^3 = 3^{\square}p^{\square}$
   d) $c^{-10} \times 8 \times c^{-2} \times c \times 4 = \square c^{\square}$
   e) $s^{-1} \times s^{-1} \times 10 \times 10 \times 10 = \square$

3) Fill in the blanks. Use positive or negative exponents.
   a) $12 \times \square \times \square \times \square \times \square = 12t^{-4}$
   b) $\frac{1}{2} \times \square \times \frac{1}{t} \times \square \times \square \times \square = 12t^{-4}$
   c) $\square^{\square} \times 3 \times \square \times t^{\square} = -12t^{-4}$
   d) $12t \times \square = -12t^{-4}$
   e) $\square \times (-2t^2) = -12t^{-4}$

4) Simplify and write answers with positive exponents:
   a) $3^{-2} \times 3^3$
   b) $3^2 \times 3^{-3}$
   c) $5p^6(-3r^0)(-2)p^{-6}$
   d) $5p^6(-3r^0)(-2)p^{-3}$

5) Simplify and give your answers with positive exponents and prime bases:
   a) $-2b \times 2b^{-3}$
   b) $-8n^{-8}.16mn^{16}(-2mn^{-2})$
   c) $-3b^{-3}.3b^{-100}.3ab^3$
   d) $\frac{-3}{3^{-4}} \times \frac{-3}{4^{-3}}$
   e) $-9x^8\,y^2.3xy^{-3}.y^{-2}$
   f) $0{,}5x^{-3}y^3 \times \frac{1}{2^{-1}x^3y^3} \times x^6$
   g) $\frac{2}{5}x^{-50}y^{50}.25x^{50}y^{-50}$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 1.2 c

Answers

Questions	Answers
1) Match the statement in column A with the correct answer in column B. If there is no matching statement in column B, you must provide the correct statement **Column A** a) $2(5m)$ b) $m(5+2)$ c) $(5m)^2$ d) $5m^{-1}$ **Column B** i) $\frac{1}{5m}$ ii) $10m$ iii) $\frac{5}{m}$ iv) $25m^2$ v) $10+2m$	1) a) ii b) No matching statement, $5m+2$ c) iv d) iii
2) Fill in the blanks: a) $2\times k^2\times k^{-1} = 2k^{\square}$ b) $n^2\times 3\times n^{-2}\times n\times n^0 = \square n^{\square}$ c) $3\times p\times 3^2\times p^3 = 3^{\square}p^{\square}$ d) $c^{-10}\times 8\times c^{-2}\times c\times 4 = \square c^{\square}$ e) $s^{-1}\times s^{-1}\times 10\times 10\times 10 = \square$	2) a) $2\times k^2\times k^{-1} = 2k^{\mathbf{1}}$ b) $n^2\times 3\times n^{-2}\times n\times n^0 = \mathbf{3}n^{\mathbf{1}}$ c) $3\times p\times 3^2\times p^3 = 3^{\mathbf{3}}p^{\mathbf{4}}$ d) $c^{-10}\times 8\times c^{-2}\times c\times 4 = \mathbf{32}c^{\mathbf{-11}}$ e) $s^{-1}\times s^{-1}\times 10\times 10\times 10 = \mathbf{1000s^{-2}}$ or $\mathbf{10^3 s^{-2}}$
3) Fill in the blanks. Use positive or negative exponents. a) $12\times\square\times\square\times\square\times\square = 12t^{-4}$ b) $\frac{1}{2}\times\square\times\frac{1}{t}\times\square\times\square\times\square = 12t^{-4}$ c) $\square^{\square}\times 3\times\square\times t^{\square} = -12t^{-4}$ d) $12t\times\square = -12t^{-4}$ e) $\square\times(-2t^2) = -12t^{-4}$	3) a) $12\times t^{-1}\times t^{-1}\times t^{-1}\times t^{-1} = 12t^{-4}$ or subs $\frac{1}{t}$ b) $\frac{1}{2}\times\mathbf{24}\times\frac{1}{t}\times\frac{1}{t}\times\frac{1}{t}\times\frac{1}{t} = 12t^{-4}$ c) Many possibilities, e.g. $\boldsymbol{t^{-6}}\times 3\times\mathbf{-4}\times t^{\mathbf{2}} = -12t^{-4}$ d) $12t\times -\frac{\mathbf{1}}{\mathbf{t^5}} = -12t^{-4}$ e) $\frac{\mathbf{6}}{\boldsymbol{t^6}}\times(-2\mathrm{t}^2) = -12t^{-4}$
4) Simplify and write answers with positive exponents. a) $3^{-2}\times 3^3$ b) $3^2\times 3^{-3}$ c) $5p^6(-3r^0)(-2)p^{-6}$ d) $5p^6(-3r^0)(-2)p^{-3}$	4) a) 3^1 b) $3^{-1} = \frac{1}{3^1}$ c) $30p^0r^0 = 30$ d) $30p^3r^0 = 30p^3$
5) Simplify and give your answers with positive exponents and prime bases: a) $-2b\times 2b^{-3}$ b) $-8n^{-8}.16mn^{16}(-2mn^{-2})$ c) $-3b^{-3}.3b^{-100}.3ab^3$ d) $\frac{-3}{3^{-4}}\times\frac{-3}{4^{-3}}$ e) $-9x^8y^2.3xy^{-3}.y^{-2}$ f) $0{,}5x^{-3}y^3\times\frac{1}{2^{-1}x^3y^3}\times x^6$ g) $\frac{2}{5}x^{-50}y^{50}.25x^{50}y^{-50}$	5) a) $-2^2b^{-2} = -\frac{2^2}{b^2}$ b) $16^2m^2n^6 = (2^4)^2m^2n^6 = 2^8m^2n^6$ c) $-3^3ab^{-100} = -\frac{3^3a}{b^{100}}$ d) $-3^5\times(-3)\times 4^3$ $= -3^6\times(2^2)^3$ $= -3^6\times 2^6$ e) $-3^2x^8y^2.3xy^{-5}$ $= -3^3x^9y^{-3} = -\frac{3^3x^9}{y^3}$ f) $\frac{1y^3}{2x^3}\times\frac{2x^6}{x^3y^3} = \frac{2x^6y^3}{2x^6y^3} = 1$ g) $\frac{50x^0y^0}{5} = 10 = 2\times 5$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 2.1 a

This worksheet focuses on dividing powers where bases have a maximum of two variables. Answers have exponents in the numerator, or denominator or both.

Questions

1)

a) Expand $\frac{x^5}{x^3}$ and then cancel like factors to show that $\frac{x^5}{x^3} = x^2$

b) Write your own question that will give an answer m^3 when two powers are divided.

c) Show that $\frac{1}{a.a.a.a.a}$ is the same as $a^1 \div a^6$.

2) Choose the correct next step to simplify $\frac{x^6}{x^2}$:

a) x^{6+2} b) x^{6-2} c) $x^{6\div2}$ d) $x^{6\times2}$

3) Fill in the blanks:

a) $x^4 \times x^{\square} = x^9$ b) $x^9 \div \square = x^3$ c) $x^4y^2 \div x^3 = \square y^2$

4) Look at the six questions in the table:

	Column A	Column B	Column C
Row 1	i) $\frac{a^4}{a^3}$	ii) $\frac{a^9}{a^5}$	iii) $\frac{a^6}{a^2}$
Row 2	iv) $\frac{a^3}{a^4}$	v) $\frac{a^5}{a^9}$	vi) $\frac{a^2}{a^6}$

a) What is the same, and what is different in the questions in:
  i) Row 1 and Row 2
  ii) Column A, Column B and Column C

b) Predict which questions will have 1 in the *numerator* of the answer. Why will this happen?

c) Predict which questions will have 1 in the *denominator* of the answer. Why will this happen?

d) Simplify each question by expanding and cancelling the like factors.

e) Predict the answers to:
  i) $\frac{k^2}{k^7}$
  ii) $\frac{k^5}{k^3}$
  iii) $\frac{k^4.\ k^2}{k^6}$

f) Now expand and cancel like factors to check if your predictions were correct.

5) Simplify:

a) $\frac{d^2}{d}$ b) $\frac{c^7}{c^3}$ c) $\frac{x^2.x^6}{x^3}$ d) $\frac{y^9a^3}{y^7a^4}$

e) $\frac{m^3n^2}{m^2n}$ f) $\frac{m^3n^2.mn^4}{m^2n^8}$ g) $\frac{y^3.a^3.ay^4}{y^7.a^8}$ h) $x^6 \div x^3 - x^3$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 2.1a

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1)
a) Expand $\frac{x^5}{x^3}$ and then cancel like factors to show that $\frac{x^5}{x^3} = x^2$
b) Write your own question that will give an answer m^3 when two powers are divided.
c) Show that $\frac{1}{a.a.a.a.a}$ is the same as $a^1 \div a^6$</td><td>1)
a) $\frac{x.x.\cancel{x.x.x}}{\cancel{x.x.x}} = x^2$
b) Many possibilities. Exponent of numerator must be 3 bigger than exponent of denominator and base must be m, e.g. $\frac{m^7}{m^4} = \frac{\cancel{m.m.m}.m.m.m.m}{\cancel{m.m.m}} = m^3$
c) $\frac{a}{a.a.a.a.a.a} = \frac{a^1}{a^6}$ or $\frac{1}{a.a.a.a.a} \times \frac{a}{a} = \frac{a^1}{a^6}$</td></tr>
<tr><td>2) Choose the correct next step to simplify $\frac{x^6}{x^2}$:
a) x^{6+2} b) x^{6-2} c) $x^{6\div 2}$ d) $x^{6\times 2}$</td><td>2)
Next step: b) x^{6-2}</td></tr>
<tr><td>3) Fill in the blanks:
a) $x^4 \times x^{\square} = x^9$ b) $x^9 \div \square = x^3$ c) $x^4y^2 \div x^3 = \square y^2$</td><td>3)
a) $x^4 \times \boldsymbol{x^5} = x^9$ b) $x^9 \div \boldsymbol{x^6} = x^3$ c) $x^4y^2 \div x^3 = \boldsymbol{x}y^2$</td></tr>
<tr><td>4) Look at the six questions in the table:
<table>
<tr><th></th><th>Column A</th><th>Column B</th><th>Column C</th></tr>
<tr><td>Row 1</td><td>i) $\frac{a^4}{a^3}$</td><td>ii) $\frac{a^9}{a^5}$</td><td>iii) $\frac{a^6}{a^2}$</td></tr>
<tr><td>Row 2</td><td>iv) $\frac{a^3}{a^4}$</td><td>v) $\frac{a^5}{a^9}$</td><td>vii) $\frac{a^2}{a^6}$</td></tr>
</table>
a) What is the same, and what is different in the questions in:
i) Row 1 and Row 2 ii) Column A, Column B and Column C
b) Predict which questions will have 1 in the <i>numerator</i> of the answer. Why will this happen?
c) Predict which questions will have 1 in the <i>denominator</i> of the answer. Why will this happen?
d) Simplify each question by expanding and cancelling the like factors.
e) Predict the answers to: i) $\frac{k^2}{k^7}$ ii) $\frac{k^5}{k^3}$ iii) $\frac{k^4.\ k^2}{k^6}$
f) Now expand and cancel like factors to check if your predictions were correct.</td><td>4)
a)
i) All bases are a. Different powers in different columns. In row 1, higher exponent is in numerator. In row 2, higher exponent is in denominator.
ii) In each column, the powers are the same but numerators and denominators are swopped around.
b) The items in row 2 will have 1 in numerator. The factors in the numerator will cancel since the degree of the denominator is bigger.
c) The items in row 1 will have 1 in denominator. The factors in the denominator will cancel since the degree of the numerator is bigger.
d)
i) $\frac{a.\cancel{a.a.a}}{\cancel{a.a.a}} = \frac{a}{1}$ ii) $\frac{a.a.a.a.\cancel{a.a.a.a.a}}{\cancel{a.a.a.a.a}} = \frac{a^4}{1}$ iii) $\frac{a.a.a.a.\cancel{a.a}}{\cancel{a.a}} = \frac{a^4}{1}$
iv) $\frac{\cancel{a.a.a}}{\cancel{a.a.a}.a} = \frac{1}{a}$ v) $\frac{\cancel{a.a.a.a.a}}{\cancel{a.a.a.a.a}.a.a.a.a} = \frac{1}{a^4}$ vi) $\frac{\cancel{a.a.}}{\cancel{a.a}.a.a.a.a} = \frac{1}{a^4}$
e) and f)
i) $\frac{\cancel{k.k}}{k.k.k.k.k.\cancel{k.k}} = \frac{1}{k^5}$ ii) $\frac{\cancel{k.k.k}.k.k}{\cancel{k.k.k}} = k^2$ iii) $\frac{\cancel{k.k.k.k.k.k}}{\cancel{k.k.k.k.k.k}} = 1$</td></tr>
<tr><td>5) Simplify:
a) $\frac{d^2}{d}$ b) $\frac{c^7}{c^3}$ c) $\frac{x^2.x^6}{x^3}$ d) $\frac{y^9a^3}{y^7a^4}$
e) $\frac{m^3n^2}{m^2n}$ f) $\frac{m^3n^2.mn^4}{m^2n^8}$ g) $\frac{y^3.a^3.ay^4}{y^7.a^8}$ h) $x^6 \div x^3 - x^3$</td><td>5)
a) d b) c^4 c) x^5 d) $\frac{y^2}{a}$
e) mn f) $\frac{m^2}{n^2}$ g) $\frac{1}{a^4}$ h) $x^3 - x^3 = 0$</td></tr>
</table>

Worksheet 2.1 b

This worksheet focuses on dividing powers with variable and numerical bases, with exponents of zero, and with exponents that that are larger in the denominator than in the numerator.

Questions

1) Which of the following is the correct solution to $\frac{3^7}{3^5}$?

a) $\frac{7}{5}$ b) 1^2 c) $0^{\frac{7}{5}}$ d) $3^{7\div5}$ e) 3^2

2) Fill in the blanks:

a) $\frac{5^4y}{5^2} = 5^{\square}y = \square$

b) $\frac{2^6n^4}{4^1n^4} = \frac{2^6n^4}{2^{\square}n^4} = \square^{\square} = \square$

c) $\frac{8a^1}{2^4} = \frac{a}{\square}$

d) $\frac{27}{9b^3} = \frac{\square}{b^3}$

3) Look at the three questions below:

i) $\frac{4^3x^2}{4^2x^5} =$	ii) $\frac{64x^2}{16x^5} =$	iii) $\frac{2^6x^2}{2^4x^5} =$

a) Simplify all 3 questions.
b) Why do they give the same answer?

4) Match the questions with the answers. Each question has only one correct answer.

Questions
a) $\frac{18a^6}{3a^2}$
b) $6a^4 \div 3a^2$
c) $\frac{6a^4}{3a^4}$
d) $6a^4 - 3a^4$
e) $6a^4 \times 3a^2$

Answers	
i) 2	vi) 3
ii) $2a^4$	vii) $3a^4$
iii) $2a^2$	viii) $3a^2$
iv) $15a^4$	ix) $18a^2$
v) $6a^4$	x) $18a^6$

5) Simplify:

a) $\frac{6m^4n}{2m^3n}$ b) $\frac{5m^3n^2}{10nm^4}$ c) $\frac{2m^4n}{8n^4m^2}$ d) $\frac{2n^4m^1}{6n^4m^1}$

e) $\frac{2^6n^4m^3}{4^2m^4n^4}$ f) $\frac{7m^4n^2}{3m^6n}$ g) $3^2b^2.3b^3$ h) $3^2b^2 \div 3b^3$

Worksheet 2.1 b

Answers

Questions	Answers
1) Which of the following is the correct solution to $\frac{3^7}{3^5}$? a) $\frac{7}{5}$ b) 1^2 c) $0^{\frac{7}{5}}$ d) $3^{7\div5}$ e) 3^2	1) Correct solution: e) 3^2
2) Fill in the blanks: a) $\frac{5^4y}{5^2}=5^{\square}y=\square$ b) $\frac{2^6n^4}{4^1n^4}=\frac{2^6n^4}{2^{\square}n^4}=\square^{\square}=\square$ c) $\frac{8a^1}{2^4}=\frac{a}{\square}$ d) $\frac{27}{9b^3}=\frac{\square}{b^3}$	2) a) $\frac{5^4y}{5^2}=5^{\mathbf{2}}y=\mathbf{25y}$ b) $\frac{2^6n^4}{4^1n^4}=\frac{2^6n^4}{2^2n^4}=\mathbf{2^4}=\mathbf{16}$ c) $\frac{8a^1}{2^4}=\frac{a}{\mathbf{2}}$ d) $\frac{27}{9b^3}=\frac{\mathbf{3}}{b^3}$
3) Look at the three questions below: i) $\frac{4^3x^2}{4^2x^5}=$ ii) $\frac{64x^2}{16x^5}=$ iii) $\frac{2^6x^2}{2^4x^5}=$ a) Simplify all3 questions. b) Why do they give the same answer?	3) a) All have same answer: $\frac{4}{x^3}$ b) The numbers all reduce to 4, the letters reduce to$\frac{1}{x^3}$
4) Match Column A with Column B. Each question has only one correct answer. **Questions** a) $\frac{18a^6}{3a^2}$ b) $6a^4\div3a^2$ c) $\frac{6a^4}{3a^4}$ d) $6a^4-3a^4$ e) $6a^4\times3a^2$ **Answers** i) 2 vi) 3 ii) $2a^4$ vii) $3a^4$ iii) $2a^2$ viii) $3a^2$ iv) $15a^4$ ix) $18a^2$ v) $6a^4$ x) $18a^6$	4) **Questions** — **Answers** a) $\frac{18a^6}{3a^2}$ — v) $6a^4$ b) $6a^4\div3a^2$ — iii) $2a^2$ c) $\frac{6a^4}{3a^4}$ — i) 2 d) $6a^4-3a^4$ — vii) $3a^4$ e) $6a^4\times3a^2$ — x) $18a^6$
5) Simplify: a) $\frac{6m^4n}{2m^3n}$ b) $\frac{5m^3n^2}{10nm^4}$ c) $\frac{2m^4n}{8n^4m^2}$ d) $\frac{2n^4m^1}{6n^4m^1}$ e) $\frac{2^6n^4m^3}{4^2m^4n^4}$ f) $\frac{7m^4n^2}{3m^6n}$ g) $3^2b^2.3b^3$ h) $3^2b^2\div3b^3$	5) a) $3m$ b) $\frac{n}{2m}$ c) $\frac{m^2}{4n^3}$ d) $\frac{1}{3}$ e) $\frac{4}{m}$ f) $\frac{7n}{3m^2}$ g) $27b^5$ h) $\frac{3}{b}$

Worksheet 2.1 c

This worksheet focuses on dividing powers having exponents of zero.

Questions

1) Choose TRUE or FALSE for each statement. If FALSE, correct the part on the right of the equal sign.
   a) $\frac{x^4}{xy^3} = \frac{x^3}{y^3}$
   b) $\frac{2^4x^2}{2x^4} = \frac{1}{x}$
   c) $3b^2.3b^2 = b$
   d) $3b^2 \div 3b^2 = 1$

2) Fill in the blanks:
   a) $\frac{x^7}{\square} = x^4$
   b) $\frac{\square}{x^7} = \frac{1}{x^5}$
   c) $\frac{x^7}{x^7} = x^{\square}$

3) Look at the four questions below:

   i) $\frac{b^4}{b} =$ ii) $\frac{b^4}{b^2} =$ iii) $\frac{b^4}{b^3} =$ iv) $\frac{b^4}{b^4} =$

   a) What is the same in each question?
   b) What is different in each question?
   c) Simplify all the questions by expanding and cancelling.
   d) Write the answer to Q3iv) as a power.
   e) Use your answers to Q3c and Q3d to write the answer to Q3iv) in two ways.

4) Using the law for dividing powers, fill in the blanks:
   a) $\frac{x^5}{x^5} = x^{\square} = \square$
   b) $\frac{2^4a^3}{2^0} = \square a^3$

5) Simplify:

   a) $\frac{y^4x^3}{y^0x^0}$ b) $\frac{a^7y^2}{7y^2a}$ c) $\frac{2^5a^0y}{2^2a^3y}$ d) $\frac{9a^7y^5}{3^2ya^2}$ e) $\frac{5a^2y^5}{15y^0a^7}$

   f) $\frac{6^2y^7a^2}{6y^7a^5}$ g) $\frac{3y^7a^1}{9y^0a^3}$ h) $\frac{36a^2b^3}{6a^2b^0}$ i) $\frac{4n^4m^3}{4n^0m^0}$ j) $\frac{n^0.8\,n^4\,m^3}{n^0.n^3m^0}$

Worksheet 2.1 c

Answers

Questions	**Answers**
1) Choose TRUE or FALSE for each statement. If FALSE, correct the part on the right of the equal sign. a) $\frac{x^4}{xy^3} = \frac{x^3}{y^3}$ b) $\frac{2^4x^2}{2x^4} = \frac{1}{x}$ c) $3b^2.3b^2 = b$ d) $3b^2 \div 3b^2 = 1$	1) a) TRUE b) FALSE, $\frac{8}{x^2}$ c) FALSE, $9b^4$ d) TRUE
2) Fill in the blanks: a) $\frac{x^7}{\square} = x^4$ b) $\frac{\square}{x^7} = \frac{1}{x^5}$ c) $\frac{x^7}{x^7} = x^{\square}$	2) a) $\frac{x^7}{\boldsymbol{x^3}} = x^4$ b) $\frac{\boldsymbol{x^2}}{x^7} = \frac{1}{x^5}$ c) $\frac{x^7}{x^7} = x^{\mathbf{0}}$
3) Look at the four questions below: i) $\frac{b^4}{b} =$ ii) $\frac{b^4}{b^2} =$ iii) $\frac{b^4}{b^3} =$ iv) $\frac{b^4}{b^4} =$ a) What is the same in each question? b) What is different in each question? c) Simplify all the questions by expanding and cancelling. d) Write the answer to Q3iv) as a power. e) Use your answers to Q3c and Q3d to write the answer to Q3iv) in two ways.	3) a) All bases are b, all numerators have exponent of 4. Exponents of the denominators are less than or equal to 4. b) Exponents of the denominators are different. c) i) $\frac{b.b.b.\cancel{b}}{\cancel{b}} = b^3$ ii) $\frac{b.b.\cancel{b}.\cancel{b}}{\cancel{b}.\cancel{b}} = b^2$ iii) $\frac{b.\cancel{b}.\cancel{b}.\cancel{b}}{\cancel{b}.\cancel{b}.\cancel{b}} = b$ iv) $\frac{\cancel{b}.\cancel{b}.\cancel{b}.\cancel{b}}{\cancel{b}.\cancel{b}.\cancel{b}.\cancel{b}} = 1$ d) b^0 e) b^0 or 1
4) Using the law for dividing powers, fill in the blanks: a) $\frac{x^5}{x^5} = x^{\square} = \square$ b) $\frac{2^4a^3}{2^0} = \square a^3$	4) a) $\frac{x^5}{x^5} = x^{\mathbf{5-5}}$ or $= \boldsymbol{x^0} = \mathbf{1}$ b) $\frac{2^4a^3}{2^0} = \mathbf{16}a^3$
5) Simplify: a) $\frac{y^4x^3}{y^0x^0}$ b) $\frac{a^7y^2}{7y^2a}$ c) $\frac{2^5a^0y}{2^2a^3y}$ d) $\frac{9a^7y^5}{3^2ya^2}$ e) $\frac{5a^2y^5}{15y^0a^7}$ f) $\frac{6^2y^7a^2}{6y^7a^5}$ g) $\frac{3y^7a^1}{9y^0a^3}$ h) $\frac{36a^2b^3}{6a^2b^0}$ i) $\frac{4n^4m^3}{4n^0m^0}$ j) $\frac{n^0.8\,n^4\,m^3}{n^0.n^3m^0}$	5) a) x^3y^4 b) $\frac{a^6}{7}$ c) $\frac{8}{a^3}$ d) a^5y^4 e) $\frac{y^5}{3a^5}$ f) $\frac{6}{a^3}$ g) $\frac{y^7}{3a^2}$ h) $6b^3$ i) m^3n^4 j) $8m^3n$

Worksheet 2.2 a

This worksheet focuses on dividing powers having negative exponents, coefficients of 1 and a maximum of two variables.

Questions

1) Rewrite without a denominator:
   a) $\frac{a^3}{a^5}$ b) $\frac{b}{b^6}$ c) $\frac{1}{c}$ d) $\frac{d^{-2}e^{-3}}{de}$

2) Fill in the blanks and give answers with positive exponents:
   a) $\frac{a^2}{a^4} = a^{2-\square} = \frac{1}{a^{\square}}$
   b) $\frac{a^6}{a^{\square}} = a^{-6} = \frac{\square}{\square}$
   c) $a^{-8}.a = a^{\square} = \square$

3) Fiona remembers her teacher saying "when you divide powers, you subtract the exponents".
   Fiona simplified $\frac{m^4}{n^6}$ as follows: $\frac{m^4}{n^6} = \frac{m^{4-6}}{n} = \frac{m^{-2}}{n}$
   a) What has Fiona done wrong?
   b) Fiona has ignored an important part of the law for dividing powers. What has she ignored?

4) Rewrite with positive exponents:
   5) $\frac{x}{y^{-3}}$ $\frac{x^{-3}}{y^{-5}}$ 7) $\frac{xy^{-3}}{x^5}$

5) Fill in the blanks:
   a) $\frac{p^3}{\square} = p^{-4}$
   b) $\frac{p^3}{\square} = \frac{1}{p^4}$
   c) You should get the same answer for Q5a and Q5b. Explain why.

6) Rewrite without a denominator:
   a) $\frac{d}{d^4}$ b) $\frac{d^0}{d^4}$ c) $\frac{d^3}{d^6}$ d) $\frac{d^{-3}}{d^6}$ e) $\frac{d^{-3}}{d^{-6}}$

7) Simplify, give answers with positive exponents:
   a) $\frac{m^4.m^5}{m^{-6}}$
   b) $\frac{m^4}{m^{-5}.n^4}$
   c) $\frac{mn\,.\,mn}{m^4n^2}$
   d) $\frac{m.n.n^2}{m^{-1}n^4}$
   e) $\frac{m^{-1}}{m^{-2}} + m$
   f) $\frac{m^{-1}}{m^{-2}} \times m$

Worksheet 2.2 a

Answers

Questions	**Answers**
1) Rewrite without a denominator: a) $\frac{a^3}{a^5}$ b) $\frac{b}{b^6}$ c) $\frac{1}{c}$ d) $\frac{d^{-2}e^{-3}}{de}$	1) a) a^{-2} b) b^{-5} c) c^{-1} d) $d^{-3}e^{-4}$
2) Fill in the blanks and give answers with positive exponents: a) $\frac{a^2}{a^4} = a^{2-\square} = \frac{1}{a^{\square}}$ b) $\frac{a^6}{a^{\square}} = a^{-6} = \frac{\square}{\square}$ c) $a^{-8}.a = a^{\square} = \square$	2) a) $\frac{a^2}{a^4} = a^{2-\mathbf{4}} = \frac{1}{\boldsymbol{a^2}}$ b) $\frac{a^6}{a^{\mathbf{12}}} = a^{-6} = \frac{\mathbf{1}}{\boldsymbol{a^6}}$ c) $a^{-8}.a = a^{\mathbf{-7}} = \frac{\mathbf{1}}{\boldsymbol{a^7}}$
3) Fiona remembers her teacher saying "when you divide powers, you subtract the exponents". Fiona simplified $\frac{m^4}{n^6}$ as follows: $\frac{m^4}{n^6} = \frac{m^{4-6}}{n} = \frac{m^{-2}}{n}$ a) What has Fiona done wrong? b) Fiona has ignored an important part of the law for dividing powers. What has she ignored?	3) a) Fiona subtracted the exponents of different bases. b) "When you divide powers <u>with the same bases</u>, you subtract the exponents." The underlined part is what she has ignored.
4) Rewrite with positive exponents: a) $\frac{x}{y^{-3}}$ b) $\frac{x^{-3}}{y^{-5}}$ c) $\frac{xy^{-3}}{x^5}$	4) a) xy^3 b) $\frac{y^5}{x^3}$ c) $\frac{1}{x^4y^3}$
5) Fill in the blanks: a) $\frac{p^3}{\square} = p^{-4}$ b) $\frac{p^3}{\square} = \frac{1}{p^4}$ c) You should get the same answer for Q5a and Q5b. Explain why.	5) a) $\frac{p^3}{\boldsymbol{p^7}} = p^{-4}$ b) $\frac{p^3}{\boldsymbol{p^7}} = \frac{1}{p^4}$ c) $p^{-4} = \frac{1}{p^4}$ both represent the same result.
6) Rewrite without a denominator: a) $\frac{d}{d^4}$ b) $\frac{d^0}{d^4}$ c) $\frac{d^3}{d^6}$ d) $\frac{d^{-3}}{d^6}$ e) $\frac{d^{-3}}{d^{-6}}$	6) a) d^{-3} b) d^{-4} c) d^{-3} d) d^{-9} e) d^3
7) Simplify, give answers with positive exponents: a) $\frac{m^4.m^5}{m^{-6}}$ b) $\frac{m^4}{m^{-5}.n^4}$ c) $\frac{mn\ .\ mn}{m^4n^2}$ d) $\frac{m.n.n^2}{m^{-1}n^4}$ e) $\frac{m^{-1}}{m^{-2}} + m$ f) $\frac{m^{-1}}{m^{-2}} \times m$	7) a) m^{15} b) $\frac{m^9}{n^4}$ c) $\frac{1}{m^2}$ d) $\frac{m^2}{n}$ e) $2m$ f) m^2

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 2.2 b

This worksheet focuses on dividing powers having negative exponents, numerical and variable bases, and positive coefficients.

Questions

1) TRUE or FALSE? If FALSE, say why.
   - a) $\frac{d^4}{d^6} = d^{4-6}$
   - b) $\frac{4^3}{8^3} = \frac{1}{2^3}$
   - b) $\frac{3^8}{3^2} = 1^{8-2}$
   - c) $\frac{2a^4}{b^4} = \frac{a^4}{b^2}$

2) Simplify, write answers with positive exponents:

a) $\frac{b^3}{b^{-4}}$	b) $\frac{2}{2^3}$	c) $\frac{3b^{-4}}{b^6}$
d) $\frac{4b}{8b^{-4}}$	e) $\frac{b^{-1}}{b^{-4}}$	f) $\frac{ab^0}{a^3b^{-4}}$

3) Simplify the expression $\frac{3b.b^4.a^3}{3^4a.b^6.b^2}$ using the two different methods specified below. Write answers with positive exponents.
   - a) Expand the expression and then cancel.
   - b) Use the exponential laws to simplify the expression.

4) Simplify, write answers with positive exponents:
   - a) $\frac{8b^3}{2b^{-4}}$
   - b) $\frac{3b^{-4}}{9b^6}$
   - c) $\frac{4b^{-1}}{12b^{-4}}$
   - d) $\frac{6b^0}{3b^{-4}}$
   - e) $\frac{2ab^{-2}}{6a^5b^4}$

Notice that sometimes the numbering goes down, then across. Sometimes it, goes across and then down to the next row.

5) Simplify, write answers with positive exponents:

a) $x^{-10} \div x^{-13}$	b) $4^{-3} \div 2^6$	c) $3^4 \div \frac{1}{3^{-3}}$
d) $\frac{p^3}{p^4} \times \frac{p}{p^{-2}}$	e) $\frac{p.p^{-2}}{p^4.p^5}$	f) $\frac{12d.e^2}{4d^{-4}.e^5}$
g) $\frac{2ab^{-2} \times 6a^5b^4}{3a^5b^5}$	h) $x^{-10} + x^{-13} \div x^{-3}$	i) $x^{-10} \times x^{-13} \div x^{-3}$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 2.2 b

Answers

Questions	**Answers**
1) TRUE or FALSE? If FALSE, say why. a) $\frac{d^4}{d^6} = d^{4-6}$ c) $\frac{4^3}{8^3} = \frac{1}{2^3}$ b) $\frac{3^8}{3^2} = 1^{8-2}$ d) $\frac{2a^4}{b^4} = \frac{a^4}{b^2}$	1) a) TRUE b) FALSE, the base should remain 3 c) TRUE d) FALSE, seems that the exponent of the denominator has been “cancelled” with the coefficient of the numerator.
2) Simplify, write answers with positive exponents: a) $\frac{b^3}{b^{-4}}$ b) $\frac{2}{2^3}$ c) $\frac{3b^{-4}}{b^6}$ d) $\frac{4b}{8b^{-4}}$ e) $\frac{b^{-1}}{b^{-4}}$ f) $\frac{ab^0}{a^3b^{-4}}$	2) a) b^7 b) $\frac{1}{2^2} = \frac{1}{4}$ c) $\frac{3}{b^{10}}$ d) $\frac{b^5}{2}$ e) b^3 f) $\frac{b^4}{a^2}$
3) Simplify the expression $\frac{3b.b^4.a^3}{3^4a.b^6.b^2}$ using the two different methods specified below. Write answers with positive exponents. a) Expand the expression and then cancel. b) Use the exponential laws to simplify the expression.	3) a) $\frac{\cancel{3}.\cancel{b}.\cancel{b}.\cancel{b}.\cancel{b}.\cancel{b}.a.a.\cancel{a}}{\cancel{3}.3.3.3.\cancel{a}.b.b.b.\cancel{b}.\cancel{b}.\cancel{b}.\cancel{b}.\cancel{b}} = \frac{a^2}{3^3.b^3}$ b) $3^{1-4}a^{3-1}b^{1+4-6-2} = 3^{-3}a^2b^{-3} = \frac{a^2}{3^3b^3}$
4) Simplify, write answers with positive exponents: a) $\frac{8b^3}{2b^{-4}}$ b) $\frac{3b^{-4}}{9b^6}$ c) $\frac{4b^{-1}}{12b^{-4}}$ d) $\frac{6b^0}{3b^{-4}}$ e) $\frac{2ab^{-2}}{6a^5b^4}$	4) a) $4b^7$ b) $\frac{1}{3b^{10}}$ c) $\frac{b^3}{3}$ d) $2b^4$ e) $\frac{1}{3a^4b^6}$
5) Simplify, write answers with positive exponents: a) $x^{-10} \div x^{-13}$ b) $4^{-3} \div 2^6$ c) $3^4 \div \frac{1}{3^{-3}}$ d) $\frac{p^3}{p^4} \times \frac{p}{p^{-2}}$ e) $\frac{p.p^{-2}}{p^4.p^5}$ f) $\frac{12d.e^2}{4d^{-4}.e^5}$ g) $\frac{2ab^{-2}\times 6a^5b^4}{3a^5b^5}$ h) $x^{-10} + x^{-13} \div x^{-3}$ i) $x^{-10} \times x^{-13} \div x^{-3}$	5) a) x^3 b) $\frac{1}{2^{12}}$ c) 3^1 d) p^2 e) $\frac{1}{p^{10}}$ f) $\frac{3d^5}{e^3}$ g) $\frac{4a}{b^3}$ h) $\frac{2}{x^{10}}$ i) $\frac{1}{x^{20}}$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 2.2 c

This worksheet focuses on dividing powers having negative and positive numerical exponents, numerical and variable bases, and negative and fraction coefficients.

Questions

1) State whether the following are TRUE or FALSE. If FALSE, give the correct answer.

a) $\frac{3^5}{3^5} = 3^0$ b) $1 = \frac{3^5}{3^5}$ c) $3^{-1} = -3$

d) $3(-1) = -3$ e) $3^{-2} = 3 \div (-2)$ f) $3^{-2} = \frac{1}{9}$

2) Compare $\frac{3}{3^{-4}}$ and $\frac{-3}{3^{-4}}$

a) What is the same?

b) What is different?

c) Simplify $\frac{3}{3^{-4}}$

3) Lindi and Shawn get the same answer for $\frac{-3}{3^{-4}}$ from Q2. They use different methods which are both mathematically correct.

Lindi's method: $\frac{-3}{3^{-4}} = -\frac{3^1}{3^{-4}} = -3^{1-(-4)} = -3^5$

Shawn's method: $\frac{-3}{3^{-4}} = \frac{(-1)3}{3^{-4}} = (-1)(3^{1+4}) = -3^5$

a) What is the difference between their methods?
b) Whose method do you prefer and why?
c) Simplify the following using the method you prefer:

i) $\frac{5}{-5^{-3}}$ ii) $\frac{-a}{a^{-3}}$

4) Simplify, give answers with positive exponents:

a) $\frac{-b}{b^{-4}}$ b) $\frac{-4b^{-1}}{b^{-4}}$ c) $\frac{-4b^{-1}}{4b^{-4}}$ d) $\frac{3ab^{-4}}{-9b^6}$ e) $\frac{-8a^4b^3}{2b^{-4}}$

5) Fill in the blanks:

a) $\frac{-21a}{\square} = \square a^{-3}$

b) $\frac{-21a^{-7}}{a^{-3}} = \square$

c) $\frac{-1a^{-3}b}{2a^{10}b^2} = -\frac{1}{2}\square$

d) $\frac{-18x^{\square}y^{\square}x^{-5}}{\square y} = \frac{-3}{x^2y}$

e) $\frac{3r^{-2}}{p^3} \times \frac{\square p^{\square}r^{\square}}{\square} = \frac{-18r^5}{4p}$

Worksheet 2.2 c

Answers

Questions	**Answers**
1) State whether the following are TRUE or FALSE. If FALSE, give the correct answer. a) $\frac{3^5}{3^5} = 3^0$ b) $1 = \frac{3^5}{3^5}$ c) $3^{-1} = -3$ d) $3(-1) = -3$ e) $3^{-2} = 3 \div (-2)$ f) $3^{-2} = \frac{1}{9}$	1) a) TRUE b) TRUE c) FALSE; $\frac{1}{3}$ d) TRUE e) FALSE; $\frac{1}{3^2}$ f) TRUE
2) Compare $\frac{3}{3^{-4}}$ and $\frac{-3}{3^{-4}}$ a) What is the same? b) What is different? c) Simplify $\frac{3}{3^{-4}}$	2) a) Both denominators are the same b) The numerators are different c) $3^{1-(-4)} = 3^5$
3) Lindi and Shawn get the same answer for $\frac{-3}{3^{-4}}$ from Q2. They use different methods which are both mathematically correct. Lindi's method: $\frac{-3}{3^{-4}} = -\frac{3^1}{3^{-4}} = -3^{1-(-4)} = -3^5$ Shawn's method: $\frac{-3}{3^{-4}} = \frac{(-1)3}{3^{-4}} = (-1)(3^{1+4}) = -3^5$ a) What is the difference between their methods? b) Whose method do you prefer and why? c) Simplify the following using the method you prefer: iii) $\frac{5}{-5^{-3}}$ iv) $\frac{-a}{a^{-3}}$	3) a) Lindi dealt with the division of a negative first, whereas Shawn factored out a -1 from -3. b) Learner should justify preference c) Lindi's Method i) $\frac{5}{-5^{-3}} = -\frac{5^1}{5^{-3}} = -5^{1-(-3)} = -5^4$ ii) $\frac{-a}{a^{-3}} = -\frac{a^1}{a^{-3}} = -a^{1-(-3)} = -a^4$ Shawn's Method i) $\frac{5}{-5^{-3}} = \frac{5}{(-1)5^{-3}} = (-1)(5^{1+3}) = -5^4$ ii) $\frac{-a}{a^{-3}} = \frac{(-1)a}{a^{-3}} = (-1)(a^{1+3}) = -a^4$
4) Simplify, give answers with positive exponents: a) $\frac{-b}{b^{-4}}$ b) $\frac{-4b^{-1}}{b^{-4}}$ c) $\frac{-4b^{-1}}{4b^{-4}}$ d) $\frac{3ab^{-4}}{-9b^6}$ e) $\frac{-8a^4b^3}{2b^{-4}}$	4) a) $-b^5$ b) $-4b^3$ c) $-b^3$ d) $\frac{a}{-3b^{10}}$ e) $-4a^4b^7$
5) Fill in the blanks: a) $\frac{-21a}{\square} = \square a^{-3}$ b) $\frac{-21\mathrm{a}^{-7}}{a^{-3}} = \square$ c) $\frac{-1a^{-3}b}{2a^{10}b^2} = -\frac{1}{2}\square$ d) $\frac{-18x^{\square}y^{\square}x^{-5}}{\square y} = \frac{-3}{x^2y}$ e) $\frac{3r^{-2}}{p^3} \times \frac{\square p^{\square}r^{\square}}{\square} = \frac{-18r^5}{4p}$	5) a) $\frac{-21a}{\boldsymbol{a^4}} = \mathbf{-21}a^{-3}$ b) $\frac{-21\mathrm{a}^{-7}}{a^{-3}} = \boldsymbol{-21a^{-4}}$ c) $\frac{-1a^{-3}b}{2a^{10}b^2} = -\frac{1}{2}\boldsymbol{a^{-13}b^{-1}}$ d) $\frac{-18x^{\mathbf{3}}y^{\mathbf{0}}x^{-5}}{\mathbf{6}y} = \frac{-3}{x^2y}$ e) $\frac{3r^{-2}}{p^3} \times \frac{\boldsymbol{-6p^2r^7}}{\mathbf{4}} = \frac{-18r^5}{4p}$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 3.1 a

This worksheet focuses on raising a power to a further power when exponents are positive, and bases are numbers or single variables.

Questions

1) Give the base and exponent for each power:
   a) 3^2
   b) $(3^2)^4$
   c) a^m
   d) $(a^m)^n$

2) Expand the following powers. Q2a is done for you.
   a) $8 = 2^3 = 2 \times 2 \times 2$ b) d^4 c) 4^2 d) w^5

3) Write the expanded forms as powers. Q3a is done for you.
   a) $7 \times 7 \times 7 \times 7 = 7^4$ b) $k \times k \times k$ c) $a \times a \times a \times a \times a$ d) $v \times v$

4) Six powers are given in the table below:

i) 5^4	ii) $(5^3)^1$
iii) $(5^2)^2$	iv) $(5^4)^1$
v) $5^3 \times 5$	vi) $(5^1)^4$

   a) Expand the powers.
   b) List the powers that have the same expansion.

5) TRUE or FALSE? If the statement is FALSE, correct the part on the right of the equal sign.
   a) $3^2 = 3 \times 3$
   b) $3 \times 3 \times 3 = (3^2)^3$
   c) $(n \times n)^2 = n^2$
   d) $(3^3)^2 = (3 \times 3 \times 3)^2$
   e) $(b^5)^2 = b^7$

6) Which of the following represents $(2^3)^2$?
   a) $(2 \times 2 \times 2)^2$ b) $(2 \times 3 \times 2)$ c) $(2 \times 3)^2$ d) $(2 \times 2)^2$

7) Which of the following DO NOT represent $(a^4\ )^3$?
   a) $(a \times a \times a \times a)^4$ b) $(a \times a \times a)^4$ c) $(a \times a \times a \times a)^3$
   d) $(a^2 \times a^2)^3$ e) $(a)^{12}$ f) $(a)^7$

8) Fill in the blanks:
   a) $(g^2)^{\square} = g^8$
   b) $(\square)^3 = d \times d \times d$
   c) $r^6 = r^{\square} \times r^2$
   d) $k^{\square} = (k^7)^2$
   e) $(m^9)^1 = m^{\square}$
   f) $y^9 = \left(y^{\square}\right)^3$

Worksheet 3.1 a

Answers

Questions	**Answers**
1) Give the base and exponent for each power: a) 3^2 b) $(3^2)^4$ c) a^m d) $(a^m)^n$	1) a) Base: 3, exponent: 2 b) Base: 3^2, exponent: 4 c) Base: a, exponent: m d) Base: a^m, exponent: n
2) Expand the following powers. Q2a is done for you. a) $8 = 2^3 = 2 \times 2 \times 2$ b) d^4 c) 4^2 d) w^5	2) b) $d^4 = d \times d \times d \times d$ c) $4^2 = 4 \times 4$ d) $w^5 = w \times w \times w \times w \times w$
3) Write the expanded forms as powers. Q3a is done for you. a) $7 \times 7 \times 7 \times 7 = 7^4$ b) $k \times k \times k$ c) $a \times a \times a \times a \times a$ d) $v \times v$	3) b) $k \times k \times k = k^3$ c) $a \times a \times a \times a \times a = a^5$ d) $v \times v = v^2$
4) Six powers are given in the table below: i) 5^4 ii) $(5^3)^1$ iii) $(5^2)^2$ iv) $(5^4)^1$ v) $5^3 \times 5$ vi) $(5^1)^4$ c) Expand the powers. d) List the powers that have the same expansion.	4) a) i) $5 \times 5 \times 5 \times 5$ ii) $(5 \times 5 \times 5)$ iii) $(5 \times 5) \times (5 \times 5)$ iv) $(5 \times 5 \times 5 \times 5)$ v) $5 \times 5 \times 5 \times 5$ vi) $(5) \times (5) \times (5) \times (5)$ b) 5^4; $(5^2)^2$; $5^3 \times 5$; $(5^4)^1$; $(5^1)^4$
5) TRUE or FALSE? If the statement is FALSE, correct the part on the right of the equal sign. a) $3^2 = 3 \times 3$ b) $3 \times 3 \times 3 = (3^2)^3$ c) $(n \times n)^2 = n^2$ d) $(3^3)^2 = (3 \times 3 \times 3)^2$ e) $(b^5)^2 = b^7$	5) a) TRUE b) FALSE; 3^3 c) FALSE; n^4 d) TRUE e) FALSE; b^{10}
6) Which of the following represents $(2^3)^2$? a) $(2 \times 2 \times 2)^2$ b) $(2 \times 3 \times 2)$ c) $(2 \times 3)^2$ d) $(2 \times 2)^2$	6) a) $(2 \times 2 \times 2)^2$
7) Which of the following DO NOT represent $(a^4)^3$? a) $(a \times a \times a \times a)^4$ b) $(a \times a \times a)^4$ c) $(a \times a \times a \times a)^3$ d) $(a^2 \times a^2)^3$ e) $(a)^{12}$ f) $(a)^7$	7) Q7a $(a \times a \times a \times a)^4$ and Q7f $(a)^7$
8) Fill in the blanks: a) $(g^2)^{\square} = g^8$ b) $(\square)^3 = d \times d \times d$ c) $r^6 = r^{\square} \times r^2$ d) $k^{\square} = (k^7)^2$ e) $(m^9)^1 = m^{\square}$ f) $y^9 = (y^{\square})^3$	8) a) $(g^2)^{\mathbf{4}} = g^8$ b) $(\boldsymbol{d})^3 = d \times d \times d$ c) $r^6 = r^{\mathbf{4}} \times r^2$ d) $k^{\mathbf{14}} = (k^7)^2$ e) $(m^9)^1 = m^{\mathbf{9}}$ f) $y^9 = (y^{\mathbf{3}})^3$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 3.1 b

This worksheet focuses on raising a power to a further power where powers have positive coefficients and exponents of 0 are included.

Questions

1) Look at the terms below:

i) $6a^c$	ii) $18(a^b)^c$

   a) Give the coefficient of each term.
   b) Give the base for exponent c in each term.

2) Expand the following powers:
   a) $9g^4$ b) $(3)^2(g)^2$ c) $9(g^2)^2$ d) 3^2g^4
   e) Which questions give the same expansion?

3) Complete. (Q3a has been done for you)
   a) $(n^3)^2 = n^{3\times 2} = n^6$ b) $(n^2)^3 = \_\_ = \_\_$ c) $3(n^2)^3 = \_\_$
   d) $(n^3)^0 = \_\_$ e) $2(n)^0 = \_\_$ f) $5(n^3)^0 = \_\_$

4) Which of the following represents $2(x^3)^2$?
   a) $4x^6$ b) x^3 c) $4x^3$ d) $2x^5$ e) $2x^6$

5) Fill in the blanks:
   a) $4 \times m^{2\times 5} = 4\left(\square^{\square}\right)^{\square}$
   b) $7 \times a^4 \times a^4 \times a^4 = 7\left(\square^{\square}\right)^{\square} = 7(\square)^{\square}$
   c) $150 = 2 \times 3 \times \square = 6 \times \square^{\square}$
   d) $9(\square)^2 = \square^2.x^2 = (\square\square)^2$
   e) $(b.b^3)^{\square} = \left(b^{\square}\right)^{\square} = b^{12}$
   f) $\left(n^{\square}\right)^0.\square.\square = 12n$

6) Which power has the greatest value if $y = 3$?
   a) $6y^2$
   b) $3^2(y)^2$
   c) $2^3(y^2)^3$
   d) $2^2(y^2)^2$
   e) $12(y)^0$

7) Simplify:
   a) $(b^2)^0(a^4)^2$ b) $2^4(b^4)^3$ c) $2(ab^2)^0.(b^4)^2$
   d) $\frac{9d^5}{3(d^3)^2}$ e) $\frac{3(d^4)^2}{9(d^3)^2}$ f) $\frac{8e^3.2(e^2)^4}{2^4(e^2)^5}$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 3.1 b

Answers

Questions	**Answers**
1) Look at the terms below: i) $6a^c$ ii) $18(a^b)^c$ a) Give the coefficient of each term. b) Give the base for exponent c in each term.	1) a) i) 6 ii) 18 b) i) a ii) a^b
2) Expand the following powers: a) $9g^4$ b) $(3)^2(g)^2$ c) $9(g^2)^2$ d) 3^2g^4 e) Which questions give the same expansion?	2) a) $3.3.g.g.g.g$ b) $3.3.g.g$ c) $3.3.g.g.g.g$ d) $3.3.g.g.g.g$ e) Q2a, Q2c and Q2d
3) Complete. Q3a has been done for you. a) $(n^3)^2 = n^{3\times2} = n^6$ b) $(n^2)^3 = \_\_ = \_\_$ c) $3(n^2)^3 = \_\_$ d) $(n^3)^0 = \_\_$ e) $2(n)^0 = \_\_$ f) $5(n^3)^0 = \_\_$	3) a) $(n^3)^2 = n^6$ b) $(n^2)^3 = \boldsymbol{n^{2\times3}} = \boldsymbol{n^6}$ c) $3(n^2)^3 = \mathbf{3}\boldsymbol{n^6}$ d) $(n^3)^0 = \mathbf{1}$ e) $2(n)^0 = \mathbf{2}$ f) $5(n^3)^0 = \mathbf{5}$
4) Which of the following represents $2(x^3)^2$? a) $4x^6$ b) x^3 c) $4x^3$ d) $2x^5$ e) $2x^6$	4) e) $2x^6$
5) Fill in the blanks: a) $4 \times m^{2\times5} = 4(\Box^{\Box})^{\Box}$ b) $7 \times a^4 \times a^4 \times a^4 = 7(\Box^{\Box})^{\Box} = 7(\Box)^{\Box}$ c) $150 = 2 \times 3 \times \Box = 6 \times \Box^{\Box}$ d) $9(\Box)^2 = \Box^2.x^2 = (\Box\Box)^2$ e) $(b.b^3)^{\Box} = (b^{\Box})^{\Box} = b^{12}$ f) $(n^{\Box})^0.\Box.\Box = 12n$	5) a) $4 \times m^{2\times5} = 4(\boldsymbol{m^2})^{\mathbf{5}}$ b) $7 \times a^4 \times a^4 \times a^4 = 7(\boldsymbol{a^4})^{\mathbf{3}} = 7(\boldsymbol{a})^{\mathbf{12}}$ c) $150 = 2 \times 3 \times \mathbf{25} = 6 \times \mathbf{5^2}$ d) $9(\boldsymbol{x})^2 = \mathbf{3}^2.x^2 = (\mathbf{3}\boldsymbol{x})^2$ e) $(b.b^3)^{\mathbf{3}} = (b^{\mathbf{4}})^{\mathbf{3}} = b^{12}$ f) Many possible choices, e.g. $(n^{\mathbf{3}})^0.\mathbf{4}\boldsymbol{n}.\mathbf{3} = 12n$ $(n^{\mathbf{8}})^0.\mathbf{24} \times \mathbf{0{,}5}\boldsymbol{n} = 12n$
6) Which power has the greatest value if $y = 3$? a) $6y^2$ b) $3^2(y)^2$ c) $2^3(y^2)^3$ d) $2^2(y^2)^2$ e) $12(y)^0$	6) a) $6(3)^2 = 54$ b) $3^2(3)^2 = 81$ c) $\mathbf{2^3(3^2)^3 = 5832}$ d) $2^2(3^2)^2 = 324$ e) $12(3)^0 = 12$
7) Simplify: a) $(b^2)^0(a^4)^2$ b) $2^4(b^4)^3$ c) $2(ab^2)^0.(b^4)^2$ d) $\frac{9d^5}{3(d^3)^2}$ e) $\frac{3(d^4)^2}{9(d^3)^2}$ f) $\frac{8e^3.2(e^2)^4}{2^4(e^2)^5}$	7) a) a^8 b) $16b^{12}$ c) $2b^8$ d) $\frac{3}{d}$ e) $\frac{d^2}{3}$ f) e

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 3.1 c

This worksheet focuses on raising a power to a further power where powers have negative coefficients.

Questions

1) Look at the four options below. Which of them expands as $-n \times -n \times -n \times -n$?
   a) $-n^4$ b) $(-n)^4$ c) $-4n$ d) n^4

2) Show that the powers below give the same answer.
   a) $((-x)^2)^3$
   b) $(-x \times -x \times -x)^2$
   c) $(-x \times -x)^3$

3)
   a) Multiply $(-3)^2$ by itself three times. What is the answer?
   b) Multiply -3^2 by itself three times. What is the answer?
   c) What do you notice about the signs of the answers in Q3a and Q3b?
   d) Why does this happen?

4) Consider the expressions in the box:

   A. $-n^6$
   B. $(-n^3)^2$
   C. $((-n)^3)^2$
   D. $(-n^2)^3$
   E. $-(n^3)^2$

   a) Predict which expressions will give the same answer.
   b) Simplify the expressions.
   c) Were your predictions correct?
   d) How is expression B different from expression E?

5) Simplify:
   a) $-(-5)^2$
   b) $((-b)^5)^2$
   c) $(-b \times b)^2$
   d) $(b^2)^0.b$
   e) $b.(b^0)^2$
   f) $(x^2)^3 \times (x^3)^2$
   g) $(x^2)^3 + (x^3)^2$
   h) $-(n^9)^0(-n^3)^4$
   i) $(-m^2)^1(-m^1)^2$

#superpowers

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 3.1 c

Answers

Questions	**Answers**
1) Look at the four options below. Which of them expands as $-n \times -n \times -n \times -n$? a) $-n^4$ b) $(-n)^4$ c) $-4n$ d) n^4	1) b) $(-n)^4$
2) Show that the powers below give the same answer. a) $((-x)^2)^3$ b) $(-x \times -x \times -x)^2$ c) $(-x \times -x)^3$	2) a) $((-x)^2)^3 = (-x)^2.(-x)^2.(-x)^2 = x^2.x^2.x^2 = x^6$ or $((-x)^2)^3 = x^6$ b) $(-x \times -x \times -x)^2 = (-x^3) \times (-x^3) = x^6$ c) $(-x \times -x)^3 = x^2.x^2.x^2 = x^6$ or $(x^2)^3 = x^6$
3) a) Multiply$(-3)^2$ by itself three times. What is the answer? b) Multiply -3^2 by itself three times. What is the answer? c) What do you notice about the signs of the answers in Q3a and Q3b? d) Why does this happen?	3) a) $(-3)^2.(-3)^2.(-3)^2 = 9.9.9 = 729$ b) $-3^2.-3^2.-3^2 = -9.-9.-9 = -729$ c) Signs of the answers are different d) In Q3a the negative is squared to give a positive number. In Q3b the number 3^2 is multiplied by -1.
4) Consider the expressions in the box: A. $-n^6$ B. $(-n^3)^2$ C. $((-n)^3)^2$ D. $(-n^2)^3$ E. $-(n^3)^2$ a) Predict which expressions will give the same answer. b) Simplify the expressions. c) Were your predictions correct? d) How is expression B different from expression E?	4) a) Same answer: A, D and E B and C b) B. $(-n^3)^2 = (-n)^3.(-n)^3 = -n^3.-n^3 = n^6$ C. $((-n)^3)^2 = (-n^3)^2 = n^6$ D. $(-n^2)^3 = -n^{2\times 3} = -n^6$ E. $-(n^3)^2 = -n^{2\times 3} = -n^6$ c) Depends on learners' predictions d) In B we get $(-n^3)(-n^3)$ so we have "negative times negative" which gives a positive product. In E, the coefficient of -1 is multiplied by $(n^3)^2$ so it is not squared.
5) Simplify: a) $-(-5)^2$ b) $((-b)^5)^2$ c) $(-b \times b)^2$ d) $(b^2)^0.b$ e) $b.(b^0)^2$ f) $(x^2)^3 \times (x^3)^2$ g) $(x^2)^3 + (x^3)^2$ h) $-(n^9)^0(-n^3)^4$ i) $(-m^2)^1(-m^1)^2$	5) a) -25 b) b^{10} c) b^4 d) b e) b f) x^{12} g) $2x^6$ h) $-n^{12}$ i) $-m^4$

Worksheet 3.2 a

This worksheet focuses on a power raised to a further power which has positive and negative exponents, and coefficients of 1.

Questions

1) Fill in the blanks:
   a) $(a^3)^2 = a^3.\square = a^{\square}$
   b) $\frac{1}{a^3.a^3} = \frac{1}{(a^{\square})^2} = (a^{\square})^2 = a^{\square}$
   c) $(3^{-1})^2 = \left(\frac{1}{\square}\right)^2 = \frac{\square}{\square}.\frac{\square}{\square} = 3^{\square}$

2) Write bases with positive exponents, then expand and write answers in simplest form.
   a) $(3^{-2})^3$ b) $(2^{-3})^2$ c) $(a^{-5})^2$ d) $(b^{-2})^5$

3)
   a) Write with negative exponents then simplify.
      i) $(5^2)^{-1}$
      ii) $(5^{-1})^2$
   b) What do you notice about the answers?
   c) Why do you think this is so?

   d) Determine the value of the following when $a = 3$.
      i) $(a^5)^{-2}$
      ii) $(a^{-2})^5$
   e) Will Q3di) and Q3dii) always have the same values if a is natural number? Why?

4) Group the powers that give the same answer.
   a) 5^{-5} b) $(5^{-3})^2$ c) $5^{-3} \times 5^{-2}$
   d) $(a^{-3})^2$ e) $(a^5)^{-1}$ f) $\left(\frac{1}{a}\right)^6$

5) Which of the following is the same as $(2^3)^{-2}$?
   a) $(2 \times 2 \times 2)^2$
   b) $(2 \times 2 \times 2)^{-2}$
   c) $\left(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}\right)^2$
   d) $(2 \times 3)^2$
   e) $(2 \times 3 \times -2)$
   f) $(2^{-1} \times 2^{-1})^3$
   g) $(2)^{-6}$

Worksheet 3.2 a

Answers

Questions	Answers
1) Fill in the blanks: a) $(a^3)^2$ $= a^3.\square$ $= a^{\square}$ b) $\frac{1}{a^3.a^3}$ $= \frac{1}{(a^{\square})^2}$ $= \left(a^{\square}\right)^2 = a^{\square}$ c) $(3^{-1})^2$ $= \left(\frac{1}{\square}\right)^2 = \frac{\square}{\square}.\frac{\square}{\square} = 3^{\square}$	1) a) $(a^3)^2 = a^3.\boldsymbol{a^3} = a^{\boldsymbol{6}}$ b) $\frac{1}{a^3.a^3} = \frac{1}{(a^3)^2} = (a^{\boldsymbol{-3}})^2 = a^{\boldsymbol{-6}}$ c) $(3^{-1})^2 = \left(\frac{1}{\mathbf{3}}\right)^2 = \frac{\mathbf{1}}{\mathbf{3}}.\frac{\mathbf{1}}{\mathbf{3}} = 3^{\mathbf{-2}}$
2) Write bases with positive exponents, then expand and write answers in simplest form. a) $(3^{-2})^3$ b) $(2^{-3})^2$ c) $(a^{-5})^2$ d) $(b^{-2})^5$	2) a) $(3^{-2})^3 = \left(\frac{1}{3^2}\right)^3 = \frac{1}{9}\times\frac{1}{9}\times\frac{1}{3} = \frac{1}{27}$ b) $(2^{-3})^2 = \left(\frac{1}{2^3}\right)^2 = \frac{1}{8}\times\frac{1}{8} = \frac{1}{64}$ c) $(a^{-5})^2 = \left(\frac{1}{a^5}\right)^2 = \frac{1}{a^5}.\frac{1}{a^5} = \frac{1}{a^{10}}$ d) $(b^{-2})^5 = \left(\frac{1}{b^2}\right)^5 = \frac{1}{b^2}.\frac{1}{b^2}.\frac{1}{b^2}.\frac{1}{b^2}.\frac{1}{b^2} = \frac{1}{b^{10}}$
3) a) Write with negative exponents then simplify. i) $(5^2)^{-1}$ ii) $(5^{-1})^2$ b) What do you notice about the answers? c) Why do you think this is so? d) Determine the value of the following when $a = 3$. iii) $(a^5)^{-2}$ iv) $(a^{-2})^5$ e) Will Q3di) and Q3dii) always have the same values if a is natural number? Why?	3) a) i) $(5^2)^{-1} = 5^{-2} = \frac{1}{25}$ ii) $(5^{-1})^2 = 5^{-2} = \frac{1}{25}$ b) Answers are the same. c) Using law 3, we get $(2)(-1) = -2$ and $(-1)(2) = -2$. So we have the same base 5 raised to the power of -2. d) i) $(a^5)^{-2} = (3^5)^{-2} = 3^{-10} = \frac{1}{3^{10}}$ ii) $(a^{-2})^5 = (3^{-2})^5 = 3^{-10} = \frac{1}{3^{10}}$ e) Yes, because the same base a is raised to the power of -10 for both Q3di) and Q3dii).
4) Group the powers that give the same answer. a) 5^{-5} b) $(5^{-3})^2$ c) $5^{-3}\times 5^{-2}$ d) $(a^{-3})^2$ e) $(a^5)^{-1}$ f) $\left(\frac{1}{a}\right)^6$	4) Q4a) and c) give $\frac{1}{5^5}$ Q4d) and f) give $\frac{1}{a^6}$
5) Which of the following is the same as $(2^3)^{-2}$? a) $(2\times 2\times 2)^2$ b) $(2\times 2\times 2)^{-2}$ c) $\left(\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\right)^2$ d) $(2\times 3)^2$ e) $(2\times 3\times -2)$ f) $(2^{-1}\times 2^{-1})^3$ g) $(2)^{-6}$	5) Q5b) c) f) and g) because $(2^3)^{-2} = (2^{-2})^3$

Worksheet 3.2 b

This worksheet focuses on a power raised to a further power having integer exponents, and positive and negative coefficients.

Questions

1) Wanda and Shikha show that $(a^{-1})^{-1} = a$ using different methods that are both correct.

Wanda's method: $(a^{-1})^{-1} = a^{-1\times-1} = a^1$ or a.

Shikha's method: $(a^{-1})^{-1} = \frac{1}{a^{-1}} = a$ or a^1

   a) Describe each method in words. Focus on what is different about the methods.
   b) Which method do you prefer? Why?
   c) Simplify the two examples below using the method you prefer. Give answers with positive exponents:
      i) $(5^{-1})^{-4}$
      ii) $(2^{-5})^{-2}$

2) Fill in the blanks:
   a) $(2^3)^{-2} = \square = \square$
   b) $(n^{-3})^{-2} = \square$
   c) $\square . 7^{-2} = 7^{-4} = \left(\frac{1}{7}\right)^{\square}$
   d) $(y^2)^{-3}.y^{\square} = \frac{1}{y^8}$
   e) $(p^0)^{-3}.p^{-3} = \square$
   f) $2(n^2)^{-2} = \square = \square$

3) Match the expansions with the powers.

Expansion	Power
a) $2y \times 2y$	i) 2^2y^{-2}
b) $\frac{2}{y} \times \frac{2}{y}$	ii) $(y^2)^2$
c) $y \times y \times y \times y$	iii) 2^2y^2
d) $\frac{1}{y^{-2}} \times \frac{1}{y^{-2}} \times \frac{1}{y^{-2}} \times \frac{1}{y^{-2}}$	iv) $(y^{-2})^4$
	v) $(y^4)^1$

4) Look at the questions below. (r has the same positive value in each question).

i) $6r^{-3}$ ii) $3^{-2}r^{-3}$ iii) $(3^2)^2(r^{-2})^2$ iv) $(3^2)^{-3}(r^2)^2$ v) $(12\,(r^{-3})^0)^{-1}$

   a) Rewrite each question in simplest form with positive exponents.
   b) If $r = 2$ in each question, work out the value of each question.
   c) Which power has the largest value?
   d) Why?

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 3.2 b

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1) Wanda and Shikha show that $(a^{-1})^{-1} = a$ using different methods that are both correct.
Wanda's method: $(a^{-1})^{-1} = a^{-1\times-1} = a^1$ or a.
Shikha's method: $(a^{-1})^{-1} = \frac{1}{a^{-1}} = a$ or a^1
a) Describe each method in words. Focus on what is different about the methods.
b) Which method do you prefer? Why?
c) Simplify the two examples below using the method you prefer. Give answers with positive exponents:
i) $(5^{-1})^{-4}$ ii) $(2^{-5})^{-2}$</td><td>1)
a) Wanda removes the brackets by multiplying the exponents. Shika writes the part in brackets with a positive exponent and gets $\frac{1}{a^{-1}}$ then writes $\frac{1}{a^{-1}}$ with a positive exponent.
b) Depends on the learner.
c) Method depends on learner
i) 5^4 ii) 2^{10}</td></tr>
<tr><td>2) Fill in the blanks:
a) $(2^3)^{-2} = \square = \square$ b) $(n^{-3})^{-2} = \square$ c) $\square . 7^{-2} = 7^{-4} = \left(\frac{1}{7}\right)^{\square}$
d) $(y^2)^{-3} . y^{\square} = \frac{1}{y^8}$ e) $(p^0)^{-3} . p^{-3} = \square$ f) $2(n^2)^{-2} = \square = \square$</td><td>2)
a) $(2^3)^{-2} = \mathbf{2^{-6}} = \frac{\mathbf{1}}{\mathbf{2^6}}$ b) $(n^{-3})^{-2} = \boldsymbol{n^6}$ c) $\mathbf{7^{-2}} . 7^{-2} = 7^{-4} = \left(\frac{1}{7}\right)^{\mathbf{4}}$
d) $(y^2)^{-3} . y^{\mathbf{-2}} = \frac{1}{y^8}$ e) $(p^0)^{-3} . p^{-3} = \frac{\mathbf{1}}{\boldsymbol{p^3}}$ f) $2(n^2)^{-2} = \boldsymbol{2n^{-4}} = \frac{\mathbf{2}}{\boldsymbol{n^4}}$</td></tr>
<tr><td>3) Match the expansions with the powers.
<table>
<tr><th>Expansion</th><th>Power</th></tr>
<tr><td>a) $2y \times 2y$
b) $\frac{2}{y} \times \frac{2}{y}$
c) $y \times y \times y \times y$
d) $\frac{1}{y^{-2}} \times \frac{1}{y^{-2}} \times \frac{1}{y^{-2}} \times \frac{1}{y^{-2}}$</td><td>i) $2^2 y^{-2}$
ii) $(y^2)^2$
iii) $2^2 y^2$
iv) $(y^2)^4$
v) $(y^4)^1$</td></tr>
</table></td><td>3)
<table>
<tr><th>Expansion</th><th>Power</th></tr>
<tr><td>a)</td><td>iii)</td></tr>
<tr><td>b)</td><td>i)</td></tr>
<tr><td>c)</td><td>ii) or v)</td></tr>
<tr><td>d)</td><td>iv)</td></tr>
</table></td></tr>
<tr><td>4) Look at the questions below. (r has the same positive value in each question).
i) $6r^{-3}$ ii) $3^{-2}r^{-3}$ iii) $(3^2)^2(r^{-2})^2$ iv) $(3^2)^{-3}(r^2)^2$ v) $(12\,(r^{-3})^0)^{-1}$
a) Rewrite each question in simplest form with positive exponents.
b) If $r = 2$ in each question, work out the value of each question.
c) Which power has the largest value?
d) Why?</td><td>4)
a) i) $\frac{6}{r^3}$ ii) $\frac{1}{9r^3}$ iii) $\frac{81}{r^4}$ iv) $\frac{r^4}{3^6}$ v) $\frac{1}{12}$
b) i) $\frac{6}{8} = \frac{3}{4}$ ii) $\frac{1}{72}$ iii) $\frac{81}{16}$ iv) $\frac{16}{729}$ v) $\frac{1}{12}$
c) iii) has the largest value
d) $\frac{81}{16}$ is the only value greater than 1</td></tr>
</table>

Worksheet 3.2 c

This worksheet focuses on a power raised to a further power having positive and negative bases and exponents, and positive, negative and fraction coefficients.

Questions

1) Use one of these signs $<$; $=$; $>$ to show the relationship between each pair of numbers.
   Give a reason for each answer
   e.g. $((-3)^1)^4$and $((-3)^1)^{-4}$ Answer:$(-3)^4 > (-3)^{-4}$ Reason: $81 > \frac{1}{81}$
   a) $((-3)^3)^1$ and $((-3)^{-3})^{-1}$
   b) $-((-3)^2)^{-3}$ and $(-(-3)^3)^{-2}$

2) Which of the following statements are correct for $((-2)^{-3})^3$?
   a) The answer is a fraction
   b) It is the same as$(-2)(-3)(3)$
   c) It can be expanded as $(-2)(-2)(-2)$ three times
   d) It can be expanded as $\frac{1}{(-2)(-2)(-2)}$ three times
   e) It has a negative answer

3) Show that the powers given below have the same solution.
   a) $((-x)^{-2})^3$
   b) $(-x \times -x \times -x)^{-2}$
   c) $(-x \times -x)^{-3}$

4) Look at the six powers in the list below. Group the powers which are the same.

$$(-y)^{-4};\ \left(\frac{1}{2}(y)^{-2}\right)^2;\ ((-y)^{-2})^2;\ (4(2y)^{-1})^2;\ (2^{-1}y^{-1})^2; y^{-4}$$

5) Simplify the following powers:
   a) -2^{-4}
   b) $(-4)^{-2}$
   c) $\left(\frac{1}{4}\right)^{-2}$
   d) $(-b)^{-2}$
   e) $((-b)^2)^{-2}$
   f) $(-b \times b)^{-2}$
   g) $(b^6)^{-2}$
   h) $(b^{-2})^6$
   i) $(b^{-2})^0 + (b^0)^2$
   j) $(-x^{-3})^3.((-x)^{-2})^3$
   k) $4(a)^{-2}.(-2a)^{-2}$
   l) $4(a)^{-2} + (-2a)^{-2}$

Worksheet 3.2

Answers

Questions	**Answers**
1) Use one of these signs $<$; $=$; $>$ to show the relationship between each pair. Give a reason for each answer e.g. $((-3)^1)^4$and $((-3)^1)^{-4}$ Answer:$(-3)^4 > (-3)^{-4}$ Reason: $81 > \frac{1}{81}$ a) $((-3)^3)^1$ and $((-3)^{-3})^{-1}$ b) $-((-3)^2)^{-3}$ and $(-(-3)^3)^{-2}$	1) a) $((-3)^3)^1 = ((-3)^{-3})^{-1}$ Reason: Both can be written as $(-3)^3 = -27$ b) $-((-3)^2)^{-3} < (-(-3)^3)^{-2}$ Reason: LHS: $-(-3)^{-6} = \frac{1}{-(-3)^6} = \frac{1}{-3^6} = -\frac{1}{3^6}$ RHS: $\frac{1}{(-(-3))^6} = \frac{1}{3^6}$
2) Which of the following statements are correct for$((-2)^{-3})^3$? a) The answer is a fraction b) It is the same as$(-2)(-3)(3)$ c) It can be expanded as $(-2)(-2)(-2)$ three times d) It can be expanded as $\frac{1}{(-2)(-2)(-2)}$ three times e) It has a negative answer	2) Correct answers: Q2a; Q2d; Q2e
3) Show that the powers given below have the same solution. a) $((-x)^{-2})^3$ b) $(-x \times -x \times -x)^{-2}$ c) $(-x \times -x)^{-3}$	3) All become $(-x)^{-6} = \frac{1}{(-x)^6} = \frac{1}{x^6}$ a) $((-x)^{-2})^3 = (-x)^{-6}$ b) $(-x \times -x \times -x)^{-2} = ((-x)^3)^{-2} = (-x^3)^{-2} = (-x)^{-6}$ c) $(-x \times -x)^{-3} = ((-x)^2)^{-3} = (-x)^{-6}$
4) Look at the six powers in the list below. Group the powers which are the same. $(-y)^{-4}; \left(\frac{1}{2}(y)^{-2}\right)^2 ; ((-y)^{-2})^2 ; (4(2y)^{-1})^2; (2^{-1}y^{-1})^2; y^{-4}$	4) $(-y)^{-4}$; $((-y)^{-2})^2$; y^{-4} No others are the same
5) Simplify the following powers: a) -2^{-4} b) $(-4)^{-2}$ c) $\left(\frac{1}{4}\right)^{-2}$ d) $(-b)^{-2}$ e) $((-b)^2)^{-2}$ f) $(-b \times b)^{-2}$ g) $(b^6)^{-2}$ h) $(b^{-2})^6$ i) $(b^{-2})^0 + (b^0)^2$ j) $(-x^{-3})^3.((-x)^{-2})^3$ k) $4(a)^{-2}.(-2a)^{-2}$ l) $4(a)^{-2} + (-2a)^{-2}$	5) a) $-\frac{1}{16}$ b) $\frac{1}{16}$ c) 16 d) $\frac{1}{b^2}$ e) $\frac{1}{b^4}$ f) $\frac{1}{b^4}$ g) $\frac{1}{b^{12}}$ h) $\frac{1}{b^{12}}$ i) 2 j) $-\frac{1}{x^{15}}$ k) $\frac{1}{a^4}$ l) $\frac{17}{4a^2}$

Worksheet 4.1 a

This worksheet focuses on a product raised to a power, exponents are positive.

Questions

1) Which are different ways of writing $(cd)^3$?
   a) c^3d b) cd^3 c) $cd.cd.cd$ d) $c.c.c.d.d.d$ e) c^3d^3

2) Fill in the blanks:
   a) $v.w.v.v.v.v.w.w.w.w.w.v = v^{\square}.w^{\square} = (vw)^{\square}$
   b) $3.5.5.3.5.3.5.3 = 3^{\square}.5^{\square} = (\square.\square)^{\square}$
   c) $(2^23)^2 = (2^23^1).\square = 2^2.\square.3.\square = 2^{\square}.3^{\square}$
   d) $7(\square)^{\square} = \square a^4b^4$

3)
   a) Work out the answers.
      i) $3^2 \times 2^2$
      ii) $(3 \times 2)^2$
   b) Is it true that $5^2 \times 13^2 = (5 \times 13)^2$? How do you know?
   c) Can we conclude that for any values of a and b: $a^2 \times b^2 = (ab)^2$?
   d) Now let's check for addition. What answers do you get for:
      i) $3^2 + 2^2$
      ii) $(3 + 2)^2$
   e) Check the addition of 2 different numbers (e.g. 5 and 7). Do they give the same answers?
   f) Can we conclude that for any values of a and b: $a^2 + b^2 = (a + b)^2$?

4) Explain why $(4x)^6$ is the same as 4^6x^6.

5) Simplify. Write answers in exponential form.
   a) $(3r)^4$ b) $(rs)^3$ c) $(3rs)^3$ d) $(3^3n)^2$
   e) $(3^2n^2)^3$ f) $(ab^3)^2$ g) $(a^0b)^4$ h) $(3ab)^2b$
   i) $(2k)^3k^4$ j) $(5 \times 2)^3$ k) $(5 + 2)^3$

Worksheet 4.1 a

Answers

Questions	**Answers**
1) Which are different ways of writing $(cd)^3$? a) c^3d b) cd^3 c) $cd.cd.cd$ d) $c.c.c.d.d.d$ e) c^3d^3	1) Q1c, Q1d, and Q1e
2) Fill in the blanks: a) $v.w.v.v.v.w.w.w.w.v = v^{\square}.w^{\square} = (vw)^{\square}$ b) $3.5.5.3.5.3.5.3 = 3^{\square}.5^{\square} = (\square.\square)^{\square}$ c) $(2^23)^2 = (2^23).\square = 2^2.\square.3.\square = 2^{\square}.3^{\square}$ d) $7(\square)^{\square} = \square a^4b^4$	2) a) $v.w.v.v.v.w.w.w.w.v = v^{\mathbf{5}}.w^{\mathbf{5}} = (vw)^{\mathbf{5}}$ b) $3.5.5.3.5.3.5.3 = 3^{\mathbf{4}}.5^{\mathbf{4}} = (3.5)^{\mathbf{4}}$ c) $(2^23)^2 = (2^23^1).(\mathbf{2^23^1}) = 2^2.\mathbf{2^2}.3.\mathbf{3} = 2^{\mathbf{4}}.3^{\mathbf{2}}$ d) $7(\boldsymbol{ab})^{\mathbf{4}} = \mathbf{7}a^4b^4$
3) a) Work out the answers. iii) $3^2 \times 2^2$ iv) $(3 \times 2)^2$ b) Is it true that $5^2 \times 13^2 = (5 \times 13)^2$? How do you know? c) Can we conclude that for any values of a and b: $a^2 \times b^2 = (ab)^2$? d) Now let's check for addition. What answers do you get for: iii) $3^2 + 2^2$ iv) $(3 + 2)^2$ e) Check the addition of 2 different numbers (e.g. 5 and 7). Do they give the same answers? f) Can we conclude that for any values of a and b: $a^2 + b^2 = (a + b)^2$?	3) a) i) 36 ii) 36 b) Yes. Because $5^2 \times 13^2 = 5 \times 5 \times 13 \times 13 = 5 \times 13 \times 5 \times 13 = (5 \times 13)^2$ c) Yes. Because $a \times a \times b \times b = a \times b \times a \times b = (ab)^2$ d) i) 13 ii) 25 e) $5^2 + 7^2 = 25 + 49 = 74$ $(5 + 7)^2 = 12^2 = 144$ The answers are not the same. f) No, they are not equal. (In Grade 9 learners will see that $(a + b)^2 = a^2 + 2ab + b^2$)
4) Explain why $(4x)^6$ is the same as 4^6x^6.	4) $(4x)^6 = 4x \times 4x \times 4x \times 4x \times 4x \times 4x$ $= 4 \times 4 \times 4 \times 4 \times 4 \times 4 \quad \times \quad x \times x \times x \times x \times x \times x = 4^6 \times x^6 = 4^6x^6$
5) Simplify. Write answers in exponential form. a) $(3r)^4$ b) $(rs)^3$ c) $(3rs)^3$ d) $(3^3n)^2$ e) $(3^2n^2)^3$ f) $(ab^3)^2$ g) $(a^0b)^4$ h) $(3ab)^2b$ i) $(2k)^3k^4$ j) $(5 \times 2)^3$ k) $(5 + 2)^3$	5) a) 3^4r^4 b) r^3s^3 c) $3^3r^3s^3$ d) 3^6n^2 e) 3^6n^6 f) a^2b^6 g) a^0b^4 h) $3^2a^2b^3$ i) 2^3k^7 j) 10^3 k) 7^3

Worksheet 4.1 b

This worksheet focuses on a quotient raised to a power, exponents are positive.

Questions

1) Fill in the blanks:

   a) $\left(\frac{5^2}{3}\right)^3 = \left(\frac{5^2}{3}\right).\square.\square = \frac{5^2.\square.\square}{3.\square.\square} = \frac{5^\square}{3^\square}$

   b) $\frac{v.v.v.v^2}{w^3.w.w} = \frac{v^\square}{w^\square} = \left(\frac{v}{w}\right)^\square$

   c) $3 \div (\square)^\square = \frac{\square}{b^6c^6}$

2) Look at the questions in the box below:

i) $(2 \times 3)^2$	ii) $\frac{(2\times3)^2}{3^3}$	iii) $\frac{6^2}{3^2}$	iv) $\left(\frac{6}{3}\right)^2$

   a) Which questions will give the same answer?

   b) Why does this happen?

3) Explain why $\left(\frac{5}{x}\right)^4$ is the same as $\frac{5^4}{x^4}$.

4) Simplify. Write answers in exponential form.

   a) $\left(\frac{m^3}{n}\right)^2$

   b) $\left(\frac{m^2}{n^2}\right)^3$

   c) $\left(\frac{a}{b^3}\right)^2$

   d) $\left(\frac{a^0}{b}\right)^4$

   e) $\left(\frac{ab}{b}\right)^2$

   f) $\left(\frac{3ab}{b}\right)^2$

   g) $\left(\frac{2k}{k^4}\right)^3$

   h) $\left(\left(\frac{5}{2}\right)^3\right)^2$

   i) $((5-2)^3)^2$

   j) $((5 \times 2)^3)^2$

   k) $\left(\frac{(5-2)^3}{(5\times2)^3}\right)^2$

   l)

   m)

   n)

   o)

Make up 4 of your own questions and answer them

Worksheet 4.1 b

Answers

Questions	**Answers**
1) Fill in the blanks: a) $\left(\frac{5^2}{3}\right)^3 = \left(\frac{5^2}{3}\right).\square.\square = \frac{5^2.\square.\square}{3.\square.\square} = \frac{5^{\square}}{3^{\square}}$ b) $\frac{v.v.v.v^2}{w^3.w.w} = \frac{v^{\square}}{w^{\square}} = \left(\frac{v}{w}\right)^{\square}$ c) $3 \div (\square)^{\square} = \frac{\square}{b^6c^6}$	1) a) $\left(\frac{5^2}{3}\right)^3 = \left(\frac{5^2}{3}\right).\left(\frac{\mathbf{5^2}}{\mathbf{3}}\right).\left(\frac{\mathbf{5^2}}{\mathbf{3}}\right) = \frac{5^2.\mathbf{5^2}.\mathbf{5^2}}{3.\mathbf{3}.\mathbf{3}} = \frac{5^{\mathbf{6}}}{3^{\mathbf{3}}}$ b) $\frac{v.v.v.v^2}{w^3.w.w} = \frac{v^{\mathbf{5}}}{w^{\mathbf{5}}} = \left(\frac{v}{w}\right)^{\mathbf{5}}$ c) $3 \div (\boldsymbol{bc})^{\mathbf{6}} = \frac{\mathbf{3}}{b^6c^6}$
2) Look at the questions in the box below: i) $(2 \times 3)^2$ ii) $\frac{(2\times3)^2}{3^3}$ iii) $\frac{6^2}{3^2}$ iv) $\left(\frac{6}{3}\right)^2$ a) Which questions will give the same answer? b) Why does this happen?	2) a) Q2iii) and Q2iv) b) Because $\frac{6^2}{3^2} = \frac{6.6}{3.3} = \frac{6}{3}.\frac{6}{3} = \left(\frac{6}{3}\right)^2$
3) Explain why $\left(\frac{5}{x}\right)^4$ is the same as $\frac{5^4}{x^4}$.	3) Because $\left(\frac{5}{x}\right)^4 = \frac{5}{x}.\frac{5}{x}.\frac{5}{x}.\frac{5}{x} = \frac{5^4}{x^4}$
4) Simplify. Write answers in exponential form. a) $\left(\frac{m^3}{n}\right)^2$ b) $\left(\frac{m^2}{n^2}\right)^3$ c) $\left(\frac{a}{b^3}\right)^2$ d) $\left(\frac{a^0}{b}\right)^4$ e) $\left(\frac{ab}{b}\right)^2$ f) $\left(\frac{3ab}{b}\right)^2$ g) $\left(\frac{2k}{k^4}\right)^3$ h) $\left(\left(\frac{5}{2}\right)^3\right)^2$ i) $((5-2)^3)^2$ j) $((5\times2)^3)^2$ k) $\left(\frac{(5-2)^3}{(5\times2)^3}\right)^2$ l) to o) Make up 4 of your own questions and answer them	4) a) $\frac{m^6}{n^2}$ b) $\frac{m^6}{n^6}$ c) $\frac{a^2}{b^6}$ d) $\frac{a^0}{b^4}$ e) a^2 f) $3^2.a^2$ g) $\frac{2^3}{k^9}$ h) $\frac{5^6}{2^6}$ i) 3^6 j) 10^6 k) $\frac{3^6}{10^6}$ l) to o) Learners answers to their own questions

Worksheet 4.1 c

This worksheet focuses on products and quotients raised to a power, exponents are positive and powers have negative coefficients.

Questions

1) Which examples represent a positive number? (i.e. which will give a positive answer when simplified)
$(-3)^2;\ \ (-3)^3;\ \ -3\times 4;\ \ (-1)(-1)^5;\ \ (-2)^{2021}$

2) Fill in the blanks:
   - a) $((5)^2)^3 = (5)^{2\times\square} = 5^{\square}$
   - b) $((-5)^2)^3 = (-5)^{2.\square.\square} = (-5)^{\square} = \square^{\square}$
   - c) $((-5)^3)^3 = (-5)^{3.\square.\square} = (\square)^{\square} = \square^{\square}$

3) Which of the following are the same as the power $(-3d)^2$?

   a) $3d^2$ b) $\frac{1}{3^2d^2}$ c) -3^2d^2 d) $9d^2$

4)
   - a) Give answers as powers:
     i) $\left(\left(\frac{1}{4}\right)^5\right)^2$ ii) $\left(\left(-\frac{1}{4}\right)^5\right)^2$ iii) $\left(\left(-\frac{1}{4}\right)^5\right)^3$ iv) $-\left(\left(-\frac{1}{4}\right)^5\right)^2$
   - b) Which answers are positive? Why?
   - c) Which answers are negative? Why?

5) Below is one way of simplifying $(-4x)^3.(-4x)^3$:

$$(-4x)^3.(-4x)^3 = (-4x)^{3+3} = (-4x)^6 = (-4)^6x^6 = 4^6x^6$$

Simplify $(-4x)^3.(-4x)^3$ in another way.

6) Simplify:

   a) $(3a)^2.(3a)^2$ b) $(3a)^2.(-3a)^2$ c) $-(3a)^2.(3a^2)$

   d) $\frac{(3a)^2}{(3a)^2}$ e) $\frac{(3a)^2}{(-3a)^2}$ f) $-\frac{(3a)^2}{(-3a^2)}$

   g) $(3a)^2 + (-3a^2)$ h) $(3a)^2 - (-3a^2)$ i) $-(3a)^2 - (-3a)^2$

   j) $(a^2b)^3$ k) $((a^2b)^3)^2$ l) $((-ab^2)^3)^2$

Worksheet 4.1 c

Answers

Questions	**Answers**
1) Which examples have a positive answer? Which examples represent a positive number? (i.e. which will give a positive answer when simplified) $(-3)^2;\ (-3)^3;\ -3 \times 4;\ (-1)(-1)^5;\ (-2)^{2021}$	1) $(-3)^2$ and $(-1)(-1)^5$
2) Fill in the blanks: a) $((5)^2)^3 = (5)^{2\times\square} = 5^{\square}$ b) $((-5)^2)^3 = (-5)^2.\square.\square = (-5)^{\square} = \square^{\square}$ c) $((-5)^3)^3 = (-5)^3.\square.\square = (\square)^{\square} = \square^{\square}$	2) a) $((5)^2)^3 = (5)^{2\times\mathbf{3}} = 5^{\mathbf{6}}$ b) $((-5)^2)^3 = (-5)^2.(\boldsymbol{-5})^{\mathbf{2}}.(\boldsymbol{-5})^{\mathbf{2}} = (-5)^{\mathbf{6}} = \mathbf{5}^{\mathbf{6}}$ c) $((-5)^3)^3 = (-5)^3.(\boldsymbol{-5})^{\mathbf{3}}.(\boldsymbol{-5})^{\mathbf{3}} = (\boldsymbol{-5})^{\mathbf{9}} = \boldsymbol{-5}^{\mathbf{9}}$
3) Which of the following are the same as the power $(-3d)^2$? a) $3d^2$ b) $\frac{1}{3^2d^2}$ c) -3^2d^2 d) $9d^2$	3) Q3d only
4) a) Give answers as powers: i) $\left(\left(\frac{1}{4}\right)^5\right)^2$ ii) $\left(\left(-\frac{1}{4}\right)^5\right)^2$ iii) $\left(\left(-\frac{1}{4}\right)^5\right)^3$ iv) $-\left(\left(-\frac{1}{4}\right)^5\right)^2$ b) Which answers are positive? Why? c) Which answers are negative? Why?	4) a) i) $\left(\frac{1}{4}\right)^{10}$ ii) $\left(\frac{1}{4}\right)^{10}$ iii) $\left(-\frac{1}{4}\right)^{15}$ iv) $-\left(\frac{1}{4}\right)^{10}$ b) i): A positive base raised to any exponent will be positive ii): A negative base raised to an even exponent is always positive c) iii): A negative base raised to an odd exponent will be negative iv): The power $\left(-\frac{1}{4}\right)^{10}$ will be positive. Then it is multiplied by negative one so the product will be negative
5) Below is one way of simplifying $(-4x)^3.(-4x)^3$: $(-4x)^3.(-4x)^3 = (-4x)^{3+3} = (-4x)^6 = (-4)^6x^6 = 4^6x^6$ Simplify $(-4x)^3.(-4x)^3$ in another way.	5) $(-4x)^3.(-4x)^3 = (-4)^3.x^3.(-4)^3.x^3 = (-4)^6.x^6 = 4^6x^6$
6) Simplify: a) $(3a)^2.(3a)^2$ b) $(3a)^2.(-3a)^2$ c) $-(3a)^2.(3a^2)$ d) $\frac{(3a)^2}{(3a)^2}$ e) $\frac{(3a)^2}{(-3a)^2}$ f) $-\frac{(3a)^2}{(-3a^2)}$ g) $(3a)^2 + (-3a^2)$ h) $(3a)^2 - (-3a^2)$ i) $-(3a)^2 - (-3a)^2$ j) $(a^2b)^3$ k) $((a^2b)^3)^2$ l) $((-ab^2)^3)^2$	6) a) $3^4.a^4$ b) $3^4.a^4$ c) $-3^4.a^4$ d) 1 e) 1 f) 3 g) $9a^2 - 3a^2 = 6a^2$ h) $9a^2 + 3a^2 = 12a^20$ i) $-9a^2 - 9a^2 = -18a^2$ j) a^6b^3 k) $(a^6b^3)^2 = a^{12}b^6$ l) $(-a^3b^6)^2 = a^6b^{12}$

Worksheet 4.2 a

This worksheet focuses on a product raised to a power, exponents are negative.

Questions

1) Which of the following statements are TRUE about $(2 \times 3)^{-5}$?

   a) It is a fraction b) $= -5(2 \times 3)$ c) $= \frac{1}{2^5 3^5}$

   d) $= \frac{1}{5(2\times3)}$ e) It has a negative answer

2) Simplify. Use the 'product raised to a power' rule if needed.

   a) $((ab)^2)^5$ b) $(a^2b^2)^5$ c) $(a^3b^{-2})^2$

   d) $((ab)^5)^{-2}$ e) $(a^5b^5)^2$ f) $(a^3b^2)^{-2}$

3) Which of the powers below simplify to give an answer of 1?

   a) $((xy)^2)^0$ b) $(x^2y^2)^0$ c) $(x^0y^{-2})^2$

   d) $((xy)^0)^{-2}$ e) $(x^0y^0)^2$ f) $(x^0y^2)^{-2}$

4) Two Grade 9 learners were given this question to simplify: $(ab^2)^{-5}.(ab^2)^{-5}$
   They used different approaches.

   a) Copy and complete each approach

Learner 1	Learner 2
$(ab^2)^{-5}.(ab^2)^{-5}$	$(ab^2)^{-5}.(ab^2)^{-5}$
$= a^{\square}b^{2\times\square}.a^{\square}b^{2\times\square}$	$= (ab^2)^{\square+\square}$
$= a^{(-5+\square)}b^{(-10+(-10))}$	$= a^{-10}b^{2\times(-10)}$
$= a^{\square}b^{\square}$	$= a^{\square}b^{\square}$
$= \dfrac{1}{a^{10}b^{20}}$	$= \dfrac{1}{a^{10}b^{20}}$

   b) Describe the main differences between the approaches of the two learners.

5) Simplify, leave answers with negative exponents.
   Try to use the 'product raised to a power' rule as often as you can.

   a) $(2 \times 3)^{-4}$ b) $(2^2 3^3)^{-3}$

   c) $(2^2x^{-3})^3$ d) $(x^0y^{-2})^2.(x^0y^2)^{-2}$

   e) $(3ab^5)^{-2}b$ f) $(a^3.b^4)^2.(a^3.b^4)^{-5}$

   g) $((cd)^{-2})^3.((cd)^{-2})^3$ h) $((cd)^{-2})^3 + ((cd)^{-2})^3$

Worksheet 4.2 a

Answers

Questions	**Answers**
1) Which of the following statements are TRUE about $(2 \times 3)^{-5}$? a) It is a fraction b) $= -5(2 \times 3)$ c) $= \frac{1}{2^5 3^5}$ d) $= \frac{1}{5(2\times3)}$ e) It has a negative answer	1) a and c
2) Simplify. Use the 'product raised to a power' rule if needed. a) $((ab)^2)^5$ b) $(a^2b^2)^5$ c) $(a^3b^{-2})^2$ d) $((ab)^5)^{-2}$ e) $(a^5b^5)^2$ f) $(a^3b^2)^{-2}$	2) a) $a^{10}b^{10}$ b) $a^{10}b^{10}$ c) a^6b^{-4} d) $a^{-10}b^{-10}$ e) $a^{10}b^{10}$ f) $a^{-6}b^{-4}$
3) Which of the powers below simplify to give an answer of 1? a) $((xy)^2)^0$ b) $(x^2y^2)^0$ c) $(x^0y^{-2})^2$ d) $((xy)^0)^{-2}$ e) $(x^0y^0)^2$ f) $(x^0y^2)^{-2}$	3) a) 1 b) 1 c) y^{-4} d) 1 e) 1 f) y^{-4}
4) Two Grade 9 learners were given this question to simplify: $(ab^2)^{-5}.(ab^2)^{-5}$ They used different approaches. a) Copy and complete each approach <u>Learner 1</u> $(ab^2)^{-5}.(ab^2)^{-5}$ $= a^{\square}b^{2\times\square}.a^{\square}b^{2\times\square}$ $= a^{(-5+\square)}b^{(-10+(-10))}$ $= a^{\square}b^{\square}$ $= \frac{1}{a^{10}b^{20}}$ <u>Learner 2</u> $(ab^2)^{-5}.(ab^2)^{-5}$ $= (ab^2)^{\square+\square}$ $= a^{-10}b^{2\times(-10)}$ $= a^{\square}b^{\square}$ $= \frac{1}{a^{10}b^{20}}$ b) Describe the main differences between the approaches of the two learners.	4) a) <u>Learner 1</u> $(ab^2)^{-5}.(ab^2)^{-5}$ $= a^{\mathbf{-5}}b^{2\times(\mathbf{-5})}.a^{\mathbf{-5}}b^{\mathbf{2\times(-5)}}$ $= a^{(\mathbf{-5-5})}b^{(\mathbf{-10}+(\mathbf{-10}))}$ $= a^{-10}b^{-20}$ $= \frac{1}{a^{10}b^{20}}$ <u>Learner 2</u> $(ab^2)^{-5}.(ab^2)^{-5}$ $= (ab^2)^{\mathbf{-5-5}}$ $= a^{-10}b^{2\times(-10)}$ $= a^{\mathbf{-10}}b^{\mathbf{-20}}$ $= \frac{1}{a^{10}b^{20}}$ b) Learner 1 first applied the law *product raised to a power*. Learner 2 first applied the law for *multiplying same bases*.
5) Simplify, leave answers with negative exponents. Try to use the 'product raised to a power' rule as often as you can. a) $(2 \times 3)^{-4}$ b) $(2^2 3^3)^{-3}$ c) $(2^2x^{-3})^3$ d) $(x^0y^{-2})^2.(x^0y^2)^{-2}$ e) $(3ab^5)^{-2}b$ f) $(a^3.b^4)^2.(a^3.b^4)^{-5}$ g) $((cd)^{-2})^3.((cd)^{-2})^3$ h) $((cd)^{-2})^3 + ((cd)^{-2})^3$	5) a) 6^{-4} b) $2^{-6}\,3^{-9}$ c) 2^6x^{-9} d) $y^{-4}.y^{-4} = y^{-8}$ e) $3^{-2}a^{-2}b^{-9}$ f) $(a^3.b^4)^{-3} = a^{-9}b^{-12}$ g) $((cd)^{-2})^6 = c^{-12}d^{-12}$ h) $2(c^{-6}d^{-6})$

Worksheet 4.2 b

This worksheet focuses on a quotient raised to a power, exponents are negative.

Questions

1) Fill in the blanks:

   a) $\left(\frac{1}{7}\right)^{-3}=\left(\frac{1^1}{7^1}\right)^{-3}=\frac{1^{-3}}{7^{\square}}=\frac{7^{\square}}{1^{\square}}=\square^{\square}$

   b) $\left(\frac{5^2}{7}\right)^{-3}=\frac{5^{2\times-3}}{7^{\square\times-3}}=\frac{5^{\square}}{7^{\square}}=\frac{7^{\square}}{5^{\square}}$

   c) $\left(\frac{6b^0}{b^4}\right)^{-2}=\left(6\square^{\square}\right)^{-2}=\frac{1}{\square^{\square}\square^{\square}}$

   d) $\left(\frac{v.v.v.v^2}{w^3.w.w}\right)^{-2}=\left(\frac{v^{\square}}{w^{\square}}\right)^{-2}=\frac{\square^{\square}}{\square^{\square}}$

   e) $4\div(ab)^{\square}=\frac{\square}{a^4b^4}$

2) Which of the following are the same as the power $\left(\frac{3}{d}\right)^{-2}$

   a) $\frac{3^{-2}}{d^{-2}}$
   b) $\frac{1}{3^2d^2}$
   c) $\frac{1}{\left(\frac{3}{d}\right)^2}$
   d) $\frac{1}{9d^2}$
   e) $\frac{d^2}{9}$

3) Three learners were asked to simplify $\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$.

   They used different approaches which have been started below.

Siphiwe	**Brian**	**Lana**
$\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$ $=(a^3b^4)^{0+2}$ $=(a^3b^4)^{+2}$ $=\cdots$	$\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$ $=\frac{1}{a^{3\times-2}b^{4\times-2}}$ $=\cdots$	$\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$ $=\frac{a^{3\times0}b^{4\times0}}{a^{3\times-2}b^{4\times-2}}$ $=\cdots$

   a) Complete each learner's approach. Give answers with positive exponents.

   b) Suggest another way to simplify $\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$

   You should get the same answer for all four approaches.

4) Simplify. Give answers with positive exponents.

   a) $\left(\frac{3}{4}\right)^{-1}$

   b) $\left(\frac{x}{x^2}\right)^{3}$

   c) $\left(\frac{1}{y^1}\right)^{-2}$

   d) $\left(\frac{g^3h}{gh^4}\right)^{-1}$

   e) $\left(\frac{a^2b^3c^0}{a^4b^2c}\right)^{-1}$

Worksheet 4.2 b

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>
1) Fill in the blanks:
a) $\left(\frac{1}{7}\right)^{-3} = \left(\frac{1^1}{7^1}\right)^{-3} = \frac{1^{-3}}{7^{\square}} = \frac{7^{\square}}{1^{\square}} = \square^{\square}$
b) $\left(\frac{5^2}{7}\right)^{-3} = \frac{5^{2\times-3}}{7^{\square\times-3}} = \frac{5^{\square}}{7^{\square}} = \frac{7^{\square}}{5^{\square}}$
c) $\left(\frac{6b^0}{b^4}\right)^{-2} = (6\square^{\square})^{-2} = \frac{1}{\square^{\square}\square^{\square}}$
d) $\left(\frac{v.v.v.v^2}{w^3.w.w}\right)^{-2} = \left(\frac{v^{\square}}{w^{\square}}\right)^{-2} = \frac{\square^{\square}}{\square^{\square}}$
e) $4 \div (ab)^{\square} = \frac{\square}{a^4b^4}$
</td><td>
1)
a) $\left(\frac{1}{7}\right)^{-3} = \left(\frac{1^1}{7^1}\right)^{-3} = \frac{1^{-3}}{7^{\boldsymbol{-3}}} = \frac{7^{\boldsymbol{3}}}{1^{\boldsymbol{3}}} = \boldsymbol{7^3}$
b) $\left(\frac{5^2}{7}\right)^{-3} = \frac{5^{2\times-3}}{7^{\boldsymbol{1}\times-3}} = \frac{5^{\boldsymbol{-6}}}{7^{\boldsymbol{-3}}} = \frac{7^{\boldsymbol{3}}}{5^{6}}$
c) $\left(\frac{6b^0}{b^4}\right)^{-2} = (6\boldsymbol{b^{-4}})^{-2} = \frac{1}{\boldsymbol{6^2 b^8}}$
d) $\left(\frac{v.v.v.v^2}{w^3.w.w}\right)^{-2} = \left(\frac{v^{\boldsymbol{5}}}{w^{\boldsymbol{5}}}\right)^{-2} = \frac{\boldsymbol{w^{10}}}{\boldsymbol{v^{10}}}$
e) $4 \div (ab)^{\boldsymbol{4}} = \frac{\boldsymbol{4}}{a^4b^4}$
</td></tr>
<tr><td>
2) Which of the following are the same as the power $\left(\frac{3}{d}\right)^{-2}$
a) $\frac{3^{-2}}{d^{-2}}$ b) $\frac{1}{3^2d^2}$ c) $\frac{1}{\left(\frac{3}{d}\right)^2}$ d) $\frac{1}{9d^2}$ e) $\frac{d^2}{9}$
</td><td>
2) Q2a, Q2c and Q2e
</td></tr>
<tr><td>
3) Three learners were asked to simplify $\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$.
They used different approaches which have been started below.
<table>
<tr><th>Siphiwe</th><th>Brian</th><th>Lana</th></tr>
<tr><td>$\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$
$= (a^3b^4)^{0+2}$
$= (a^3b^4)^{+2}$
$= \cdots$</td><td>$\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$
$= \frac{1}{a^{3\times-2}b^{4\times-2}}$
$= \cdots$</td><td>$\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$
$= \frac{a^{3\times0}b^{4\times0}}{a^{3\times-2}b^{4\times-2}}$
$= \cdots$</td></tr>
</table>
a) Complete each learner’s approach. Give answers with positive exponents.
b) Suggest another way to simplify $\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$
You should get the same answer for all four approaches.
</td><td>
3)
a)
<table>
<tr><th>Siphiwe</th><th>Brian</th><th>Lana</th></tr>
<tr><td>$\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$
$= (a^3b^4)^{0+2}$
$= (a^3b^4)^{2}$
$= \boldsymbol{a^6b^8}$</td><td>$\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$
$= \frac{1}{a^{3\times-2}b^{4\times-2}}$
$= \frac{1}{\boldsymbol{a^{-6}b^{-8}}}$
$= \boldsymbol{a^6b^8}$</td><td>$\frac{(a^3b^4)^0}{(a^3b^4)^{-2}}$
$= \frac{a^{3\times0}b^{4\times0}}{a^{3\times-2}b^{4\times-2}}$
$= \frac{\boldsymbol{1}}{\boldsymbol{a^{-6}b^{-8}}}$
$= \boldsymbol{a^6b^8}$</td></tr>
</table>
b) $(a^3b^4)^0.(a^3b^4)^2$
$= a^{3\times0}b^{4\times0}.a^{3\times2}b^{4\times2} = a^{0+6}b^{0+8} = a^6b^8$
</td></tr>
<tr><td>
4) Simplify. Give answers with positive exponents.
a) $\left(\frac{3}{4}\right)^{-1}$ b) $\left(\frac{x}{x^2}\right)^{3}$ c) $\left(\frac{1}{y^1}\right)^{-2}$ d) $\left(\frac{g^3h}{gh^4}\right)^{-1}$ e) $\left(\frac{a^2b^3c^0}{a^4b^2c}\right)^{-1}$
</td><td>
4)
a) $\frac{4}{3}$ b) $\left(\frac{1}{x}\right)^3 = \frac{1}{x^3}$ c) $\frac{1}{y^{-2}} = y^2$ d) $\left(\frac{g^2}{h^3}\right)^{-1} = \frac{h^3}{g^2}$ e) $\left(\frac{b}{a^2c}\right)^{-1} = \frac{a^2c}{b}$
</td></tr>
</table>

Worksheet 4.2 c

This worksheet focuses on products and quotients raised to a power, exponents and coefficients are positive or negative.

Questions

1) Write with positive exponents:
   a) $(x)^{-3}$
   b) $(2^2x)^{-3}$
   c) $\left(\frac{1}{b}\right)^{-4}$
   d) $\left(\frac{x}{b}\right)^{-4}$
   e) $-(b^{-2})^{-2}$

2) Fill in the blanks:
   e.g. $(-2)^3 = (-2).\square.\square = \square^{\square}$ will be: $(-2)^3 = (-2).(-\mathbf{2}).(-\mathbf{2}) = -\mathbf{2^3}$
   a) $((-c)^2)^3 = (-c)^2.\square.\square = (-c)^{\square} = \square^{\square}$
   b) $((-c)^3)^3 = (-c)^3.\square.\square = (\square)^{\square} = \square^{\square}$
   c) $-((-c)^3)^3 = (-1)(-c)^3.\square.\square = (-1)(\square)^{\square} = \square^{\square}$

3)
   a) Predict whether $\left(\frac{-3}{3^{-4}}\right)^3$ will have a positive or negative answer.
   b) Complete these two Grade 9 learners' responses to $\left(\frac{-3}{3^{-4}}\right)^3$.

Lindi's response	**Shawn's response**
$\left(\frac{-3}{3^{-4}}\right)^3$ $= \frac{(-3)^3}{3^{-12}}$ $= \cdots$ $= \cdots$	$\left(\frac{-3}{3^{-4}}\right)^3$ $= \left(\frac{(-1)(3)}{3^{-4}}\right)^3$ $= \cdots$ $= \cdots$

   c) Check that you get the same answer for both learners.
   d) Was your prediction correct about the sign of the answer?

4) Simplify. Give answers with positive exponents.
   a) $((-2)^2 3^3)^3$
   b) $(-2^2.-3^3)^3$
   c) $((-a)^2(-b)^5)^3$
   d) $((-a)^2(-b)^5)^{-3}$
   e) $((-2)^2 3^3)^{-3}$
   f) $\left(\frac{-3}{4}\right)^{-1}$
   g) $\left(-\frac{x}{x^2}\right)^5$
   h) $-\left(\frac{x\,y^4}{x^2y}\right)^5$
   i) $-((a)^{-1})^4 + (b^{-4})^1$

Worksheet 4.2 c

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr>
<td>1) Write with positive exponents:
a) $(x)^{-3}$ b) $(2^2x)^{-3}$ c) $\left(\frac{1}{b}\right)^{-4}$ d) $\left(\frac{x}{b}\right)^{-4}$ e) $-(b^{-2})^{-2}$</td>
<td>1)
a) $\frac{1}{x^3}$ b) $\frac{1}{2^6x^3}$ c) b^4 d) $\frac{b^4}{x^4}$ e) $-b^4$</td>
</tr>
<tr>
<td>2) Fill in the blanks: e.g. $(-2)^3 = (-2).\square.\square = \square^{\square}$will be:
$(-2)^3 = (-2).(\mathbf{-2}).(\mathbf{-2}) = \mathbf{-2^3}$
a) $((-c)^2)^3 = (-c)^2.\square.\square = (-c)^{\square} = \square^{\square}$
b) $((-c)^3)^3 = (-c)^3.\square.\square = (\square)^{\square} = \square^{\square}$
c) $-((-c)^3)^3 = (-1)(-c)^3.\square.\square = (-1)(\square)^{\square} = \square^{\square}$</td>
<td>2)
a) $((-c)^2)^3 = (-c)^2.(\boldsymbol{-c})^{\mathbf{2}}.(\boldsymbol{-c})^{\mathbf{2}} = (-c)^{\mathbf{6}} = \mathbf{c^6}$
b) $((-c)^3)^3 = (-c)^3.(\boldsymbol{-c})^{\mathbf{3}}.(\boldsymbol{-c})^{\mathbf{3}} = ((\boldsymbol{-c})^{\mathbf{3}})^3 = \mathbf{-c^9}$
c) $-((-c)^3)^3 = (-1)(-c)^3.(\boldsymbol{-c})^{\mathbf{3}}.(\boldsymbol{-c})^{\mathbf{3}} = (-1)((\boldsymbol{-c})^{\mathbf{3}})^3 = \mathbf{c^9}$</td>
</tr>
<tr>
<td>3)
a) Predict whether $\left(\frac{-3}{3^{-4}}\right)^3$ will have a positive or negative answer.
b) Complete these two Grade 9 learners' responses to $\left(\frac{-3}{3^{-4}}\right)^3$.
<table>
<tr><th>Lindi's response</th><th>Shawn's response</th></tr>
<tr><td>$\left(\frac{-3}{3^{-4}}\right)^3$
$= \frac{(-3)^3}{3^{-12}}$
$= \cdots$
$= \cdots$</td><td>$\left(\frac{-3}{3^{-4}}\right)^3$
$= \left(\frac{(-1)(3)}{3^{-4}}\right)^3$
$= \cdots$
$= \cdots$</td></tr>
</table>
c) Check that you get the same answer for both learners.
d) Was your prediction correct about the sign of the answer?</td>
<td>3)
a) Negative
b)
<table>
<tr><th>Lindi's response</th><th>Shawn's response</th></tr>
<tr><td>$\left(\frac{-3}{3^{-4}}\right)^3$
$= \frac{(-3)^3}{3^{-12}}$
$= \mathbf{-3^3.3^{12}}$
$= \mathbf{-3^{15}}$</td><td>$\left(\frac{-3}{3^{-4}}\right)^3$
$= \left(\frac{(-1)(3)}{3^{-4}}\right)^3$
$= (\mathbf{-1})(\mathbf{3})^3(\mathbf{3^4})^3$
$= (\mathbf{-1})(\mathbf{3^{3+12}})$
$= \mathbf{-3^{15}}$</td></tr>
</table>
c) Learners check answers for both learners are the same..
d) Correctness of Q3a depends on learner.</td>
</tr>
<tr>
<td>4) Simplify. Give answers with positive exponents.
a) $((-2)^2 3^3)^3$ b) $(-2^2.-3^3)^3$ c) $((-a)^2(-b)^5)^3$
d) $((-a)^2(-b)^5)^{-3}$ e) $((-2)^2 3^3)^{-3}$ f) $\left(\frac{-3}{4}\right)^{-1}$
g) $\left(-\frac{x}{x^2}\right)^5$ h) $-\left(\frac{x\,y^4}{x^2y}\right)^5$ i) $-((a)^{-1})^4 + (b^{-4})^1$</td>
<td>4)
a) $2^6 3^9$ b) $2^6 3^9$ c) $a^6(-b)^{15} = -a^6b^{15}$
d) $a^{-6}(-b)^{-15} = -\frac{1}{a^6b^{15}}$ e) $2^{-6}3^{-9} = \frac{1}{2(2^6 3^9)}$ f) $\frac{4}{-3}$ or $-\frac{4}{3}$
g) $\left(-\frac{1}{x}\right)^5 = -\frac{1}{x^5}$ h) $-\left(\frac{y^3}{x}\right)^5 = -\frac{y^{15}}{x^5}$ i) $-a^{-4} + b^{-4} = -\frac{1}{a^4} + \frac{1}{b^4}$</td>
</tr>
</table>

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 5.1 a

This worksheet focuses on multiplying and dividing powers.

Questions

1) Look at each pair of expressions and do the following:
   - Say what is the *same* and what is *different* about the expressions in each pair
   - Simplify each expression
   - Are the answers the same or different (in each pair)?
   - Say why the answers are the same or different

a) $2^3 \times 2^4$	c) $2a^4 \div a^3$	e) $\frac{a^2b}{b}$	g) $9a \times b^2$
b) $2^3 \div 2^4$	d) $2a^3 \div a^4$	f) $a \times a.b^0$	h) $3^2a \times (-b)^2$

2) Each statement in the box has an error.

i) $5^4 \times 5^2 = 25^6$
ii) $ab.ab = ab^2$
iii) $ab + ab = (ab)^2$
iv) $7^2 \times 3^4 = 7^6$
v) $ab + ab = 2a2b$

   a) Describe the error in each statement.
   b) Change the part on the right-side of the equal sign to make the statement true.

3) Simplify:

   a) $2.2^3.2^4$
   b) $a^3 \times a^4$
   c) $2a^3 \times a^4$
   d) $a.b^3.a^4$
   e) $\frac{2^3.2^4}{2^5}$
   f) $\frac{2a^3}{6b^2}$
   g) $\frac{m^6 \times m^5}{m^3}$
   h) $\frac{2^3.2}{2.3 \times 2.3^2}$
   i) $\frac{c^2d}{cd \times cd^2}$
   j) $\frac{16e^8f^7}{32e^7f}$
   k) $\frac{16x^8 \times 4x^0y^7}{32x^7y^9}$
   l) $ab^3 + ab^3$

Worksheet 5.1 a

Answers

Questions	**Answers**
1) Look at each pair of expressions and do the following: - Say what is the *same* and what is *different* about the expressions in each pair - Simplify each expression - Are the answers the same or different (in each pair)? - Say why the answers are the same or different a) $2^3 \times 2^4$ b) $2^3 \div 2^4$ c) $2a^4 \div a^3$ d) $2a^3 \div a^4$ e) $\frac{a^2b}{b}$ f) $a \times a.b^0$ g) $9a \times b^2$ h) $3^2a \times (-b)^2$ **Answers for <u>a) and b)</u>:** **<u>a) and b):</u>** Both have the same powers but a) focuses on multiplication whereas b) focuses on division. a) 2^7, b) $\frac{1}{2}$. The answers are different because in a) the exponents were added together and in b) the exponents were subtracted from each other.	**1) Answers continued** **<u>c) and d):</u>** Both required the division of powers but the exponents were swopped in d). c) $2a$, d) $\frac{2}{a}$. The answers are different because c) the leading power had a greater exponent whereas d) the leading power had a smaller exponent resulting in a fraction answer. **<u>e) and f):</u>** The variables are the same. The operations are different. e) a^2, f) a^2. The answers are the same because $a^2 = a \times a$ and in e) dividing b by b gives 1 and in f) b^0results in 1 as well. **<u>g) and h):</u>** Both require multiplication. The coefficients of a are written differently (9 and3^2), and the base in g) is b but in h) the base is – b. g) $9ab^2$, h) $9ab^2$. The answers are the same because in h) $3^2$is the same as 9 and a negative squared is the same as a positive squared i.e.$3^2a \times (-b)^2 = 9a \times b^2$, this is the same as g).
2) Each statement in the box has an error. i) $5^4 \times 5^2 = 25^6$ ii) $ab.ab = ab^2$ iii) $ab + ab = (ab)^2$ iv) $7^2 \times 3^4 = 7^6$ v) $ab + ab = 2a2b$ a) Describe the error in each statement. b) Change the part on the right-side of the equal sign to make the statement true.	2) a) i) Multiplication of the bases ii) No multiplication of a and a. iii) Multiplication was done instead of addition. iv) Exponents may not be added, bases are not the same. v) The like terms are ab should be added not separated into a and b, then added. b) i) 5^6 ii) a^2b^2 iii) $2ab$ iv) $7^2 \times 3^4$ v) $2ab$
3) Simplify: a) $2.2^3.2^4$ b) $a^3 \times a^4$ c) $2a^3 \times a^4$ d) $a.b^3.a^4$ e) $\frac{2^3.2^4}{2^5}$ f) $\frac{2a^3}{6b^2}$ g) $\frac{m^6 \times m^5}{m^3}$ h) $\frac{2^3.2}{2.3 \times 2.3^2}$ i) $\frac{c^2d}{cd \times cd^2}$ j) $\frac{16e^8f^7}{32e^7f}$ k) $\frac{16x^8 \times 4x^0y^7}{32x^7y^9}$ l) $ab^3 + ab^3$	3) a) 2^8 b) a^7 c) $2a^7$ d) a^5b^3 e) 2^2 f) $\frac{a^3}{3b^2}$ g) m^8 h) $\frac{2^2}{3^3}$ i) $\frac{1}{d}$ j) $\frac{ef^6}{2}$ k) $\frac{2x}{y^2}$ l) $2ab^3$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 5.1 b

This worksheet focuses on raising a power, a product and a quotient to a further power.

Questions

1)

Column A	Column B
i) $(p^5)^3$	A. p^{12}
ii) $(p^2)^2$	B. p^8
iii) $(p^2)^6$	C. p^4
iv) $(p^1)^2$	D. p^{15}
	E. p^2

a) Match the powers which are the same in the two columns.

b) One of the items in column B will not have a match from column A. Create an item for column A that will match with this item from column B.

2) Which of these terms produce the same answer when they are simplified?

5^2p^2; $5 \times 2p^2$; $2(5p)^2$; $25p^2$ $(5p)^2$

3) Choose *positive* or *negative* to make these statements true:

a) The product of 6 negative numbers is positive/negative

b) The product of 13 negative numbers is positive/negative

c) The product of an odd number of negative numbers is positive/negative.

d) The product of an even number negative numbers is positive/negative.

4)

a) Simplify:
i) $(4^{10})^3$ ii) $(4^2)^{15}$ iii) $-(4^6)^5$ iv) $(-4^5)^6$

b) Which question does not give the same answer as the others?

c) Explain why.

5) Simplify. Do Q5c and Q5h in more than one way.

a) $(5^2)^6$

b) $(3.2)^6$

c) $(3.2)^6 + 3(3.2)^6$

d) $(2d^3e^2)^4 \times 3d^2e^3$

e) $\left(\frac{1}{5^2}\right)^6$

f) $-\left(\frac{5}{a^2}\right)^6$

g) $\left(\frac{-6}{12b^2}\right)^5 \times (2b^5)^2$

h) $\frac{(2d^2e^3)^4 \times (de^4)^3}{(2d^2e^3)^2}$

#super$^{\text{powers}}$

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 5.1 b

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1)
a) Match the powers which are the same in the two columns.
<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>i) $(p^5)^3$</td><td>A. p^{12}</td></tr>
<tr><td>ii) $(p^2)^2$</td><td>B. p^8</td></tr>
<tr><td>iii) $(p^2)^6$</td><td>C. p^4</td></tr>
<tr><td>iv) $(p^1)^2$</td><td>D. p^{15}</td></tr>
<tr><td>v)</td><td>E. p^2</td></tr>
</table>
b) One of the items in column B will not have a match from column A. Create an item for column A that will match with this item from column B.</td>
<td>1) a)
<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>i) $(p^5)^3$</td><td>D</td></tr>
<tr><td>ii) $(p^2)^2$</td><td>C</td></tr>
<tr><td>iii) $(p^2)^6$</td><td>A</td></tr>
<tr><td>iv) $(p^1)^2$</td><td>E</td></tr>
<tr><td>No match</td><td>B</td></tr>
</table>
b) Possible match for B:$(p^4)^2$</td></tr>
<tr><td>2) Which of these terms produce the same answer when they are simplified?
5^2p^2 $5 \times 2p^2$ $2(5p)^2$ $25p^2$ $(5p)^2$</td>
<td>2)
5^2p^2 $25p^2$ $(5p)^2$</td></tr>
<tr><td>3) Choose <i>positive</i> or <i>negative</i> to make these statements true:
e) The product of 6 negative numbers is positive/negative
f) The product of 13 negative numbers is positive/negative
g) The product of an odd number of negative numbers is positive/negative.
h) The product of an even number negative numbers is positive/negative.</td>
<td>3)
a) Positive
b) Negative
c) Negative
d) Positive</td></tr>
<tr><td>4)
a) Simplify:
i) $(4^{10})^3$ ii) $(4^2)^{15}$ iii) $-(4^6)^5$ iv) $(-4^5)^6$
b) Which question does not give the same answer as the others?
c) Explain why.</td>
<td>4)
a) i) 4^{30} ii) 4^{30} iii) -4^{30} iv) 4^{30}
b) iii) -4^{30}
c) The negative is outside the brackets and not raised to30.</td></tr>
<tr><td>5) Simplify. Do Q5c and Q5h in more than one way.
a) $(5^2)^6$ b) $(3.2)^6$ c) $(3.2)^6 + 3(3.2)^6$ d) $(2d^3e^2)^4 \times 3d^2e^3$
e) $\left(\frac{1}{5^2}\right)^6$ f) $-\left(\frac{5}{a^2}\right)^6$ g) $\left(\frac{-6}{12b^2}\right)^5 \times (2b^5)^2$ h) $\frac{(2d^2e^3)^4 \times (de^4)^3}{(2d^2e^3)^2}$</td>
<td>5) c) and h) are done in 2 ways
<table>
<tr><td>a) 5^{12} b) 6^6 d) $48d^{14}e^{11}$
e) $\frac{1}{5^{12}}$ f) $-\frac{5^6}{a^{12}}$ g) $\frac{-1}{2^3}$</td>
<td>c) $4(6)^6$
$= 4.6^6$ or
$1.(3^6 2^6)$
$+ 3.(3^6 2^6)$
$= 4.3^6 2^6$</td>
<td>h) $\frac{2^4.d^{11}e^{24}}{2^2d^4e^6} = 2^2d^7e^{18}$ or
$(2d^2e^3)^{4-2}.(de^4)^3$
$= 2^4.d^{11}e^{24}2^{-2}d^{-4}e^{-6}$
$= 2^2d^7e^{18}$</td></tr>
</table></td></tr>
</table>

Worksheet 5.1 c

This worksheet focuses on multiplying and dividing powers.

Questions

1) Look at each pair of expressions and do the following:
   - Say what is the *same* and what is *different* about the expressions in each pair
   - Simplify each expression
   - Are the answers the same or different (in each pair)?
   - Say why the answers are the same or different

a) $7^3 \times 7^5$
b) $7^3 \div 7^5$

c) $7p^5 \div p^3$
d) $7p^3 \div p^5$

e) $(a^2b)^2$
f) a^2b^2

g) $8r \times s^2$
h) $2^3r \times (-s)^2$

2) Each statement in the box has an error.

A. $3^4 \times 3^2 = 9^6$
B. $(2xy)^2 = 4xy^2$
C. $xy + xy + xy = (xy)^3$
D. $4^2 + 3^2 = 7^2$
E. $xy + 3xy = 4x2y$

   a) Describe the error in each statement.
   b) Change the part on the right-side of the equal sign to make the statement true.

3) Simplify:

a) $7.7^4.7$

b) $m^3 \times m^5$

c) $2m^3 \times m^5$

d) $m.n^3.m^5$

e) $\frac{7^3.7^5}{7^6}$

f) $\frac{8x^3}{3y^2}$

g) $\frac{p^4 \times p^6}{p^3}$

h) $\frac{7^3.7}{7.3 \times 7.3^2}$

i) $\frac{5d.5e^2}{ed \times (5ed)^2}$

j) $\frac{16c^4f^3}{32c^3f}$

k) $\frac{14a^8 \times 4a^0b^7}{28a^7b^9}$

l) $ed^3 - d^2.e.d$

Worksheet 5.1 c

Answers

Questions	**Answers**
1) Look at each pair of expressions and do the following: - Say what is the *same* and what is *different* about the expressions in each pair - Simplify each expression - Are the answers the same or different (in each pair)? - Say why the answers are the same or different a) $7^3 \times 7^5$ b) $7^3 \div 7^5$ c) $7p^5 \div p^3$ d) $7p^3 \div p^5$ e) $(a^2b)^2$ f) a^2b^2 g) $8r \times s^2$ h) $2^3r \times (-s)^2$ **Answers to:** **a) and b):** Both have the same powers but a) focuses on multiplication whereas b) focuses on division. a) 7^8, b) $\frac{1}{7^2}$. The answers are different because in a) the exponents are added together and in b) three pairs of $7s$ cancel $\frac{7.7.7}{7.7.7.7.7}$ leaving 1 in the numerator and 7^2 in the denominator.	1) **Answers continued** **c) and d):** Both are division and use p^3;p^5 and 7 but in c) the divisor is p^3 and in d) the divisor is p^5. c) $7p^2$, d) $\frac{7}{p^2}$. The answers are different because in c) the leading power had a greater exponent whereas in d) the leading power had a smaller exponent resulting in a fraction answer. **e) and f):** Both have a, b and squares but in e) the whole product a^2b is squared and in f) each variable is squared. e) a^4b^2, f) a^2b^2. The answers are different, squaring the product a^2b we get a^4b^2. **g) and h):** Both require multiplication however the structures of the powers are different. g) $8rs^2$, h) $8rs^2$. The answers are the same: $2^3r = 8r$, and $(-s)^2 = s^2$.
2) Each statement in the box has an error. A. $3^4 \times 3^2 = 9^6$ B. $(2xy)^2 = 4xy^2$ C. $xy + xy + xy = (xy)^3$ D. $4^2 + 3^2 = 7^2$ E. $xy + 3xy = 4x2y$ a) Describe the error in each statement. b) Change the part on the right-side of the equal sign to make the statement true.	2) a) A. The bases were multiplied B. The x was not squared C. The operation is addition and so the answer should not be cubed D. The exponential laws do not work with addition. E. x and y were added separately, like terms xy were not added. b) A. 3^6 B. $4x^2y^2$ C. $3xy$ D. $16 + 9 = 25$ E. $4xy$
3) Simplify: a) $7.7^4.7$ b) $m^3 \times m^5$ c) $2m^3 \times m^5$ d) $m.n^3.m^5$ e) $\frac{7^3.7^5}{7^6}$ f) $\frac{8x^3}{3y^2}$ g) $\frac{p^4 \times p^6}{p^3}$ h) $\frac{7^3.7}{7.3 \times 7.3^2}$ i) $\frac{5d.5e^2}{ed \times (5ed)^2}$ j) $\frac{16c^4f^3}{32c^3f}$ k) $\frac{14a^8 \times 4a^0b^7}{28a^7b^9}$ l) $ed^3 - d^2.e.d$	3) a) 7^6 b) m^8 c) $2m^8$ d) m^6n^3 e) 7^2 f) $\frac{8x^3}{3y^2}$ g) p^7 h) $\frac{7^2}{3^3}$ i) $\frac{1}{d^2}$ j) $\frac{cf^2}{2}$ k) $\frac{2a}{b^2}$ l) 0

#super$^{\text{powers}}$
PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 5.1 d

This worksheet focuses on raising a power, a product and a quotient to a further power.

Questions

1)
   a) Match the powers which are the same in the two columns.

Column A	Column B
i) $(a^2z^5)^3$	A. $a^9(z^5)^3$
ii) $(az^2)^2$	B. a^6z^{15}
iii) $(a^3)^3z^{15}$	C. a^2z^4
iv) $a(z^1)^4$	D. a^5z^8
	E. az^4

   b) One of the items in column B will not have a match from column A. Create an item for column A that will match with this item from column B.

2) Which of these terms produce the same answer when they are simplified?

$(2 \times 3)^2ap^2 \qquad 6a \times 2p^2 \qquad 2a(6p)^2 \qquad 2(3^2)p^2 \times 2a \qquad \dfrac{(6ap)^2}{a}$

3) State whether the following are TRUE or FALSE. Give a reason for your answer
   a) $a^3.a^4 = a^{12}$
   b) $(a^3)^4 = a^7$
   c) $a^3 + a^3 = 2a^3$

4)
   a) Simplify:
      i) $(6^{10})^4$ ii) $(6^2)^{20}$ iii) $-(6^8)^5$ iv) $(-6^8)^5$
   b) Which question does not give the same answer as the others?
   c) Explain why.

5)
   a) Simplify. All numbers can be left as powers.
      i) $(1+2)^6$
      ii) $(4 \times 5)^{3+1}$
      iii) $(5 \times 4)^4 + 4(5 \times 4)^4$
      iv) $\left(\frac{4}{4^3}\right)^8$
      v) $-\left(\frac{4}{a^3}\right)^8$
      vi) $\left(\frac{2}{c^2}\right)^5 \times (c^2)^6$
      vii) $(3a^3b^2)^4 \times 2.3b^2a^3$
      viii) $\frac{3\left(d^4e^2\right)^5 \times \left(de^4\right)^3}{(-3d^2e^3)^2}$
   b) Try to do Q5a(iii) in another way.
   c) Look at Q5a(vi). Explain why you can't cancel c^2 in the brackets before applying the laws of exponents.

Worksheet 5.1 d

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>1)
a) Match the powers which are the same in the two columns.
<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>i) $(a^2z^5)^3$</td><td>A. $a^9(z^5)^3$</td></tr>
<tr><td>ii) $(az^2)^2$</td><td>B. a^6z^{15}</td></tr>
<tr><td>iii) $(a^3)^3z^{15}$</td><td>C. a^2z^4</td></tr>
<tr><td>iv) $a(z^1)^4$</td><td>D. a^5z^8</td></tr>
<tr><td></td><td>E. az^4</td></tr>
</table>
b) One of the items in column B will not have a match from column A.
Create an item for column A that will match with this item from column B.</td>
<td>1) a)
<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>i) $(a^2z^5)^3$</td><td>B</td></tr>
<tr><td>ii) $(az^2)^2$</td><td>C</td></tr>
<tr><td>iii) $(a^3)^3z^{15}$</td><td>A</td></tr>
<tr><td>iv) $a(z^1)^4$</td><td>E</td></tr>
<tr><td>No match</td><td>D</td></tr>
</table>
b) Possible match for D:
$a^2.a^3.(z^2)^2$</td></tr>
<tr><td>2) Which of these terms produce the same answer when they are simplified?
$(2\times 3)^2ap^2$ $6a\times 2p^2$ $2a(6p)^2$ $2(3^2)p^2\times 2a$ $\frac{(6ap)^2}{a}$</td>
<td>2)
$(2\times 3)^2ap^2$ $2(3^2)p^2\times 2a$ $\frac{(6ap)^2}{a}$</td></tr>
<tr><td>3) State whether the following are TRUE or FALSE. Give a reason for your answer
a) $a^3.a^4 = a^{12}$
b) $(a^3)^4 = a^7$
c) $a^3 + a^3 = 2a^3$</td>
<td>3)
a) False. When multiplying powers with same bases, you *add* exponents.
b) False. When you raise to a power you *multiply* exponents not *add*.
c) True. These are like terms so you can add them. Don’t change the exponent.</td></tr>
<tr><td>4)
a) Simplify:
i) $(6^{10})^4$ ii) $(6^2)^{20}$ iii) $-(6^8)^5$ iv) $(-6^8)^5$
b) Which question does not give the same answer as the others?
c) Explain why.</td>
<td>4)
a) i)6^{40} ii) 6^{40} iii) -6^{40} iv) 6^{40}
b) Q4a(iii)
c) Because the negative is outside the bracket which means you get $6^{40}\times(-1)$.</td></tr>
<tr><td>5)
a) Simplify. All numbers can be left as powers.
i) $(1+2)^6$ ii) $(4\times 5)^{3+1}$ iii) $(5\times 4)^4 + 4(5\times 4)^4$
iv) $\left(\frac{4}{4^3}\right)^8$ v) $-\left(\frac{4}{a^3}\right)^8$ vi) $\left(\frac{2}{c^2}\right)^5\times(c^2)^6$
vii) $(3a^3b^2)^4\times 2.3b^2a^3$ viii) $\frac{3(d^4e^2)^5\times(de^4)^3}{(-3d^2e^3)^2}$
b) Try to do Q5a(iii) in another way.
c) Look at Q5a(vi). Explain why you can’t cancel c^2 in the brackets before applying the laws of exponents.</td>
<td>5)
a)
i) 3^6 ii) 20^5 iii) $5(20^4)$ iv) $\frac{1}{4^{16}}$
v) $-\frac{4^8}{a^{24}}$ vi) 2^5c^2 vii) $2.3^5a^{15}b^{10}$ viii) $\frac{d^{19}e^{16}}{3}$
b) $5^44^4 + 4.5^44^4 = 1.5^44^4 + 4.5^44^5 = 5.5^44^4 = 5(5^44^4) = 5.20^4$
or $1(5\times 4)^4 + 4(5\times 4)^4 = 5(5\times 4)^4 = 5.20^4$
c) Cancelling involves dividing like factors. But first we need to deal with the exponents of 5 and 6 which are applied to the different c^2 factors.</td></tr>
</table>

Worksheet 5.2 a

This worksheet focuses on multiplying and dividing powers, exponents & bases are positive or negative, coefficients include fractions.

Questions

1) Each expression below uses $2x$; x and 3. The expressions are grouped into 3 clusters.

A.	$2x \times 3x$
B.	$2x^3 \times x$
C.	$x^2 \times x^3$
D.	$2x \div 3x$
E.	$x^2 \div x^3$
F.	$2x + 3 - x$

a) What is different between A, B and C?
b) What is different between D and E?
c) In what way/s is F different from the other 5 expressions?
d) Simplify A to F.

2) Write the following in symbols:

a) A base x has an exponent of negative three.

b) A base negative x has an exponent of three.

c) The power k cubed has a coefficient of four.

d) b to the power of two divided by b to the negative two.

3) Learners were asked to simplify $(-5a)a^{-5} \times \frac{1}{a^5}$.
Four learners used different approaches and each approach is correct.
We have given the first line of each approach below.

A. $-5.a^{1+(-5)} \times \frac{1}{a^5}$ B. $(-5a) \times \frac{1}{a^5} \times \frac{1}{a^5}$

C. $(-5).a^{1+(-5)+(-5)}$ D. $-5.a.a^{-5}.a^{-5}$

a) Describe what each learner did.
b) Complete the simplification for one of the approaches.

4) Simplify. Give answers with positive exponents.

a) $2d^{-9}$ b) $\frac{6}{m^{-5}}$ c) $4c^{-2}b$

d) $g^{-4} \times g^3$ e) $h^3 j^4 \div h^2 j^7$ f) $\frac{3a^3b^4}{5a^2b^5}$

g) $\frac{-3a^3b^4}{5a^2b^5}$ h) $\frac{u^{-3}x^{-5} \times u^5 x^{-2}}{u^2 x^6}$ i) $\frac{2^2 d^4 e^6 \times d^3 e^{12}}{2^4 d^{12} e^8}$

Worksheet 5.2 a

Answers

Questions	**Answers**
1) Each expression below uses $2x$; x and 3. The expressions are grouped into 3 clusters. A. $2x \times 3x$ B. $2x^3 \times x$ C. $x^2 \times x^3$ D. $2x \div 3x$ E. $x^2 \div x^3$ F. $2x + 3 - x$ a) What is different between A, B and C? b) What is different between D and E? c) In what way/s is F different from the other 5 expressions? d) Simplify A to F.	1) a) A, B and C have different powers of x, C has no coefficient b) In D, 2 and 3 are coefficients of x. In E, 2 and 3 are exponents of x. c) F focuses on the addition and subtraction of terms. d) A. $6x^2$ B. $2x^4$ C. x^5 D. $\frac{2}{3}$ E. $\frac{1}{x}$ F. $x + 3$
2) Write the following in symbols: a) A base x has an exponent of negative three. b) A base negative x has an exponent of three. c) The power k cubed has a coefficient of four. d) b to the power of two divided by b to the negative two. 2) **Answers** a) x^{-3} b) $(-x)^3$ c) $4k^3$ d) $\frac{b^2}{b^{-2}}$	3) a) A: Learner used Law 1 with the a's that are next to each other. B: Learner converted $a^{-5}\ to\ \frac{1}{a^5}$ C: Learner grouped powers of a after converting $\frac{1}{a^5}$to $a^{-5}$and used the law for multiplying powers of the same base. D: Learner separated the factors and converted $\frac{1}{a^5}$ to a^{-5}. b) Answer based on learners' option A. $-5.a^{1+(-5)} \times \frac{1}{a^5}$ $= -5a^{-4}.a^{-5} = \frac{-5}{a^9}$ B. $(-5a) \times \frac{1}{a^5} \times \frac{1}{a^5}$ $= \frac{-5a}{a^{10}} = \frac{-5}{a^9}$ C. $(-5).a^{1+(-5)+(-5)}$ $= -5.a^{-9} = \frac{-5}{a^9}$ D. $-5.a.a^{-5}.a^{-5}$ $= -5a^{-9} = \frac{-5}{a^9}$ or$-\frac{5}{a^9}$
3) Learners were asked to simplify $(-5a)a^{-5} \times \frac{1}{a^5}$. Four learners used different approaches and each approach is correct. We have given the first line of each approach below. A. $-5.a^{1+(-5)} \times \frac{1}{a^5}$ B. $(-5a) \times \frac{1}{a^5} \times \frac{1}{a^5}$ C. $(-5).a^{1+(-5)+(-5)}$ D. $-5.a.a^{-5}.a^{-5}$ a) Describe what each learner did. b) Complete the simplification for one of the approaches.	
4) Simplify. Give answers with positive exponents. a) $2d^{-9}$ b) $\frac{6}{m^{-5}}$ c) $4c^{-2}b$ d) $g^{-4} \times g^3$ e) $h^3j^4 \div h^2j^7$ f) $\frac{3a^3b^4}{5a^2b^5}$ g) $\frac{-3a^3b^4}{5a^2b^5}$ h) $\frac{u^{-3}x^{-5} \times u^5x^{-2}}{u^2x^6}$ i) $\frac{2^2d^4e^6 \times d^3e^{12}}{2^4d^{12}e^8}$	4) a) $\frac{2}{d^9}$ b) $6m^5$ c) $\frac{4b}{c^2}$ d) $\frac{1}{g}$ e) $\frac{h}{j^3}$ f) $\frac{3a}{5b}$ g) $\frac{-3a}{5b}$ h) $= u^2x^{-7}.u^{-2}x^{-6} = \frac{1}{x^{13}}$ i) $= \frac{d^7e^{18}}{2^2d^{12}e^8} = \frac{e^{10}}{2^2d^5}$

Worksheet 5.2 b

This worksheet focuses on raising a power, a product and a quotient to a further power, exponents & bases are positive or negative, coefficients include fractions and non-prime bases.

Questions

1)

Column A	**Column B**
i) $(p^5)^{-3}$	A. p^{12}
ii) $(p^{-2})^2$	B. p^{-25}
iii) $(p^2)^6$	C. p^{-4}
iv) $(p^1)^{-25}$	D. p^{-15}
	E. p^{-8}

a) Match the powers which are the same in the two columns.

b) One of the items in column B will not have a match from column A. Create an item for column A that will match with this item.

2) Look at the questions in the box below.

A. $-(m^6)^2$
B. $(-m^3)^4$
C. $((-m)^6)^2$
D. $(-m^4)^3$
E. $((-m)^3)^4$

a) Predict which expressions will give the same answer.
b) Work out the answers.
c) Were your predictions correct?
d) How is expression B different to expression E?

3) Answer the following questions about $\frac{6a(a^3.b^4)^{-2}}{-2d(a^3.b^4)^5}$

a) Look at the numerator. What is the exponent of the base $a^3.b^4$?

b) Is $6a(a^3.b^4)^{-2}$ the same as $(6a.a^3.b^4)^{-2}$? Explain.

c) Look at the denominator. What is the base that has an exponent of 5?

d) Can we use the law of dividing powers to simplify $(a^3.b^4)^{-2}$ and $(a^3.b^4)^5$? Explain.

e) Simplify $\frac{6a(a^3.b^4)^{-2}}{-2d(a^3.b^4)^5}$

4) Simplify. Give answers with positive exponents.

a) $3a^2b^3 \times (a^{-1}b^2)^2$

b) $\left(\frac{g^{-6}h^7}{2g^{-2}}\right)^{-3}$

c) $\frac{3^{-1}}{5^{-1}}$

d) $\left(\frac{c^2d^3}{d^{-2}}\right)^2 \times \left(\frac{c^0d^4}{c^2}\right)^{-2}$

e) $\left(\frac{3^{-1}}{5^{-1}}\right)^{-2}$

f) $\frac{-2(xy)^3}{(-xy)^2} \times \frac{(4xy)^2}{xy}$

#superpowers

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 5.2 b

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>
1)
<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>i) $(p^5)^{-3}$</td><td>A. p^{12}</td></tr>
<tr><td>ii) $(p^{-2})^2$</td><td>B. p^{-25}</td></tr>
<tr><td>iii) $(p^2)^6$</td><td>C. p^{-4}</td></tr>
<tr><td>iv) $(p^1)^{-25}$</td><td>D. p^{-15}</td></tr>
<tr><td>v)</td><td>E. p^{-8}</td></tr>
</table>
a) Match the powers which are the same in the two columns.
b) One of the items in column B will not have a match from column A. Create an item for column A that will match with this item..
</td><td>
1)
a)
<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>i) $(p^5)^{-3}$</td><td>D</td></tr>
<tr><td>ii) $(p^{-2})^2$</td><td>C</td></tr>
<tr><td>iii) $(p^2)^6$</td><td>A</td></tr>
<tr><td>iv) $(p^1)^{-25}$</td><td>B</td></tr>
<tr><td>No match</td><td>E</td></tr>
</table>
b) Possible match for E: $p^{-8} = (p^2)^{-4}$ Multiple answers based on the factors of 8. One number has to be positive and the other negative.
</td></tr>
<tr><td>
2) Look at the questions in the box below
A. $-(m^6)^2$
B. $(-m^3)^4$
C. $((-m)^6)^2$
D. $(-m^4)^3$
E. $((-m)^3)^4$
a) Predict which expressions will give the same answer.
b) Work out the answers.
c) Were your predictions correct?
d) How is expression B different to expression E?
</td><td>
2)
a) A and D are the same and B, C and E are the same.
b)
A. $-m^{12}$ B. m^{12} C. m^{12} D. $-m^{12}$ E. m^{12}
c) Based on learners answers.
d) In B, only m is raised to the power of 3.In E,$-m$ is raised to the power of 3.
</td></tr>
<tr><td>
3) Answer the following questions about: $\frac{6a(a^3.b^4)^{-2}}{-2d(a^3.b^4)^5}$
a) Look at the numerator. What is the exponent of the base $a^3.b^4$?
b) Is $6a(a^3.b^4)^{-2}$ the same as $(6a.a^3.b^4)^{-2}$? Explain.
c) Look at the denominator. What is the base that has an exponent of 5?
d) Can we use the law of dividing powers to simplify $(a^3.b^4)^{-2}$ and $(a^3.b^4)^5$? Explain.
f) Simplify $\frac{6a(a^3.b^4)^{-2}}{-2d(a^3.b^4)^5}$
</td><td>
3)
a) -2
b) No, the $6a$ is not raised to the power of-2.
c) $a^3.b^4$
d) Yes, the bases are the same.
e) $= \frac{6a}{-2d(a^3.b^4)^7} = -\frac{3a}{d.a^{21}b^{28}} = -\frac{3}{a^{20}b^{28}d}$
</td></tr>
<tr><td>
4) Simplify. Give answers with positive exponents.
a) $3a^2b^3 \times (a^{-1}b^2)^2$
b) $\frac{3^{-1}}{5^{-1}}$
c) $\left(\frac{3^{-1}}{5^{-1}}\right)^{-2}$
d) $\left(\frac{g^{-6}h^7}{2g^{-2}}\right)^{-3}$
e) $\left(\frac{c^2d^3}{d^{-2}}\right)^2 \times \left(\frac{c^0d^4}{c^2}\right)^{-2}$
f) $\frac{-2(xy)^3}{(-xy)^2} \times \frac{(4xy)^2}{xy}$
</td><td>
4)
a) $3b^7$
b) $\frac{5}{3}$
c) $\left(\frac{3}{5}\right)^2 = \frac{3^2}{5^2}$
d) $\frac{2^3g^{12}}{h^{21}}$
e) c^8d^2
f) $32x^2y^2$
</td></tr>
</table>

Worksheet 5.2 c

This worksheet focuses on multiplying and dividing powers, exponents & bases are positive or negative, coefficients include fractions.

Questions

1) Each expression below uses 4; a and 6 in various ways. The expressions are grouped into 3 clusters.

A.	$4a \times 6a$
B.	$4a \times 6$
C.	$(a^4)^6$
D.	$a^6 \div a^4$
E.	$a^4 \div a^6$
F.	$6a - 4a$
G.	$6a \div 4a$

a) What is the same and what is different between:
   - i) A, B and C
   - ii) D and E
   - iii) F and G

b) Simplify A to G.

2) Write the following in symbols:
   - a) A base p has an exponent of negative five.
   - b) A base negative $3m$ has an exponent of five.
   - c) The power b squared has a coefficient of a half.
   - d) c to the power of three divided by c to the negative four.
   - e) Five y squared is raised to the power of five.

3) Learners were asked to simplify $(-7a)a^{-7} \times \frac{1}{a^7}$. Two learners used different approaches and each approach is correct. We have given the first line of each approach below.

   A. $-7.a^{1+(-7)} \times \frac{1}{a^7}$ B. $(-7a) \times \frac{1}{a^7} \times \frac{1}{a^7}$

   - a) Describe what each learner did.
   - b) Complete the simplification for both approaches

4) Simplify. Give answers with positive exponents.

   a) $3d^{-8}$ b) $\frac{12}{6m^{-6}}$ c) $5c^{-3}k$

   d) $g^{-5} \times (-3)g^4$ e) $h^2j^3 \div (h^3j^8)$ f) $\frac{4a^4b^6}{7a^2b^5}$

   g) $\frac{-7a^3c^4}{5a^2c^{-5}}$ h) $\frac{u^{-3}w^{-5} \times u^1w^{-2}}{u^3w^6}$ i) $\frac{3^2d^4d^6 \times d^0d^{12}}{3^{-4}d^{12}d^8}$

Worksheet 5.2 c

Answers

Questions	**Answers**
1) Each expression below uses 4; a and 6 in various ways. The expressions are grouped into 3 clusters. A. $4a \times 6a$ B. $4a \times 6$ C. $(a^4)^6$ D. $a^6 \div a^4$ E. $a^4 \div a^6$ F. $6a - 4a$ G. $6a \div 4a$ a) What is the same and what is different between: i) A, B and C ii) D and E iii) F and G b) Simplify A to G.	1) a) In A to G the numbers 4 and 6 and letter a is used. i) All involve multiplication. The difference is that in A and B , 4 and/or 6 are coefficients but in C, 4 and 6 are exponents. ii) Both D and E involve division. The difference is that the exponents in D and E are swopped around. iii) Both use $4a$ and $6a$ but in F the operation is subtraction and in G the operation is division b) A. $24a^2$ B. $24a$ C. a^{24} D. a^2 E. $\frac{1}{a^2}$ F. $2a$ G. $\frac{3}{2}$
2) Write the following in symbols: a) A base p has an exponent of negative five. b) A base negative $3m$ has an exponent of five. c) The power b squared has a coefficient of a half. d) c to the power of three divided by c to the negative four. e) Five y squared is raised to the power of five.	2) **Answers** a) p^{-5} b) $(-3m)^5$ c) $\frac{1}{2}b^2$ d) $\frac{c^3}{c^{-4}}$ e) $(5y^2)^5$
3) Learners were asked to simplify $(-7a)a^{-7} \times \frac{1}{a^7}$. Two learners used different approaches and each approach is correct. We have given the first line of each approach below. A. $-7.a^{1+(-7)} \times \frac{1}{a^7}$ B. $(-7a) \times \frac{1}{a^7} \times \frac{1}{a^7}$ a) Describe what each learner did. b) Complete the simplification for both approaches.	3) **Answers** a) A added the exponents of a and a^{-7} as they are multiplied and have the same bases. B used the law $a^{-m} = \frac{1}{a^m}$ to rewrite a^{-7} with a positive exponent. b) A. $= -\frac{7a^{-6}}{a^7} = -\frac{7}{a^{13}}$ B. $-\frac{7a}{a^{14}} = -\frac{7}{a^{13}}$
4) Simplify. Give answers with positive exponents. a) $3d^{-8}$ b) $\frac{12}{6m^{-6}}$ c) $5c^{-3}k$ d) $g^{-5} \times (-3)g^4$ e) $h^2j^3 \div (h^3j^8)$ f) $\frac{4a^4b^6}{7a^2b^5}$ g) $\frac{-7a^3c^4}{5a^2c^{-5}}$ h) $\frac{u^{-3}w^{-5}\times u^1w^{-2}}{u^3w^6}$ i) $\frac{3^2d^4d^6\times d^0d^{12}}{3^{-4}d^{12}d^8}$	4) a) $\frac{3}{d^8}$ b) $2m^6$ c) $\frac{5k}{C^3}$ d) $-\frac{3}{g}$ e) $\frac{1}{hj^5}$ f) $\frac{4a^2b}{7}$ g) $-\frac{7ac^9}{5}$ h) $\frac{1}{u^5w^{13}}$ i) $\frac{3^6}{d^2}$

#super^{powers}

PRACTICE IN WORKING WITH POWERS & EXPONENTS

Worksheet 5.2 d

This worksheet focuses on raising a power, a product and a quotient to a further power, exponents & bases are positive or negative, coefficients include fractions and non-prime bases.

Questions

1)

Column A	**Column B**
i) $2.6^2a^{-4} \div 4a^8$	A. $8a^4$
ii) $2a^4 \times 6a^4$	B. $3^8 2^{16} a^8$
iii) $(3.2^2a)^8$	C. $18a^8$
iv) $2a^4 + 6a^4$	D. $12a^8$
	E. $18a^4$

a) Match the powers which are the same in the two columns.
b) One of the items in column B will not have a match from column A. Create an item for column A that will match with this item.

2)

a) Explain why $(5.2^2x)^6$ is not the same as $20x^6$
b) Explain why $3a^4 \times 7a^4$ does not equal $21a^{16}$
c) Explain why $3a^4 \times 7a^4$ does not equal $21a^4$
d) What errors were made in the following response:
$(6a^4 \times 4a^5)^3 = 10a^{20\times 3} = 10a^{24}$?
Give the correct answer.

3) Answer the following questions about: $\frac{7z^2(x^3y^4)^{-5}}{-14z(x^3y^4)^7}$

a) Look at the numerator.
What is the exponent of the base x^3y^4?
b) Look at the denominator.
What is the base that has an exponent of -7?
c) Is $7z^2(x^3y^4)^{-5}$ the same as $(7z^2x^3y^4)^{-5}$?
Explain.
d) Simplify to have only positive exponents:
$(x^3y^4)^{-5}$
e) Simplify $\frac{7z^2(x^3y^4)^{-5}}{-14z(x^3y^4)^7}$

4) Simplify. Give answers with positive exponents.

a) $4a^{-2}b^6 + (a^1b^{-3})^{-2}$

b) $\frac{14^{-1}}{7^{-1}}$

c) $\left(\frac{14^{-1}}{7^{-1}}\right)^{-2}$

d) $\left(\frac{k^{-6}j^7}{2k^{-2}}\right)^{-3} \times \left(\frac{k^{-6}j^7}{2k^{-2}}\right)^3$

e) $\left(\frac{m^0n^3}{m^2}\right)^2 \times \left(\frac{m^0n^3}{m^2}\right)^{-6}$

f) $\frac{-3(ab)^3}{(ab)^2} \times \frac{(5ab)^2}{-ab}$

Worksheet 5.2 d

Answers

<table>
<tr><th>Questions</th><th>Answers</th></tr>
<tr><td>
1)

<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>v) $2.6^2a^{-4} \div 4a^8$</td><td>F. $8a^4$</td></tr>
<tr><td>vi) $2a^4 \times 6a^4$</td><td>G. $3^8 2^{16} a^8$</td></tr>
<tr><td>vii) $(3.2^2a)^8$</td><td>H. $18a^8$</td></tr>
<tr><td>viii) $2a^4 + 6a^4$</td><td>I. $12a^8$</td></tr>
<tr><td></td><td>J. $18a^4$</td></tr>
</table>

a) Match the powers which are the same in the two columns.
b) One of the items in column B will not have a match from column A. Create an item for column A that will match with this item.
</td><td>
1)

a)

<table>
<tr><th>Column A</th><th>Column B</th></tr>
<tr><td>i) $2.6^2a^{-4} \div 4a^8$</td><td>E</td></tr>
<tr><td>ii) $2a^4 \times 6a^4$</td><td>D</td></tr>
<tr><td>iii) $(3.2^2a)^8$</td><td>B</td></tr>
<tr><td>iv) $2a^4 + 6a^4$</td><td>A</td></tr>
<tr><td>No match</td><td>C</td></tr>
</table>

b) Possible match for C:
$3^2.2a^2.(a^3)^2$
</td></tr>
<tr><td>
2)
a) Explain why $(5.2^2x)^6$ is not the same as $20x^6$
b) Explain why $3a^4 \times 7a^4$ does not equal $21a^{16}$
c) Explain why $3a^4 \times 7a^4$ does not equal $21a^4$
d) What errors were made in the following response:
$(6a^4 \times 4a^5)^3 = 10a^{20\times 3} = 10a^{24}$?
Give the correct answer.
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2)
a) In $(5.2^2x)^6$ the coefficient 20 is raised to the power of 6.
b) When you multiply powers with the same base, you <u>add</u> exponents.
c) When you multiply powers with the same base, you <u>add</u> exponents.
d) The coefficients of 6 and 4 should be <u>multiplied</u> to get 24. The exponents 4 and 5 must be <u>added</u> to get 9. This gives $(24a^9)^3$ which gives $24^3.a^{27}$
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3) Answer the following questions about: $\frac{7z^2(x^3y^4)^{-5}}{-14z(x^3y^4)^7}$
a) Look at the numerator.
What is the exponent of the base x^3y^4?
b) Look at the denominator. What is the base that has an exponent of -7?
c) Is $7z^2(x^3y^4)^{-5}$ the same as $(7z^2x^3y^4)^{-5}$?
Explain.
d) Simplify to have only positive exponents: $(x^3y^4)^{-5}$
e) Simplify $\frac{7z^2(x^3y^4)^{-5}}{-14z(x^3y^4)^7}$
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3)
a) -5
b) x^3y^4
c) No because in the first expression we are not raising the $7z^2$ to the power of -5
d) $(x^3y^4)^{-5}$ with positive exponents is $\frac{1}{(x^3y^4)^5}$ or $\frac{1}{x^{15}y^{20}}$
e) $\frac{z}{-2(x^3y^4)^{7+5}} = -\frac{z}{2(x^3y^4)^{12}} = -\frac{z}{2x^{36}y^{48}}$
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4) Simplify. Give answers with positive exponents.
a) $4a^{-2}b^6 + (a^1b^{-3})^{-2}$
b) $\frac{14^{-1}}{7^{-1}}$
c) $\left(\frac{14^{-1}}{7^{-1}}\right)^{-2}$
d) $\left(\frac{k^{-6}j^7}{2k^{-2}}\right)^{-3} \times \left(\frac{k^{-6}j^7}{2k^{-2}}\right)^3$
e) $\left(\frac{m^0n^3}{m^2}\right)^2 \times \left(\frac{m^0n^3}{m^2}\right)^{-6}$
f) $\frac{-3(ab)^3}{(ab)^2} \times \frac{(5ab)^2}{-ab}$
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4)
a) $\frac{5b^6}{a^2}$
b) $\frac{1}{2}$
c) 4
d) 1
e) $\left(\frac{m^0n^3}{m^2}\right)^{-4} = \frac{m^8}{n^{12}}$
f) $-3ab.(-5)ab = 15a^2b^2$
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