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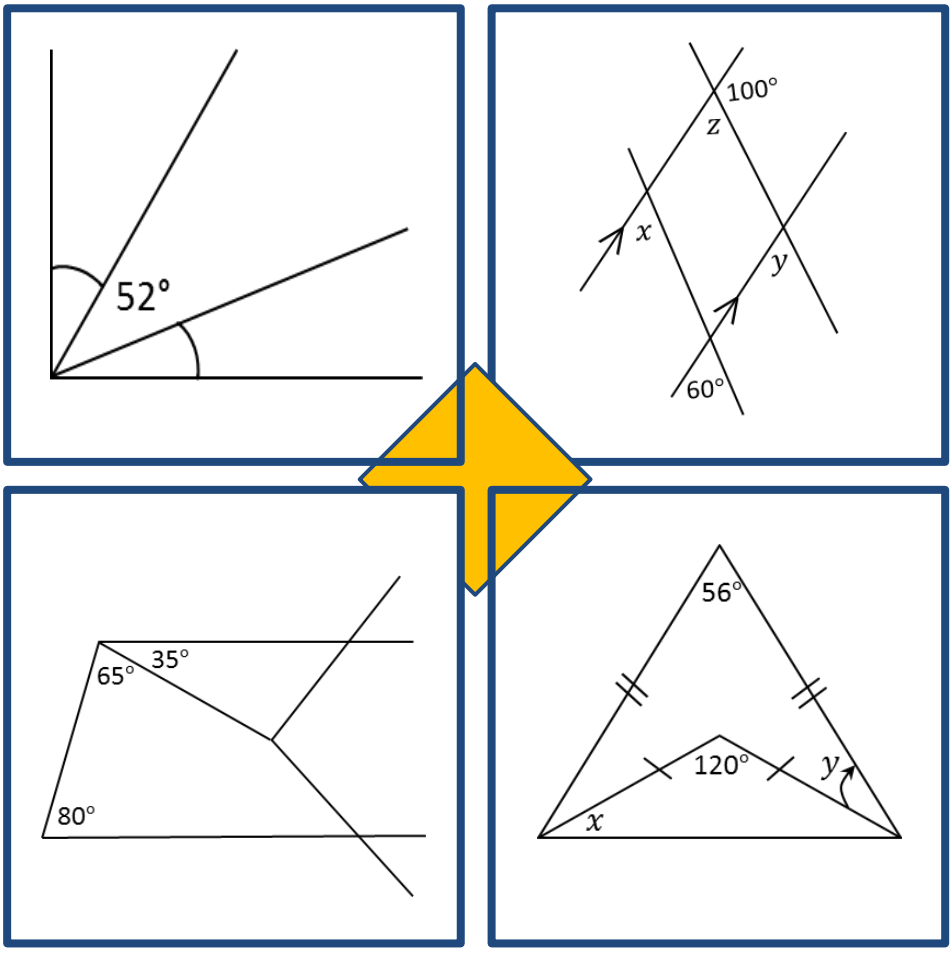
#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

VERSION 1.1



UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
FIRSTRAND
NRF National Research Foundation
science & innovation
Department:
Science and Innovation
REPUBLIC OF SOUTH AFRICA

#TRY–ângles: Practice in solving geometry problems

These materials were produced by the Wits Maths Connect Secondary (WMCS) project at the University of the Witwatersrand.
Visit us at www.witsmathsconnectsecondary.co.za

Team members:
Craig Pournara (Project leader)
Micky Lavery, Wanda Masondo, Yvonne Sanders and Fatou Sey
with thanks to Vasantha Moodley and Iresha Ratnayake

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PRACTICE IN SOLVING GEOMETRY PROBLEMS

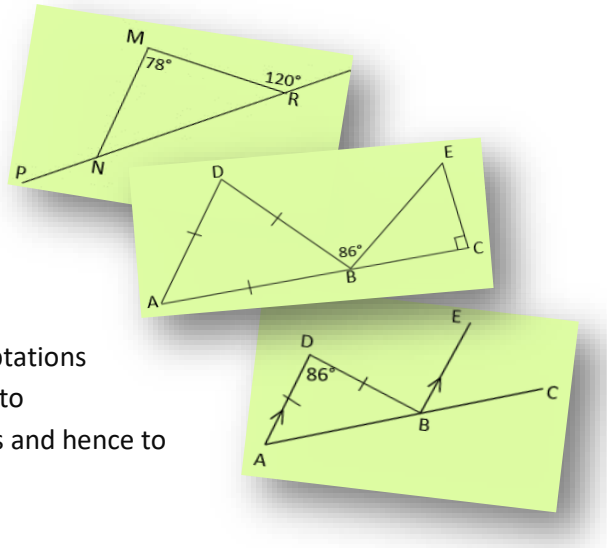
About this booklet

The 22 worksheets in this booklet provide practice in solving simple geometry problems (or riders). They focus on Grade 8 geometry content and include solutions for each question.

The pack is called ***#TRY–ângles*** because we know that geometry is difficult to learn and to teach. Nevertheless, we challenge everyone to *try*!! However, it's difficult to convince learners to try if the riders are too difficult from the outset. Our worksheets begin with examples that require only single statements to determine the answer. From there, we build up to examples that require two statements and then more. All riders involve numeric calculations of angles only.

We assume learners have been taught the content so that they can use these worksheets to practise. We do, however, provide a 2-page summary of the basics of angles, lines and triangles. The summary includes definitions and theorems but we don't emphasise the difference between the two. We also include the accepted abbreviations of geometry reasons distributed by the Department of Basic Education. While we are concerned that too much emphasis is being placed on formal geometric reasoning in Grades 8 and 9, we provide reasons in all our solutions to assist the teacher.

Worksheets begin with simple recall or knowledge tasks which direct learners to the properties or theorems that form the focus of the worksheet.



Riders begin with simple diagrams and gradually include more lines and angles. Often we use the same diagram in different orientations, with different labels and slight adaptations of the features as shown alongside. This will help learners to develop confidence in making sense of geometry diagrams and hence to cope with more complex diagrams in higher grades.

The worksheets are arranged in 3 sections with each worksheet in a section being slightly more difficult than the previous one and/or focusing on a different aspect.

Section	#wksts	Content
1	8	Simple riders involving right angles, angles on straight lines, angles around a point and vertically opposite angles
2	6	Angles formed when parallel lines are cut by a transversal, including "converses" and the content of section 1
3	8	Properties of triangles with several worksheets that include parallel lines

This pack does not include the Theorem of Pythagoras, similarity or congruence.

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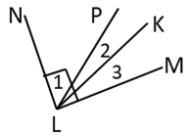
PRACTICE IN SOLVING GEOMETRY PROBLEMS

In geometry we need to give reasons for the statements we make about lines, angles and shapes.
There are specific reasons and specific abbreviations which you can use in tests and exams. We are introducing them in Grade 8 so that you can begin to learn them.

LINES AND ANGLES

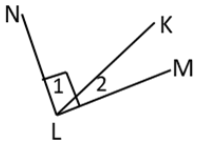
Two or more **adjacent angles in a right angle** add up to 90°.

$\hat{L}_1 + \hat{L}_2 + \hat{L}_3 = 90°$ NLM is a right ∠

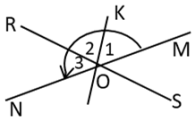


TWO adjacent **angles in a right angle** are complementary.

$\hat{L}_1 + \hat{L}_2 = 90°$ complementary $\angle s$



Two or more **adjacent angles on a straight line** add up to 180°.



We are given: a line NM and adjacent angles.

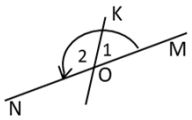
$K\hat{O}M + K\hat{O}R + R\hat{O}N = 180°$ $\angle s$ on a str line

OR $\hat{O}_1 + \hat{O}_2 + \hat{O}_3 = 180°$ $\angle s$ on a str line

TWO adjacent **angles on a straight line** are supplementary.

$K\hat{O}M + K\hat{O}N = 180°$

OR $\hat{O}_1 + \hat{O}_2 = 180°$ $\angle s$ on a str line

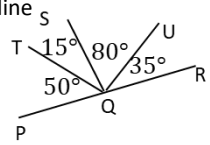


If two or more **adjacent angles add up to 180°**, the outer arms of these angles form a straight line.

$50° + 15° + 80° + 35° = 180°$

∴ PQR is a straight line adj $\angle s$ on a straight line

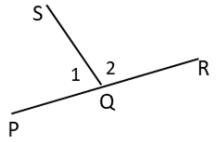
We are given adjacent angles that add up to 180°. This is the opposite (or converse) of ∠s on a str line.



If TWO **adjacent angles are supplementary**, the outer arms of these two angles form a straight line.

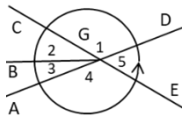
$\hat{Q}_1 + \hat{Q}_2 = 180°$ given

∴ PQR is a straight line adj $\angle s$ supp

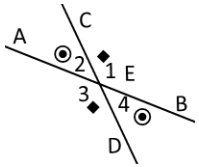


The **adjacent angles in a revolution** add up to 360°.
OR The **angles around a point** form a full turn which is 360°.

$\hat{G}_1 + \hat{G}_2 + \hat{G}_3 + \hat{G}_4 + \hat{G}_5 = 360°$ $\angle s$ in a rev
OR $\angle s$ round a pt



Vertically opposite angles are equal.



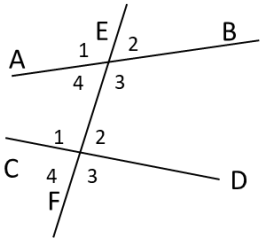
$\hat{E}_1 = \hat{E}_3$ vert opp $\angle s$
AND
$\hat{E}_2 = \hat{E}_4$ vert opp $\angle s$

NOTE: These angles are **opposite** each other.
They are not necessarily in a vertical orientation.

NOTE

- Complementary angles add up to 90°.
- Supplementary angles add up to 180°.
- The terms c*omplementary* and *supplementary* apply to the sum of **two** angles only.

ANGLES FORMED WHEN LINES ARE CUT BY TRANSVERSALS



When 2 lines are cut by a transversal, three important pairs of angles are formed:

Pairs of corresponding angles:
$\hat{E}_1$ and $\hat{F}_1$
$\hat{E}_2$ and $\hat{F}_2$
$\hat{E}_3$ and $\hat{F}_3$
$\hat{E}_4$ and $\hat{F}_4$

Pairs of alternate angles:
$\hat{E}_4$ and $\hat{F}_2$
$\hat{E}_3$ and $\hat{F}_1$

Pairs of co-interior angles:
$\hat{E}_4$ and $\hat{F}_1$
$\hat{E}_3$ and $\hat{F}_2$

NOTE

- AB and CD are <u>not</u> parallel

PARALLEL LINES CUT BY A TRANSVERSAL

When *parallel* lines are cut by a transversal, these pairs of angles have special relationships.

- Pairs of corresponding $\angle s$ are *equal*
- Pairs of alternate $\angle s$ are *equal*
- Pairs of co-interior $\angle s$ are *supplementary*

See next page for more details

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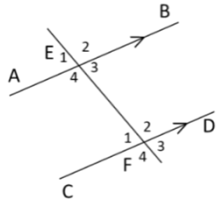
GIVEN: Parallel lines cut by transversals

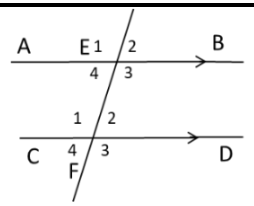
If $AB//CD$, then the **corresponding** angles are **equal**

$\hat{E}_1 = \hat{F}_1$
AND
$\hat{E}_2 = \hat{F}_2$
AND
$\hat{E}_3 = \hat{F}_3$
AND
$\hat{E}_4 = \hat{F}_4$ corresp ∠s, $AB//C$



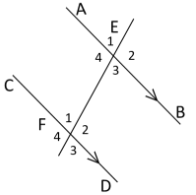
If $AB//CD$, then the **alternate** angles are **equal**.

$\hat{E}_4 = \hat{F}_2$ alt ∠s, $AB//CD$
AND
$\hat{E}_3 = \hat{F}_1$ alt ∠s, $AB//CD$



If $AB//CD$, then the **co-interior** angles are **supplementary** (i.e. add up to 180°)

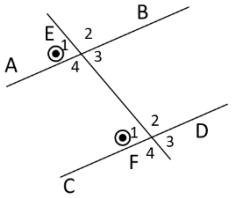
$\hat{E}_4 + \hat{F}_1 = 180°$ co-int ∠s, $AB//CD$
AND
$\hat{E}_3 + \hat{F}_2 = 180°$ co-int ∠s, $AB//CD$



GIVEN: Equal corresponding ∠s, equal alternate ∠s and supplementary co-interior ∠s

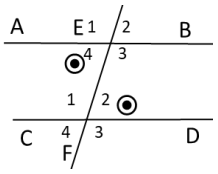
If the **corresponding** angles are **equal**, then the lines are parallel.

$\hat{E}_1 = \hat{F}_1$ given
$\therefore AB//CD$ corresp ∠s =



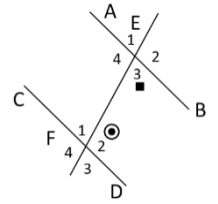
If the **alternate** angles are **equal**, then the lines are parallel.

$\hat{E}_4 = \hat{F}_2$ given
$\therefore AB//CD$ alt ∠s =



If the **co-interior** angles are **supplementary**, then the lines are parallel.

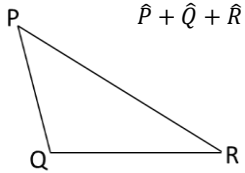
$\hat{E}_3 + \hat{F}_2 = 180°$ given
$\therefore AB//CD$ co-int ∠s sup



TRIANGLES

The **interior angles of a triangle** add up to 180°.

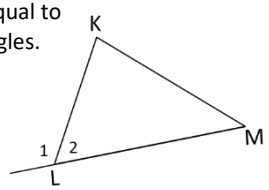
$\hat{P} + \hat{Q} + \hat{R} = 180°$ int ∠s Δ
OR sum of ∠s in Δ
OR ∠ sum in Δ



We don't say of the angles of a Δ are *supplementary* because there are *3* ∠s.

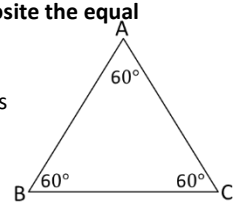
The **exterior angle of a triangle** is equal to the sum of the interior opposite angles.

$\hat{L}_1 = \hat{K} + \hat{M}$ ext ∠ of Δ



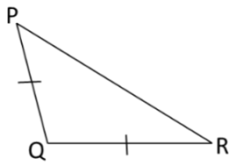
In an equilateral triangle, the **angles opposite the equal sides** are equal.

$\hat{A} = \hat{B} = \hat{C} = 60°$ given
$\therefore BC = AC = AB$ sides opp equal ∠s



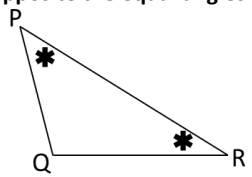
In an isosceles triangle, the **angles opposite the equal sides** are equal.

$PQ = QR$ given
$\therefore \hat{R} = \hat{P}$ ∠s opp equal sides



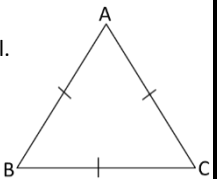
In an isosceles triangle, the **sides opposite the equal angles** are equal.

$\hat{R} = \hat{P}$ given
$\therefore PQ = QR$ sides opp equal ∠s



In an equilateral triangle, the **sides opposite the equal angles** are equal.

$BC = AC = AB$ given
$\therefore \hat{A} = \hat{B} = \hat{\hat{C}} = 60°$ ∠s opp equal sides



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PRACTICE IN SOLVING GEOMETRY PROBLEMS

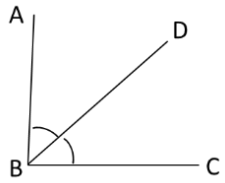
Worksheet 1.1

This worksheet focuses on right angles.

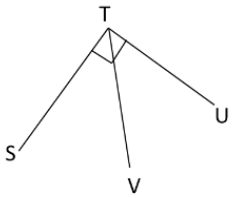
1) Complete: The size of a right angle is ___

2) Which of the following diagrams indicates a right angle?

A.

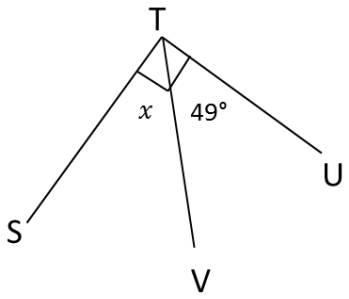


B.

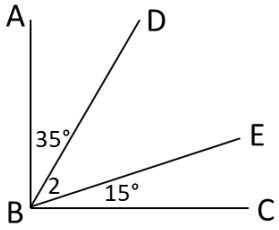


C.

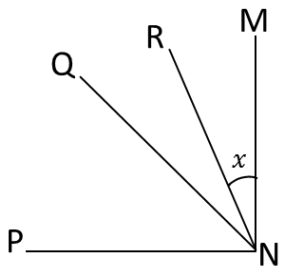
3) If $S\hat{T}U = 90°$, determine x.



4) If $A\hat{B}C = 90°$, determine $\hat{B}_2$.



5) Given: $M\hat{N}P = 90°$. $R\hat{N}Q$ is double $M\hat{N}R$. $Q\hat{N}P$ is triple $M\hat{N}R$.



a) Determine the value of x.

b) Determine the size of $Q\hat{N}P$.

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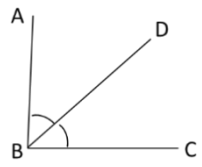
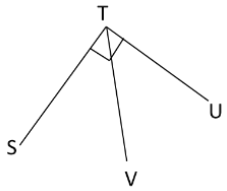
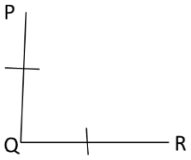
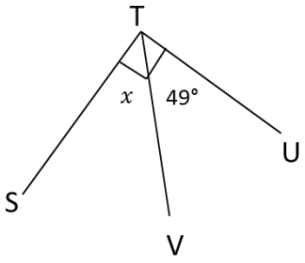
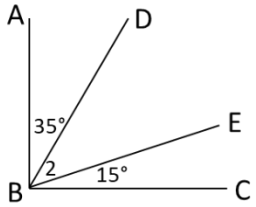
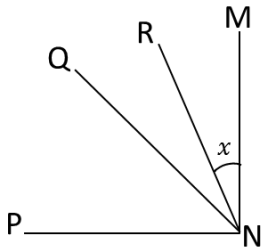
PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.1

Answers

Questions	**Answers**
1) Complete: The size of a right angle is ___	1) $90°$
2) Which of the following diagrams indicates a right angle? A.  B.  C. 	2) B
3) If $S\hat{T}U = 90°$, determine x. 	3) $x + 49° = 90°$ right ∠ $x = 41°$
4) If $A\hat{B}C = 90°$, determine $\hat{B}_2$ 	4) $35° + \hat{B}_2 + 15° = 90°$ right ∠ $x = 40°$
5) Given: $M\hat{N}P = 90°$. $R\hat{N}Q$ is double $M\hat{N}R$. $Q\hat{N}P$ is triple $M\hat{N}R$.  a) Determine the value of x. b) Determine the size of $Q\hat{N}P$.	5) We know that $M\hat{N}R = x$, so $R\hat{N}Q = 2x$ and $Q\hat{N}P = 3x$. a) We know that $M\hat{N}P = 90°$. $\therefore\ x + 2x + 3x = 90°$ right ∠ $6x = 90°$ $x = 15°$ b) $Q\hat{N}P = 3(15°) = 45°$

#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

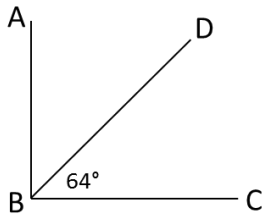
Worksheet 1.2

This worksheet focuses on right angles.

Questions

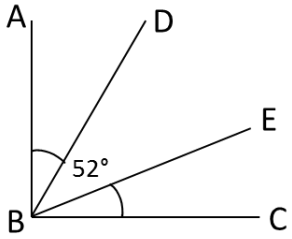
1) Complete: Complementary angles add up to ___

2) Given: $A\hat{B}C = 90°$.
Determine the size of $A\hat{B}D$.



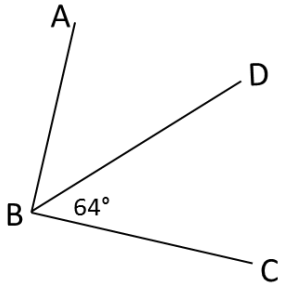
3) Given: $A\hat{B}C = 90°$ and $A\hat{B}D = C\hat{B}E$.
Determine the sizes of $A\hat{B}D$ and $C\hat{B}E$.

$A\hat{B}D$, $D\hat{B}E$, and $C\hat{B}E$ are not called *complementary* angles. Why?



4) If $A\hat{B}C > 90°$, which statement about $A\hat{B}D$ is always true?

A. $A\hat{B}D = 26°$
B. $A\hat{B}D = 27°$
C. $A\hat{B}D > 26°$
D. $A\hat{B}D < 26°$

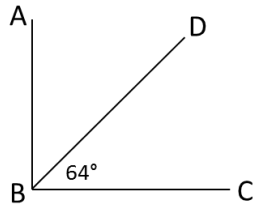
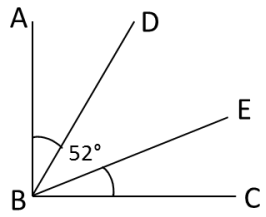
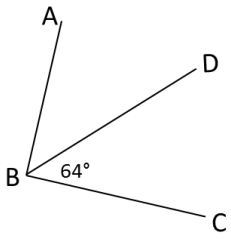


#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.2

Answers

Questions	Answers
1) Complete: Complementary angles add up to ___	1) $90°$
2) Given: $A\hat{B}C = 90°$ Determine the size of $A\hat{B}D$.  	2) $A\hat{B}D + 64° = 90°$ given $A\hat{B}D = 26°$
3) Given: $A\hat{B}C = 90°$ and $A\hat{B}D = C\hat{B}E$ Determine the size of $A\hat{B}D$ and $C\hat{B}E$.  	3) $A\hat{B}D + 52° + E\hat{B}C = 90°$ given $A\hat{B}D = E\hat{B}C$ given $2A\hat{B}D = 38°$ **OR** $2E\hat{B}C = 38°$ $A\hat{B}D = E\hat{B}C$ $= 19°$ They are not complementary angles because there are **3** angles that add up to $90°$
4) If $A\hat{B}C > 90°$, which statement about $A\hat{B}D$ is always true? A. $A\hat{B}D = 26°$ B. $A\hat{B}D = 27°$ C. $A\hat{B}D > 26°$ D. $A\hat{B}D < 26°$  	4) C is definitely true. It is possible that $A\hat{B}D = 27°$ because then $A\hat{B}C > 90°$. However, $A\hat{B}D$ could also be 28° or 29° etc. In fact, if $A\hat{B}C = 26{,}1°$ then $A\hat{B}C = 90{,}1°$ which is greater than $90°$. This means only C is <u>always</u> true.

#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

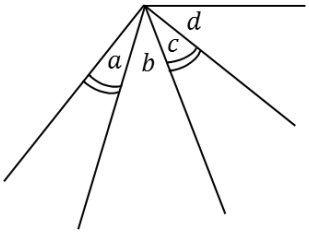
Worksheet 1.3

This worksheet focuses on right angles

Questions

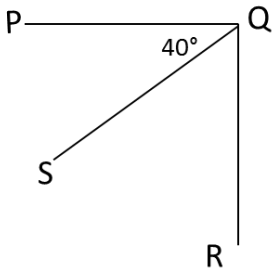
1) How many degrees in a right angle?

2) Which angles are equal in the diagram below?



3) Given: $P\hat{Q}R = 90°$

Determine the size of $S\hat{Q}R$.



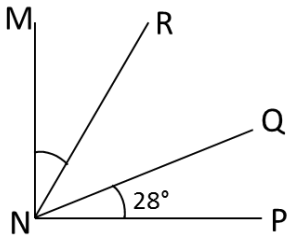
4) $M\hat{N}P$ is a right angle.

Determine the size of:

a) $M\hat{N}R$

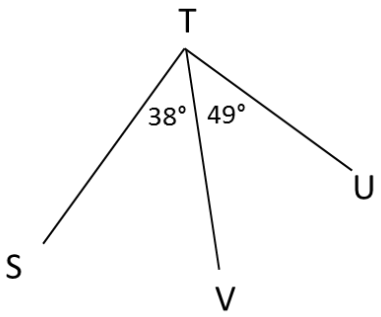
b) $Q\hat{N}R$

c) $P\hat{N}R$



5) Is $S\hat{T}U$ a right angle?

If not, what type of angle is it?

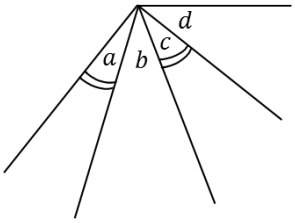
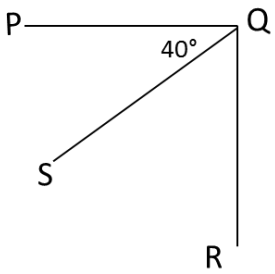
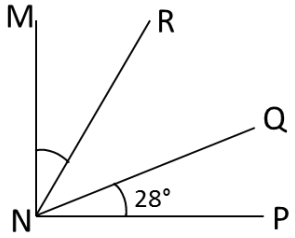
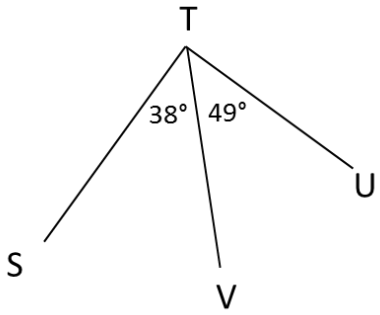


#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.3

Answers

Questions	**Answers**
1) How many degrees in a right angle?	1) $90°$
2) Which angles are equal in the diagram below? 	2) *a* and *c* are equal because they have the same markings on them
3) Given: $P\hat{Q}R = 90°$ Determine the size of $S\hat{Q}R$. 	3) $S\hat{Q}R = 90° - 40° = 50°$
4) $M\hat{N}P$ is a right angle. Determine the size of: a) $M\hat{N}R$ b) $Q\hat{N}R$ c) $P\hat{N}R$ 	4) a) $M\hat{N}R = Q\hat{N}P$ given $= 28°$ b) $Q\hat{N}R = 90° - 2(28°) = 34°$ c) $P\hat{N}R = 28° + 34° = 62°$
5) Is $S\hat{T}U$ a right angle? If not, what type of angle is it? 	5) $S\hat{T}U = 38° + 49° = 87°$ $\therefore$ $S\hat{T}U$ is not a right angle $S\hat{T}U$ is an acute angle

#TRY–ângles

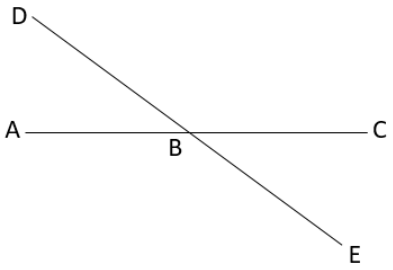
PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.4

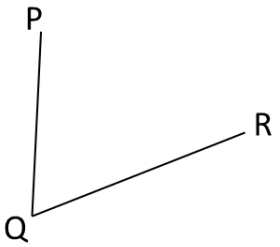
This worksheet focuses on angles around a point, angles on a straight line and vertically opposite angles

Questions

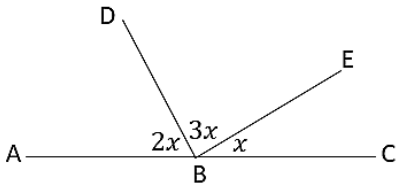
1) Line AC intersects line DE at B.
Complete and give reasons for each answer:
   a) $C\hat{B}E =$ ____
   b) $A\hat{B}E =$ ____
   c) $C\hat{B}E + A\hat{B}E =$ ___



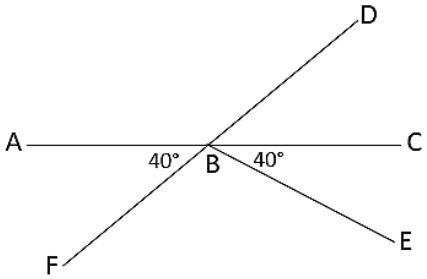
2) The diagram shows line segments PQ and QR forming an acute angle and a reflex angle.
   a) Indicate *acute* angle $P\hat{Q}R$ (draw an arc and label it)
   b) Indicate *reflex* angle $P\hat{Q}R$, using a different colour.



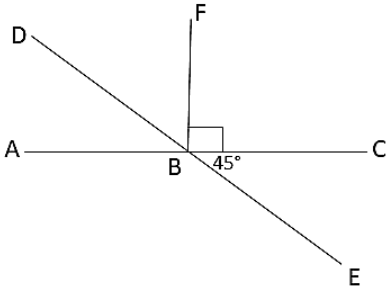
3) ABC is a straight line.
   a) Determine the value of x.
   b) Write down the sizes of the following angles:
      $E\hat{B}C$; $A\hat{B}D$; $D\hat{B}E$
   c) Explain why $E\hat{B}C, A\hat{B}D$ and $D\hat{B}E$ are not supplementary angles.



4) AC and FD intersect at B.
Determine the sizes of the following, giving reasons:
   a) $D\hat{B}E$
   b) $A\hat{B}D$
   c) $F\hat{B}E$



5) Consider the diagram below.
Determine the sizes of the following and give reasons:
   a) $D\hat{B}F$
   b) $A\hat{B}E$ when it is a reflex angle
   c) $A\hat{B}E$ when it is an obtuse angle

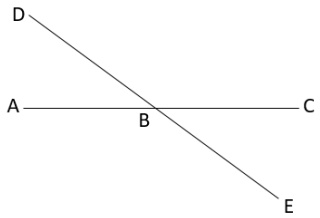
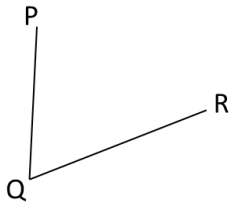
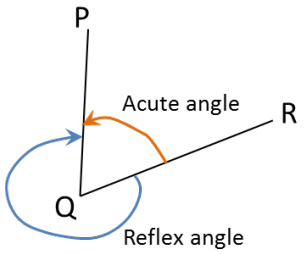
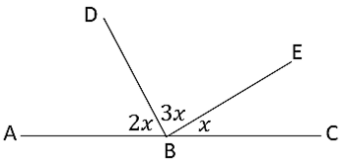
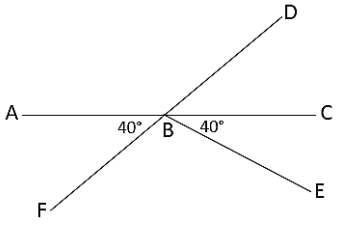
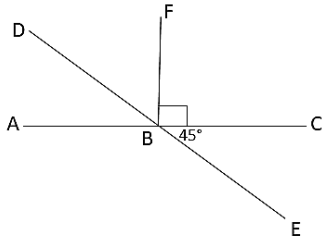


#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.4

Answers

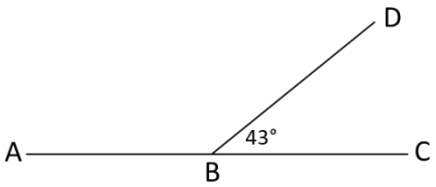
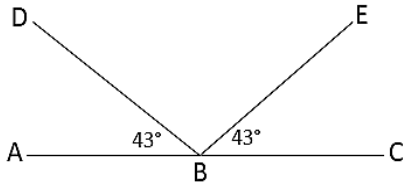
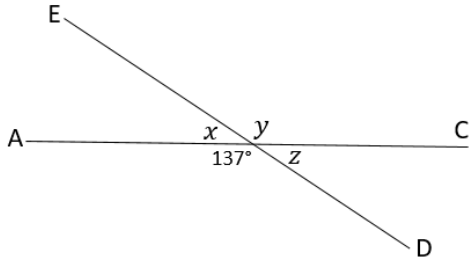
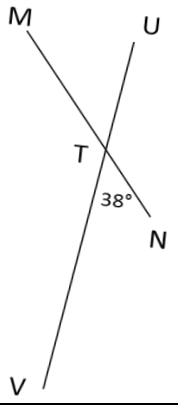
Questions	Answers
1) Line AC intersects line DE at B. Complete and give reasons for each answer: a) $C\hat{B}E =$ ____ b) $A\hat{B}E =$ ____ c) $C\hat{B}E + A\hat{B}E =$ ___ 	1) a) $A\hat{B}D$ vert opp $\angle s$ b) $D\hat{B}C$ vert opp $\angle s$ c) $C\hat{B}E + A\hat{B}E = 180°$ $\angle s$ on a str line
2) The diagram shows line segments PQ and QR forming an acute angle and a reflex angle. a. Indicate *acute* angle $P\hat{Q}R$ (draw an arc and label it) b. Indicate *reflex* angle $P\hat{Q}R$, using a different colour. 	2) 
3) ABC is a straight line. a) Determine the value of x. b) Write down the sizes of the following angles: $E\hat{B}C$; $A\hat{B}D$; $D\hat{B}E$ c) Explain why $E\hat{B}C, A\hat{B}D$ and $D\hat{B}E$ are not supplementary angles. 	3) a) $2x + 3x + x = 180°$ $\angle s$ on a str line $6x = 180°$ $x = 30°$ b) $E\hat{B}C = 30°$ $A\hat{B}D = 60°$ $D\hat{B}E = 90°$ c) *Supplementary* refers to only 2 angles that add up to $180°$
4) AC and FD intersect at B. Determine the sizes of the following, giving reasons: a) $D\hat{B}E$ b) $A\hat{B}D$ c) $F\hat{B}E$ 	4) a) $D\hat{B}E = D\hat{B}C + 40°$ $D\hat{B}C = 40°$ vert opp $\angle s$ $\therefore D\hat{B}E = 80°$ b) $A\hat{B}D = 180° - D\hat{B}C$ $\angle s$ on a str line $= 180° - 40° = 140°$ c) $F\hat{B}E = 180° - (40° + 40°)$ $\angle s$ on a str line $= 100°$
5) Consider the diagram below. Determine the sizes of the following and give reasons: a) $D\hat{B}F$ b) $A\hat{B}E$ when it is a reflex angle c) $A\hat{B}E$ when it is an obtuse angle 	5) a) $D\hat{B}F = 45°$ $\angle s$ on a str line b) $D\hat{B}A = 45°$ $\angle s$ on a str line Reflex $A\hat{B}E = D\hat{B}A + 180°$ $= 225°$ c) $A\hat{B}E = 360° - 225°$ $\angle s$ around a pt $= 135°$

#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.5

This worksheet focuses on angles on a straight line

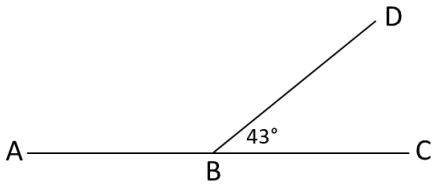
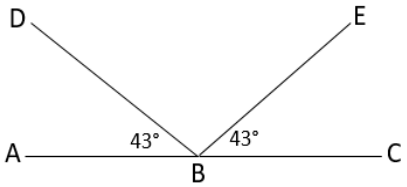
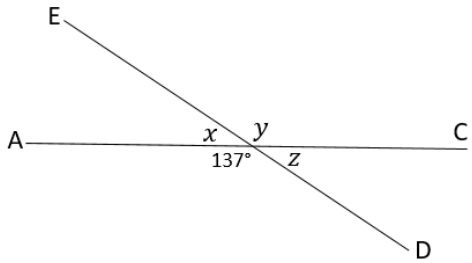
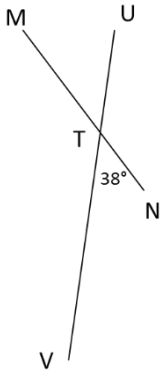
Questions
1) Complete: The sum of angles on a straight line is ____
2) ABC is a straight line. Determine the size of $A\hat{B}D$. 
3) ABC is a straight line. Is $D\hat{B}E$ equal to $90°$? Justify your answer. 
4) Line AC intersects line ED. Determine x, y and z <u>without using the fact</u> that vertically opposite angles are equal. 
5) MN intersects UV at T. Determine $M\hat{T}U$, $M\hat{T}V$ and $N\hat{T}U$ <u>without using the fact</u> that angles on a straight line add up to 180°. 

#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.5

Answers

Questions	**Answers**
1) Complete: The sum of angles on a straight line is ____	1) $180°$
2) ABC is a straight line. Determine the size of $A\hat{B}D$  	2) $A\hat{B}D + 43° = 180°$ $\angle s$ on a str line $A\hat{B}D = 137°$
3) ABC is a straight line. Is $D\hat{B}E$ equal to $90°$? Justify your answer.  	3) $D\hat{B}E + 2(43°) = 180°$ $\angle s$ on a str line $D\hat{B}E = 94°$ So, $D\hat{B}E$ is not $90°$ **OR** The $43°$ angles need to be $45°$ for $D\hat{B}E$ to equal $90°$
4) Line AC intersects line ED. Determine x, y and z <u>without using</u> the fact that vertically opposite angles are equal.  	4) $x = 43°$ ∠s on a str line ED $y = 137°$ ∠s on a str line AC $z = 43°$ ∠s on a str line ED or AC
5) MN intersects UV at T. Determine $M\hat{T}U$, $M\hat{T}V$ and $N\hat{T}U$ <u>without using</u> the fact that angles on a straight line add up to 180°.  	5) $M\hat{T}U = 38°$ vert opp ∠s $M\hat{T}V + N\hat{T}U + 2 \times 38° = 360°$ ∠s around a pt $M\hat{T}V + N\hat{T}U = 284°$ $M\hat{T}V = N\hat{T}U$ vert opp ∠s So, $M\hat{T}V = N\hat{T}U = \frac{284°}{2} = 142°$

#TRY−ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.6

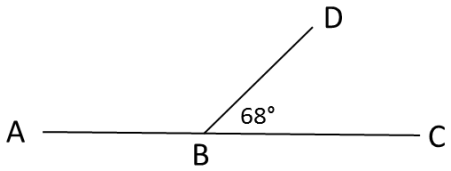
This worksheet focuses on angles on a straight line and includes showing that a straight line is formed

Questions

1) Complete: Adjacent angles on a straight line add up to ___

2) Given: $A\hat{B}C = 180°$

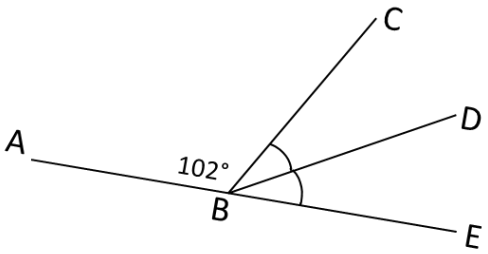
Determine the size of $A\hat{B}D$.



3) Assume that ABE is a straight line.

Determine the size of $C\hat{B}D$ and $D\hat{B}E$.

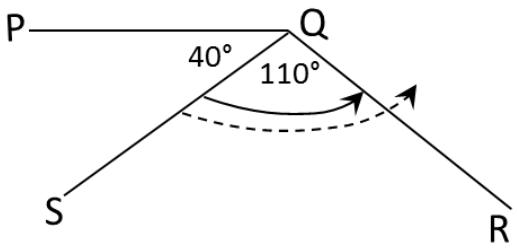
Give reasons for each statement.



4) $P\hat{Q}S = 40°$. QR is rotating anticlockwise so that $S\hat{Q}R = 110°$.

a) How many more degrees must QR rotate so that PQR forms a straight line?

b) When PQR forms a straight line, will the angles be supplementary?

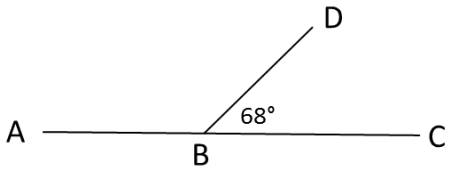
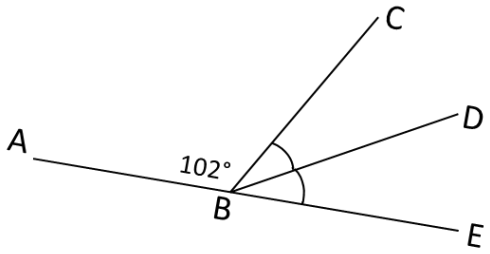
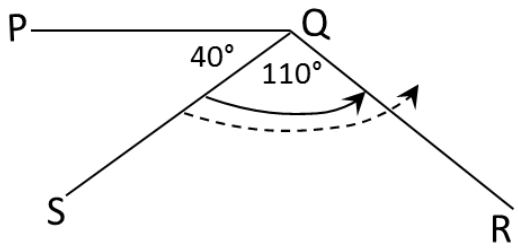


#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.6

Answers

Questions	Answers
1) Complete: Adjacent angles on a straight line add up to ___	1) $180°$
2) Given: $A\hat{B}C = 180°$ Determine the size of $A\hat{B}D$.  	2) $A\hat{B}D = 180° - 68°$ $\angle s$ on a str line $= 112°$
3) Assume that ABE is a straight line. Determine the size of $C\hat{B}D$ and $D\hat{B}E$. Give reasons for each statement.  	3) $C\hat{B}D = D\hat{B}E$ given $2(C\hat{B}D) + 102° = 180°$ $\angle s$ on a str line $2(C\hat{B}D) = 78°$ $C\hat{B}D = 39°$ $= D\hat{B}E$ **OR** $C\hat{B}E = 180° - 102$ $\angle s$ on a str line $= 78°$ $\therefore\ C\hat{B}D = D\hat{B}E = 39°$
4) $P\hat{Q}S = 40°$. QR is rotating anticlockwise so that $S\hat{Q}R = 110°$. a) How many more degrees must QR rotate so that PQR forms a straight line? b) When PQR forms a straight line, will the angles be supplementary?  	5) a) For PQR to be a straight line, $P\hat{Q}R$ must equal $180°$. $180° - 40° - 110° = 30°$ $\therefore$ QR must rotate by $30°$ b) Yes, there are 2 angles and they will add up to $180°$.

#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

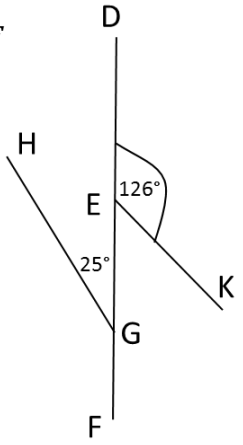
Worksheet 1.7

This worksheet focuses on angles on a straight line and complementary angles

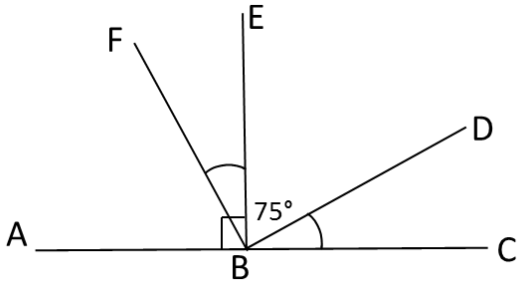
Questions

1) True or false:
   a) Adjacent supplementary angles add up to 360°.
   b) Complementary angles have a common arm and add up to 90°.

2) DEGF is a straight line.
   Determine the size of $K\hat{E}F$ and $H\hat{G}F$

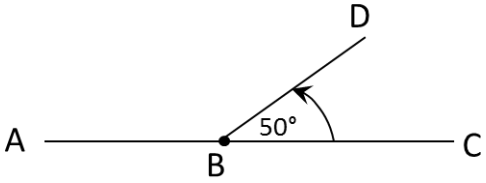


3) ABC is a straight line.
   a) Determine the sizes of $C\hat{B}D$, $E\hat{B}F$ and $A\hat{B}F$. Give reasons for each statement.
   b) Name as many pairs of complementary angles as possible.
   c) Are there any supplementary angles in the diagram?



4) If $A\hat{B}C < 180°$, which statement about $A\hat{B}D$ is always true?

   A. $A\hat{B}D = 129°$
   B. $A\hat{B}D = 130°$
   C. $A\hat{B}D > 130°$
   D. $A\hat{B}D < 130°$

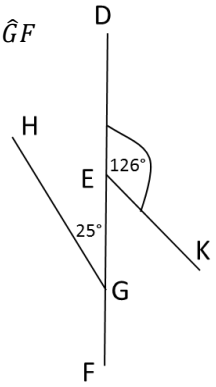
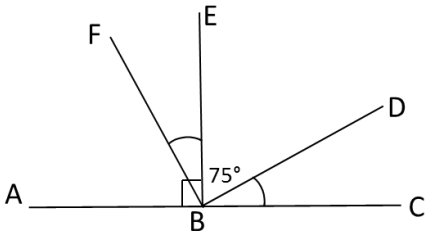
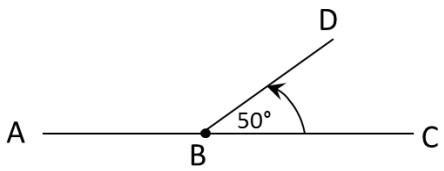


#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.7

Answers

Questions	Answers
1) True or false: a) Adjacent supplementary angles add up to 360°. b) Complementary angles have a common arm and add up to 90°.	1) a) False, they total 180°. b) False. Complementary angles are just 2 angles that add up to 90°. They do not need to have a common vertex or a common arm.
2) DEGF is a straight line. Determine the size of $K\hat{E}F$ and $H\hat{G}F$ 	2) $K\hat{E}F = 54°$ $\angle s$ on a str line $H\hat{G}F = 155°$ $\angle s$ on a str line
3) ABC is a straight line. a) Determine the sizes of $C\hat{B}D$, $E\hat{B}F$ and $A\hat{B}F$. Give reasons for each statement. b) Name as many pairs of complementary angles as possible. c) Are there any supplementary angles in the diagram? 	3) a) $E\hat{B}C = 90°$ $\angle s$ on a str line $C\hat{B}D = 90° - 75° = 15°$ $E\hat{B}F = C\hat{B}D$ given $= 15°$ $A\hat{B}F = 90° - 15° = 75°$ **OR** by $\angle s$ on a str line b) $A\hat{B}F$ & $E\hat{B}F$; $E\hat{B}F$ & $E\hat{B}D$; $E\hat{B}D$ & $C\hat{B}D$; $A\hat{B}F$ & $C\hat{B}D$ c) There are several pairs of angles that add up to $180°$. e.g. $A\hat{B}E$ and $E\hat{B}C$, $A\hat{B}F$ and $F\hat{B}C$, $A\hat{B}D$ and $D\hat{B}C$,
4) If $A\hat{B}C < 180°$, which statement about $A\hat{B}D$ is always true? A. $A\hat{B}D = 129°$ B. $A\hat{B}D = 130°$ C. $A\hat{B}D > 130°$ D. $A\hat{B}D < 130°$ 	4) D is always true. It is possible that $A\hat{B}D = 129°$ because this would make $A\hat{B}C < 180°$. However, $A\hat{B}D$ could also be 128° or 127° etc. So we can't say that A is <u>always</u> true but we can say that D is always true.

#TRY–ângles

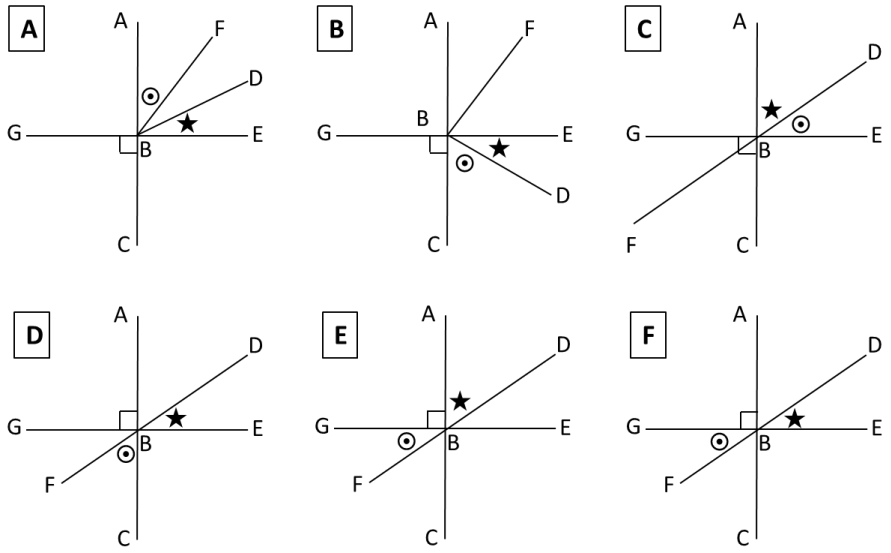
PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.8

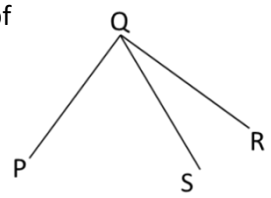
This worksheet focuses on angles on a straight line, complementary angles and vertically opposite angles

Questions

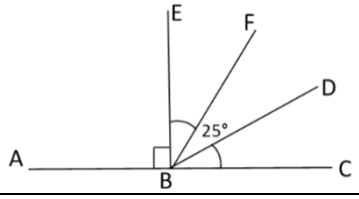
1) In each diagram below AC intersects GE at B. In diagrams C – F, FD intersects AC and GE at B.
We have marked 2 angles with symbols ★ and ⊙.
Which diagrams show the angle relationships in i – iii? Write the letter of the diagram/s.
   i) Adjacent complementary angles
   ii) Complementary angles
   iii) Vertically opposite angles



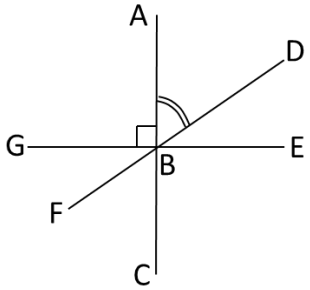
2) $P\hat{Q}S$ and $S\hat{Q}R$ are complementary angles.
If $P\hat{Q}S$ is three times the size of $S\hat{Q}R$, what is the size of each angle?



3) ABC is a straight line. What is the size of $A\hat{B}D$? Give reasons for your answer.



4) AC, DF and GE intersect at B. $A\hat{B}D = 50°$.
Indicate whether the following statements are TRUE or FALSE.
Support your answers with reasons (and calculations if necessary).
   a) $F\hat{B}C = 50°$
   b) $C\hat{B}E$ is a right angle
   c) $G\hat{B}D = F\hat{B}E$
   d) $A\hat{B}D$ and $G\hat{B}F$ are complementary angles
   e) $G\hat{B}A$, $F\hat{B}C$ and $D\hat{B}E$ are supplementary angles
   f) $G\hat{B}D - F\hat{B}A = 10°$



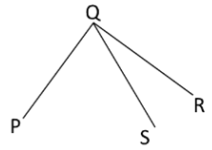
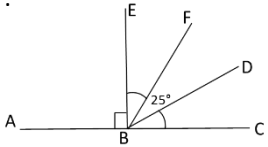
5) Read the following description of angles:
$A\hat{B}C + C\hat{B}D + D\hat{B}E = 180°$.
$A\hat{B}C$ is twice the size of $D\hat{B}E$ and of $C\hat{B}D$.
   a) Draw a diagram to represent this situation.
   b) Determine the size of each angle, giving reasons for your answers.

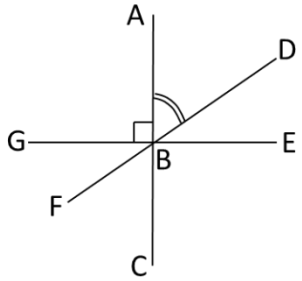
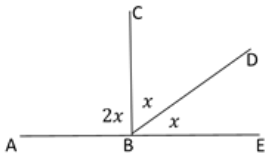
#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 1.8

Answers

Questions	1) In each diagram below, we have marked 2 angles with symbols ★ and ⊙. Which diagrams show the angle relationships in i – iii? Write the letter of the diagram/s. i) Adjacent complementary angles ii) Complementary angles iii) Vertically opposite angles	2) $P\hat{Q}S$ and $S\hat{Q}R$ are complementary angles. If $P\hat{Q}S$ is three times the size of $S\hat{Q}R$ what is the size of each angle? 	3) ABC is a straight line. What is the size of $A\hat{B}D$? Give reasons for your answer. 
Answers	1) i) B;C ii) B;C;D;E iii) F	2) $P\hat{Q}S$+$S\hat{Q}R = 90°$ complementary $\angle s$ **OR** given $P\hat{Q}S$ =3 $S\hat{Q}R$ given $\therefore 4\ S\hat{Q}R = 90°$ $S\hat{Q}R = 22{,}5°$ $\therefore P\hat{Q}S\ = 3(22{,}5°) = 67{,}5°$	3) $A\hat{B}E + E\hat{B}C = 180°$ $\angle s$ on a str line $A\hat{B}E = 90°$ given So, $2(E\hat{B}F) + 25° = 90°$ $E\hat{B}F = 65° \div 2 = 32{,}5°$ $A\hat{B}D = 90° + 32{,}5° + 25° = 147{,}5°$

	Questions	**Answers**
	4) AC, DF and GE intersect at B. $A\hat{B}D = 50°$. State whether the following statements are TRUE or FALSE. Support your answers with reasons (and calculations if necessary). a) $F\hat{B}C = 50°$ b) $C\hat{B}E$ is a right angle c) $G\hat{B}D = F\hat{B}E$ d) $A\hat{B}D$ and $G\hat{B}F$ are complementary angles e) $G\hat{B}A$, $F\hat{B}C$ and $D\hat{B}E$ are supplementary angles f) $G\hat{B}D - F\hat{B}A = 10°$ 	4) a) True: vert opp $\angle s$ b) True: vert opp $\angle s$ c) True: both angles consist of $90° + 50°$ d) True: They add up $90°$. We can deduce this from $\angle s$ on a str line: where $A\hat{B}G = 90°$. e) False: Supplementary angles are TWO angles that add to $180°$ f) True: $90° + 50° - (90° + 40°) = 10°$
	5) Read the following description of angles: $A\hat{B}C + C\hat{B}D + D\hat{B}E = 180°$. $A\hat{B}C$ is twice the size of $D\hat{B}E$ and of $C\hat{B}D$. a) Draw a diagram to represent this situation. b) Determine the size of $A\hat{B}C$, giving reasons for your answer.	5 a)  b) $4x = 180°$ given $2x = 90°$ $\therefore\ A\hat{B}C = 90°$

#TRY−ângles

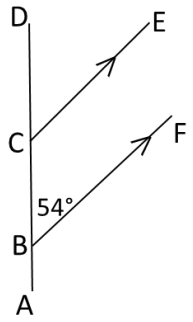
PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 2.1

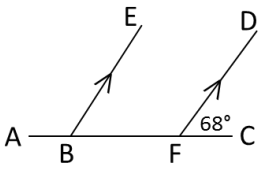
This worksheet deals mainly with relationships between alternate, corresponding and co-interior angles when parallel lines are cut by a transversal. It draws on earlier work involving angles around a point, and angles on a straight line.

1) Complete the statements:
   a) The sum of angles around a point is _____
   b) If two parallel lines are cut by a transversal, then their co-interior angles _____

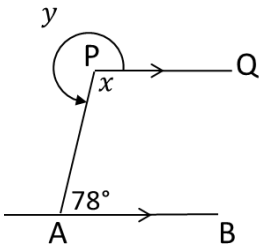
2) DA is a straight line. Determine the size of $D\hat{C}E$ and $E\hat{C}B$. Give a reason for each statement.



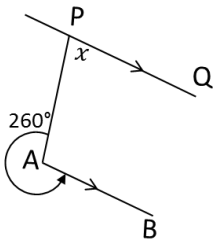
3) ABFC is a straight line. Determine the size of $E\hat{B}A$. Give reasons.



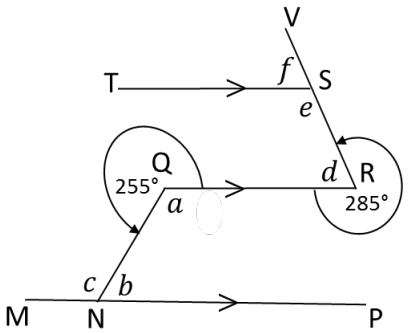
4) Determine the sizes of x and y.
   Give reasons.



5) Determine the size of x.
   Give reasons.

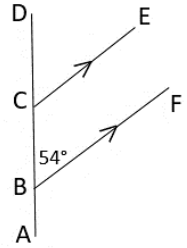
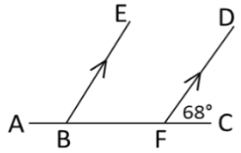
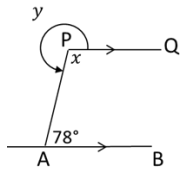
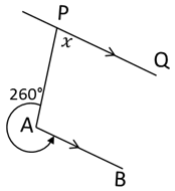
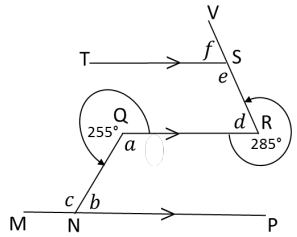


6) MP is a straight line. S lies on VR.
   Determine a, b, c, d, e and f (preferably) in this order. Give reasons.



Worksheet 2.1

Answers

Questions	1) Complete the statements: a) The sum of angles around a point is ___ b) If two parallel lines are cut by a transversal, then their co-interior angles _____	2) DA is a straight line. Determine the size of $D\hat{C}E$ and $E\hat{C}B$. Give a reason for each statement. 	3) ABFC is a straight line. Determine the size of $E\hat{B}A$. Give reasons. 
Answers	1) a) $360°$ b) Are supplementary **OR** add up to $180°$	2) $D\hat{C}E = 54°$ corresp ∠s, CE//BF $E\hat{C}B = 126°$ co-int ∠s, CE//BF **OR** ∠s on a str line	3) $E\hat{B}F = 68°$ corresp ∠s, BE//DF $E\hat{B}A = 112°$ ∠s on a str line
Questions	4) Determine the sizes of x and y. Give reasons. 	5) Determine the size of x. Give reasons. 	6) MP is a straight line. S lies on VR. Determine a, b, c, d, e and f (preferably) in this order. Give reasons. 
Answers	4) $x = 102°$ co-int ∠s, PQ//AB $y = 258°$ ∠s around a pt	5) $P\hat{A}B = 100°$ ∠s around a pt $x = 80°$ co-int ∠s, AB//PQ	6) $a = 105°$ ∠s around a pt $b = 75°$ co-int ∠s, MP//QR $c = 105°$ alt ∠s, MP//QR **OR** ∠s on a str line $d = 75°$ ∠s around a pt $e = 105°$ co-int ∠s, TS//QR $f = 75°$ corres ∠s, TS//QR **OR** ∠s on a str line

Worksheet 2.2

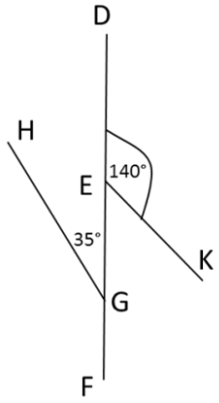
In this worksheet you will

- Use your knowledge about alternate, corresponding and co-interior angles to state whether lines cut by a transversal are parallel or not
- Work with angles on a straight line, vertically opposite angles and angle relationships when parallel lines are cut by a transversal.

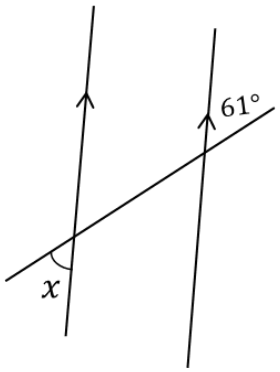
1) Is this statement TRUE or FALSE: *Alternate angles are always equal*.

2) If a transversal intersects 5 parallel lines,
   a) How many angles will be formed?
   b) How many pairs of co-interior angles will be supplementary?

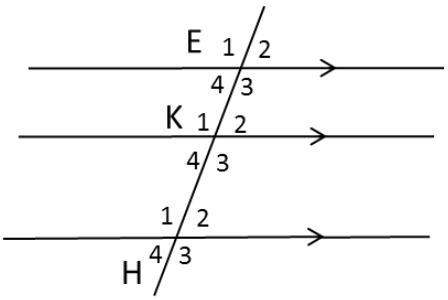
3) DEGF is a straight line. Is GH ∥ EK?
   Justify your answer.



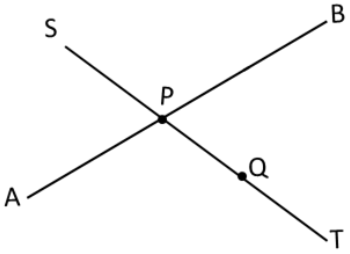
4) Is $x = 61°$? Justify your answer.



5) The diagram shows 3 parallel lines cut by a transversal.
   If $\widehat{K}_2 = 78°$, determine the size of all the other angles in the diagram.
   Copy the diagram and write in the angle sizes.

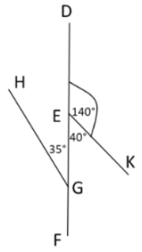
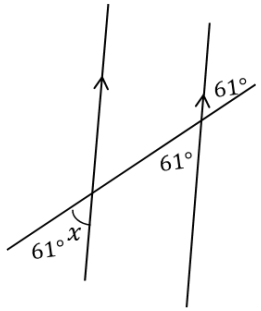
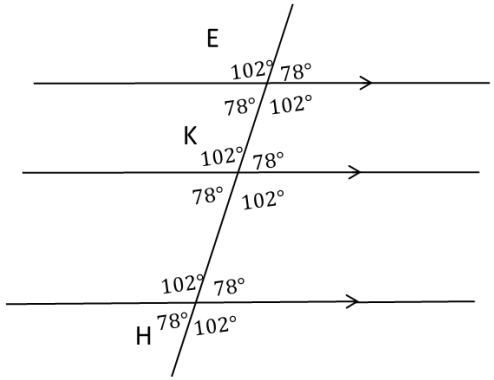
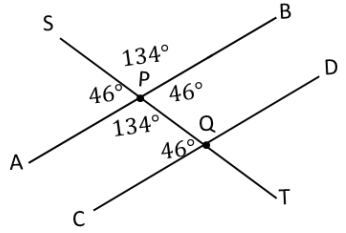


6) AB intersects SPQT at P.
   a) If S$\hat{P}B = 134°$, determine the sizes of the other 3 angles.
   b) Draw line CD so that it intersects ST at Q and is parallel to AB. Determine the size of $P\hat{Q}C$.



Worksheet 2.2

Answers

<table>
<tr><td>Questions</td><td>1) Is this statement TRUE or FALSE: Alternate angles are always equal.</td><td>2) If a transversal intersects 5 parallel lines,
a) How many angles will be formed?
b) How many pairs of co-interior angles will be supplementary?</td><td>3) DEGF is a straight line. Is GH ∥ EK?
Justify your answer.</td></tr>
<tr><td>Answers</td><td>1)
False. They will only be equal if the lines are parallel.</td><td>2)
a) 20 angles. At each intersection 4 angles are formed. Imagine the diagram for Q5 with 2 more parallel lines.
b) 8 pairs, lying on both sides of the transversal. See the 4 pairs in the diagram for Q5 and imagine the diagram with 5 parallel lines.</td><td>3)
NO, because the alternate angles are not equal, and so the lines will not be parallel
 
</td></tr>
<tr><td>Questions</td><td>4) Is $x = 61°$? Justify your answer.</td><td>5) The diagram shows 3 parallel lines cut by a transversal. If $\widehat{K}_2 = 78°$, determine the size of all the other angles in the diagram.
Copy the diagram and write in the angle sizes.</td><td>6) AB intersects SPQT at P.
a) If $\text{S}\widehat{\text{P}}\text{B} = 134°$, determine the sizes of the other 3 angles.
b) Draw line CD so that it intersects ST at Q and is parallel to AB. Determine the size of $\text{P}\widehat{\text{Q}}\text{C}$.</td></tr>
<tr><td>Answers</td><td>4)
Yes, because it is vertically opposite to the corresponding angle to $61°$ <u>and</u> the lines are parallel. See diagram.
 
</td><td>5)
 
</td><td>6)
a) $A\hat{P}Q = 134°$ vert opp ∠s
$S\hat{P}A = 46°$ ∠s on a str line
$STB = 46°$ vert opp ∠s
b) $A\hat{P}Q = S\hat{P}B = 134°$ vert opp ∠s
$P\hat{Q}C = 46°$ co-int ∠s, AB//CD
 

OR swop C and D:
$S\hat{P}B = P\hat{Q}C = 134°$ corresp ∠s, AB//CD</td></tr>
</table>

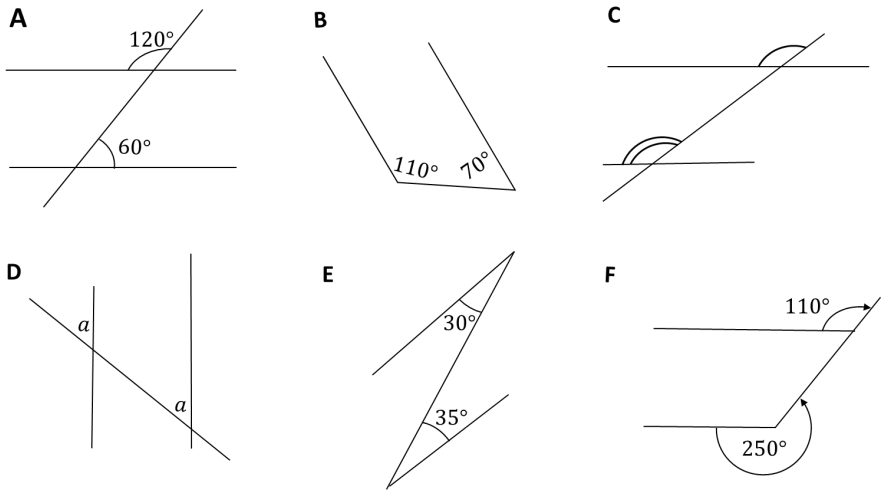
#TRY−ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 2.3

This worksheet focuses on corresponding, alternate and co-interior angles when pairs of lines are cut by a transversal.

1) Which of the following diagrams do not represent parallel lines?

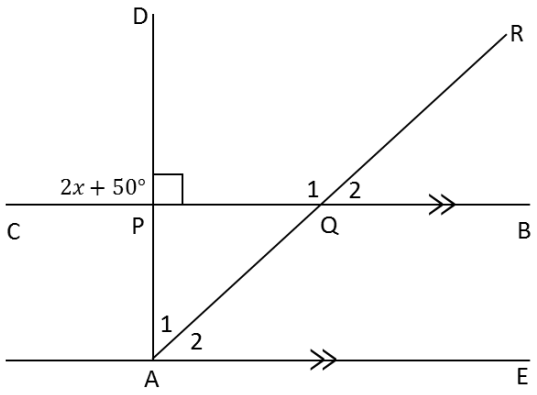


2) AE ∥ BC. DA ⊥ BC and intersects BC at P.
AR cuts BC at Q.
Use the diagram to answer the following questions.
Give reasons for your answers.

a) Calculate the value of x.

b) If $\hat{A}_2 = 3x$, what is the size of $\hat{Q}_2$?

c) Calculate the size of $\hat{Q}_1$.



3) In the diagram YZ, XW and YX are straight lines.

a) Is YZ is parallel to XW? Give a reason for your answer.

b) State whether the following are TRUE or FALSE:

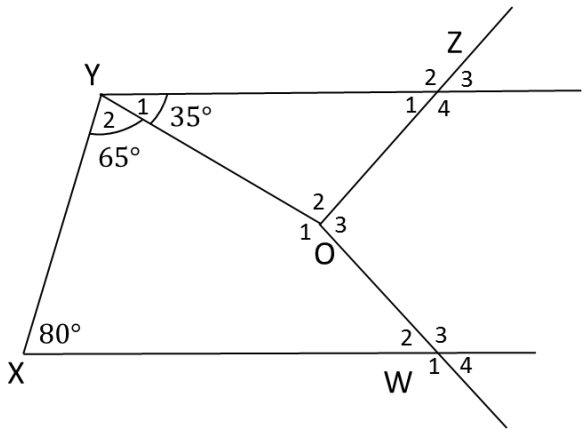
i) $\hat{O}_1 + \hat{O}_2 + \hat{O}_3 = 360^o$

ii) $\widehat{W}_1 = \widehat{W}_3$

iii) $\hat{Z}_4 + \widehat{W}_3 = 180°$

iv) $\hat{O}_1 = \hat{O}_3$

c) If $\widehat{W}_4 = \hat{Z}_3 = 35°$, determine the size of all the angles around W and Z. Give reasons.

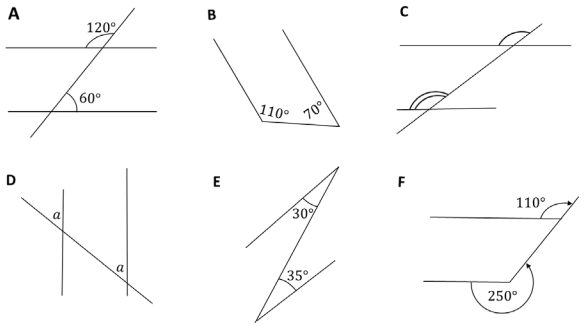


Worksheet 2.3

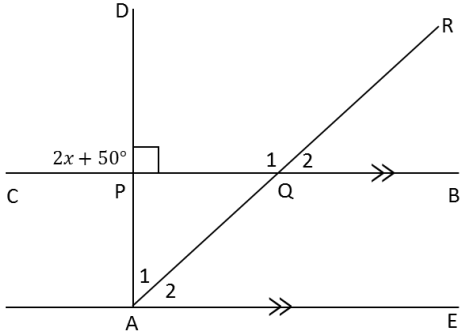
Answers

Questions

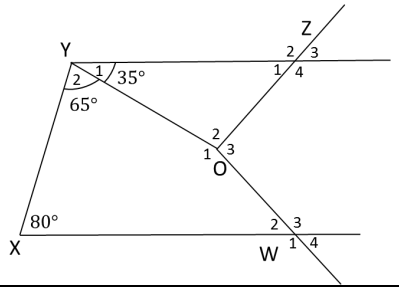
1) Which of the following diagrams do not represent parallel lines?



2) AE ∥ BC. DA ⊥ BC and intersects BC at P. AR cuts BC at Q.
Use the diagram to answer the following questions. Give reasons for your answers.
a) The value of x.
b) If $\hat{A}_2 = 3x$, what is the size of $\hat{Q}_2$?
c) Determine the size of $\hat{Q}_1$.



3) In the diagram YZ, XW and YX are straight lines.
a) Is YZ is parallel to XW? Give a reason for your answer.
b) State whether the following is TRUE or FALSE:
i) $\hat{O}_1 + \hat{O}_2 + \hat{O}_3 = 360^o$
ii) $\widehat{W}_1 = \widehat{W}_3$
iii) $\hat{Z}_4 + \widehat{W}_3 = 180°$
iv) $\hat{O}_1 = \hat{O}_3$
c) If $\widehat{W}_4 = \hat{Z}_3 = 35°$, determine the size of all the angles around W and Z. Give reasons.



Answers

1)
C : The marked angles are in corresponding positions but the markings are different which means the angles are not equal
E : Angles in alternate positions are not equal

2)
a) $2x + 50° + 90° = 180°$ ∠s on a str line
$x = 20°$

b) $\hat{A}_2 = 3(20°) = 60^o$
$\hat{Q}_2 = 60^o$ corresp ∠s AE//CB

c) $\hat{Q}_1 = 120^o$ ∠s on a str line

3)
a) Yes, co-int ∠s sup
b)
i) True angles around a point
ii) True vert opp ∠s
iii) False ZOW is not a transversal
iv) False $\hat{O}_1 > \hat{O}_3$

c) $\widehat{W}_2 = 35°$ vert opp∠s
$\widehat{W}_1 = 145°$ ∠s on a str line
$\widehat{W}_3 = 145°$ vert opp ∠s
$\hat{Z}_1 = 35°$ vert opp ∠s
$\hat{Z}_4 = 145°$ ∠s on a str line
$\hat{Z}_2 = 145°$ vert opp ∠s

#TRY−ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 2.4

This worksheet focuses on determining angle sizes or values of variables given parallel lines and includes proving that lines are parallel.

1) Which of the following symbols represent parallel lines?

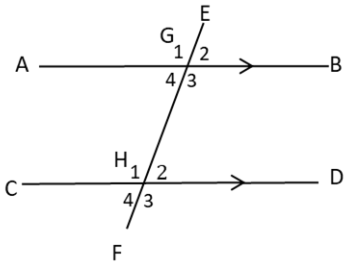
There may be more than 1 correct answer in Q1 and Q2.

A. ≡
B. =
C. ‖
D. |||
E. //
F. ⊥

2) We know that lines in a geometry diagram are parallel when …

A. they are equal in length
B. they are the same distance apart
C. they don't intersect
D. they have arrows like this: →
E. they have short lines like this: —|—

3) You must use the diagram below three times. Each time the size of the given angle will change.



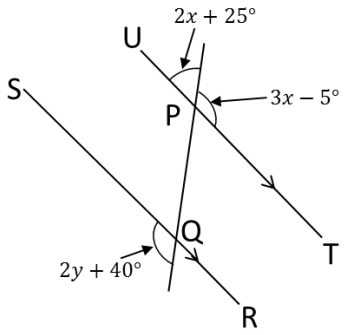
a) $\hat{H}_2 = 70°$. Write down the sizes of all angles in the diagram. You do not need to give reasons.

Now we will focus on writing reasons.

b) If $\hat{H}_2 = 75°$, determine the size of $\hat{G}_3$, $\hat{G}_2$, $\hat{G}_1$ and $\hat{G}_4$ in the order they are listed here. Give reasons for each statement.

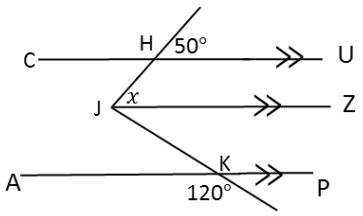
c) If $\hat{G}_1 = 115°$, determine the size of $\hat{G}_3$, $\hat{H}_1$, and $\hat{H}_2$ in the order they are listed here. Give reasons for each statement.

4) You will use algebra to answer this question



a) Determine the value of x. Give reasons.
b) Determine the value of y. Give reasons.
c) Copy the diagram and fill in the sizes of all 8 angles on the diagram.

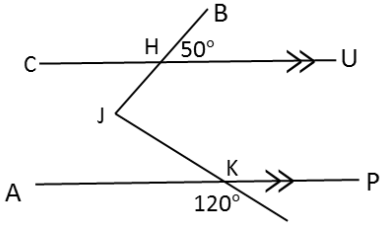
5) Three parallel lines are indicated on the diagram.



a) Determine x, giving reasons.
b) Is $H\hat{J}K$ a right angle? Explain.

6)

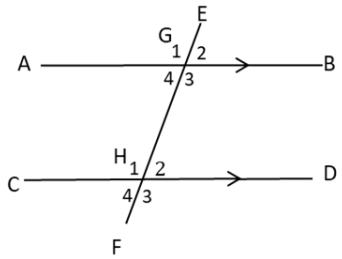
a) Fill in the sizes of as many angles as possible.



b) Join HP. When you do this $J\hat{H}P = 85°$. Is HP// JK? Explain.

Worksheet 2.4
Answers

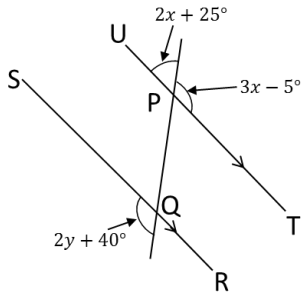
Question	Answer	Question	Answer
1) Which of the following symbols represent parallel lines? A. ≡ B. = C. ‖ D. ||| E. // F. ⊥	1) C and E	2) We know that lines in a geometry diagram are parallel when … A. they are equal in length B. they are the same distance apart C. they don't intersect D. they have arrows like this:  E. they have short lines like this: 	2) B, C and D

Question	Answer
3) You must use the diagram below three times. Each time the size of the given angle will change.  a) $\hat{H}_2 = 70°$. Write down the sizes of all angles in the diagram. You do not need to give reasons. Now we will focus on writing reasons. b) If $\hat{H}_2 = 75°$, determine the size of $\hat{G}_3$, $\hat{G}_2$, $\hat{G}_1$ and $\hat{G}_4$ in the order they are listed here. Give reasons for each statement. c) If $\hat{G}_1 = 115°$, determine the size of $\hat{G}_3$, $\hat{H}_1$, and $\hat{H}_2$ in the order they are listed here. Give reasons for each statement.	3) a) $\hat{H}_2 = \hat{H}_4 = \hat{G}_4 = \hat{G}_2 = 70°$ and $\hat{H}_1 = \hat{H}_3 = \hat{G}_3 = \hat{G}_1 = 110°$ b) $\hat{G}_3 = 105°$ co-int $\angle s$, AB//CD $\hat{G}_2 = 75°$ $\angle s$ on a str line **OR** corresp $\angle s$, AB//CD $\hat{G}_1 = 105°$ vert opp $\angle s$ $\hat{G}_4 = 75°$ alt $\angle s$, AB//CD c) $\hat{G}_3 = 115°$ vert opp $\angle s$ $\hat{H}_1 = 115°$ alt $\angle s$, AB//CD $\hat{H}_2 = 65°$ $\angle s$ on a str line **OR** co-int $\angle s$, AB//CD

Worksheet 2.4
Answers continued

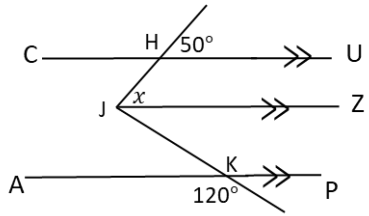
Questions

4) You will use algebra to answer this question.



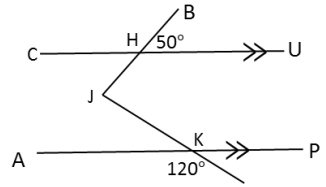
a) Determine the value of x. Give reasons.
b) Determine the value of y. Give reasons.
c) Copy the diagram and fill in the sizes of all 8 angles on the diagram.

5) Three parallel lines are indicated on the diagram.



a) Determine x, giving reasons.
b) Is $H\hat{J}K$ a right angle? Explain.

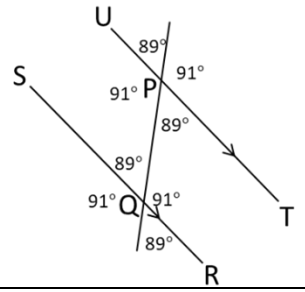
6)
a) Fill in the sizes of as many angles as possible on the diagram.



b) Join HP. When you do this $J\hat{H}P = 85°$.
Is HP// JK? Explain.

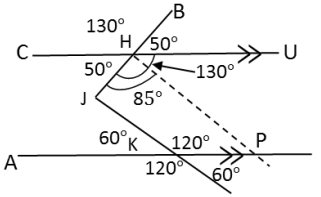
Answers

4)
a) $3x - 5° + 2x + 25° = 180°$ $\angle s$ on a str line
$x = 32°$
b) $U\hat{P}Q = 3x - 5°$ vert opp $\angle s$
$= 3(32°) - 5°$
$= 91°$
$U\hat{P}Q = 2y + 40°$ corresp $\angle s$, UT//RS
$2y + 40° = 91°$
$2y = 51°$
$y = 25{,}5°$
c)



5)
a) $x = 50°$ corresp $\angle s$, HU//JZ
b) $A\hat{K}J$
$= 180° - 120°$ $\angle s$ on a str line
$= 60°$
$K\hat{J}Z = 60°$ alt $\angle s$, AP//JZ
$H\hat{J}K = x + K\hat{J}Z$
$= 50° + 60°$
$= 110°$
$\therefore$ $H\hat{J}K$ is not a right angle because it is not $90°$

6)
a) Diagram for a) should exclude HP and $J\hat{H}P = 85°$.



b) $J\hat{H}P = 85°$ Given
But $J\hat{H}U = C\hat{H}B$ vert opp $\angle s$
$= 130°$
$\therefore$ $P\hat{H}U = 180° - 50° - 85°$ $\angle s$ on a str line
$= 45°$
$A\hat{P}H = 45°$ alt $\angle s$, CU//AP
$A\hat{P}H + J\hat{K}P = 120° + 45° \neq 180°$
$\therefore$ HP and JK are not parallel because the co-interior angles are not supplementary.

#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

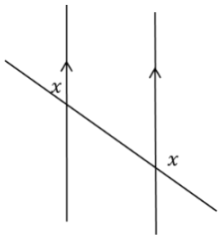
Worksheet 2.5

This worksheet focuses on understanding alternate, corresponding and alternate angles when lines are or are not parallel.

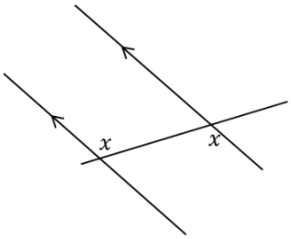
1) Say whether these statements are TRUE or FALSE, give reasons for your answers:
   a) Corresponding angles are *always* equal
   b) Co-interior angles are *sometimes* equal

2) In which diagram/s do the angles marked with x represent alternate angles?

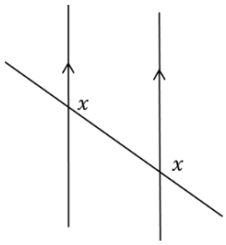
A.



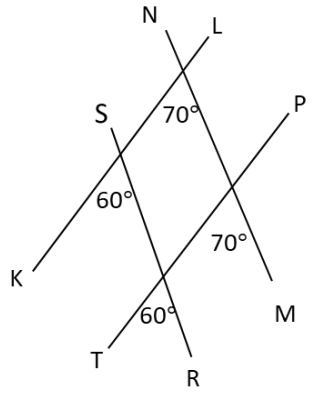
B.



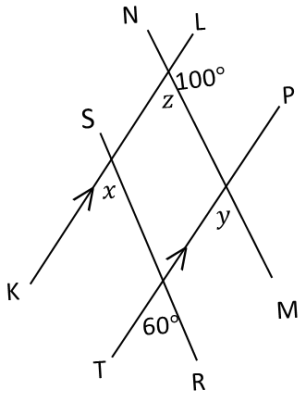
C.



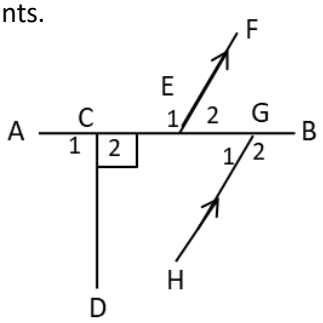
3) The diagram contains 2 pairs of lines. The sizes of 4 angles are given. Use this information to decide which pairs of lines are parallel. Give reasons for your answer.



4) The diagram contains 2 pairs of lines.
   a) Which pair of lines is parallel?
   b) Determine x, y and z. Give reasons for each statement.

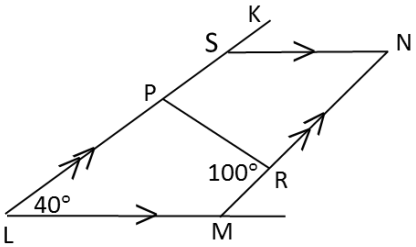


5) AB is a straight line. It intersects CD, EF and GH at C, E and G respectively. Give reasons for all your statements.



   a) Which angle is corresponding but not equal to $\hat{C}_2$?
   b) Which angle is alternate and equal to $\hat{E}_2$?
   c) If you are now told that $\hat{G}_1 = 45°$, determine the sizes of $\hat{E}_2$, $\hat{G}_2$, $\hat{E}_1$ and $\hat{C}_1$ in this order.

6) LK ∥ MN and SN ∥ LM.
   Give reasons for all your statements.



   a) If $M\hat{R}P = 100°$ and $M\hat{L}K = 40°$, determine the sizes of:
      i) $K\hat{S}N$
      ii) $S\hat{N}M$
      iii) $R\hat{P}S$
   b) Join K to N. This will make $S\hat{K}N = 100°$. Is $PR \parallel KN$? Justify your answer.

Worksheet 2.5

Answers

Questions and answers

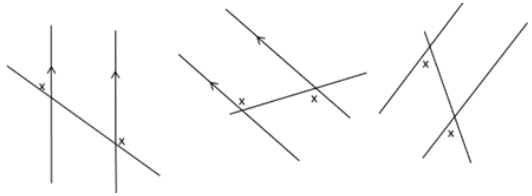
1) Say whether these statements are TRUE or FALSE, give reasons for your answers:
   a) Corresponding angles are *always* equal
   b) Co-interior angles are *sometimes* equal

Answer

1)
a) False, only if the lines are parallel
b) True, co-interior angles are equal when the lines are parallel and when the two co-interior angles are each 90°

2) In which diagram/s do the angles marked with x represent alternate angles?

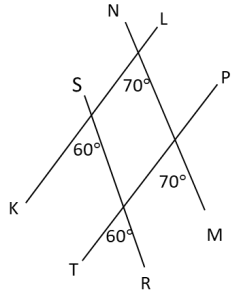
A. B. C.



Answer

2) B only

3) The diagram contains 2 pairs of lines. The sizes of 4 angles are given. Use this information to decide which pairs of lines are parallel. Give reasons for your answer.

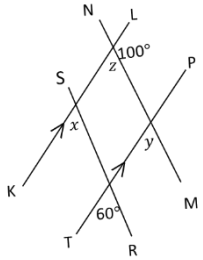


Answer

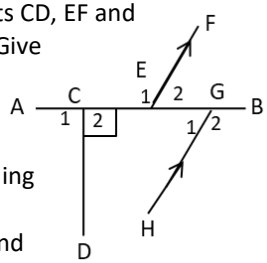
2) KL//TP because the corresponding angles are equal.

Questions

4) The diagram contains 2 pairs of lines.
   a) Which pair of lines is parallel?
   b) Determine x, y and z. Give reasons for each statement.

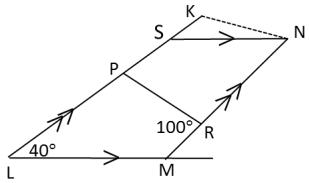


5) AB is a straight line. It intersects CD, EF and GH at C, E and G respectively. Give reasons for all your statements.



   a) Which angle is corresponding but not equal to $\hat{C}_2$?
   b) Which angle is alternate and equal to $\hat{E}_2$?
   c) If you are now told that $\hat{G}_1 = 45°$, determine the sizes of $\hat{E}_2$, $\hat{G}_2$, $\hat{E}_1$ and $\hat{C}_1$ in this order.

6) LK ∥ MN and SN ∥ LM. Give reasons for all your statements.



   a) If $M\hat{R}P = 100°$ and $M\hat{L}K = 40°$ determine the sizes of:
      i) $K\hat{S}N$ ii) $S\hat{N}M$ iii) $R\hat{P}S$
   b) Join K to N. This will make $S\hat{K}N = 100°$. Is $PR \parallel KN$? Justify your answer.

Answers

4)
a) KL//TP
b) $z = 80°$ ∠s on a str line
$y = z = 80°$ corresp ∠s KL//TP
$x = 60°$ corresp ∠s KL//TP

5)
a) $\hat{G}_2$ b) $\hat{G}_1$
b) $\hat{G}_1 = 45°$ given
$\hat{E}_2 = 45°$ alt ∠s, $EF \parallel GH$
$\hat{G}_2 = 180° - 45° = 135°$ ∠s on a str line
$\hat{E}_1 = 135°$ ∠s on a str line
$\hat{C}_1 = 90°$ ∠s on a str line

6)
a)
i) $K\hat{S}N = 40°$ corresp ∠s, $LM \parallel SN$
ii) $S\hat{N}M = 40°$ alt ∠s, $KL \parallel NM$
iii) $R\hat{P}S = 100°$ alt ∠s, $KL \parallel MN$

b) $R\hat{P}S = 100°$ from iii above and $S\hat{K}N = 100°$ is given. They are co-interior angles which sum to 200° not 180°. So $PR \nparallel KN$.

#TRY−ângles

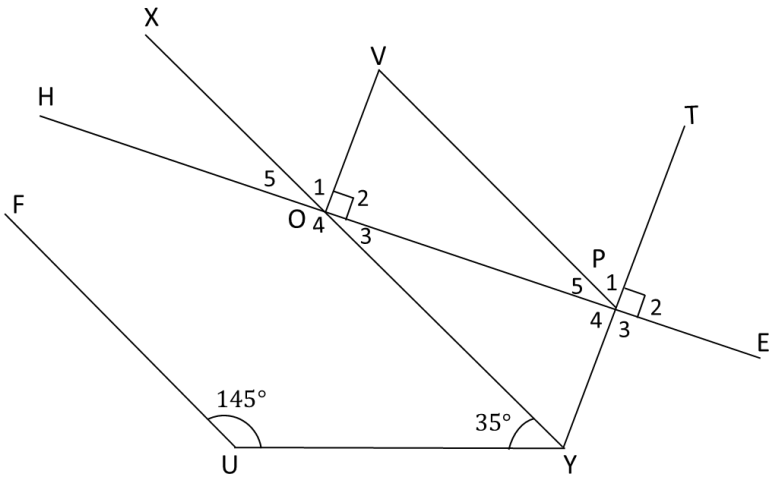
PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 2.6

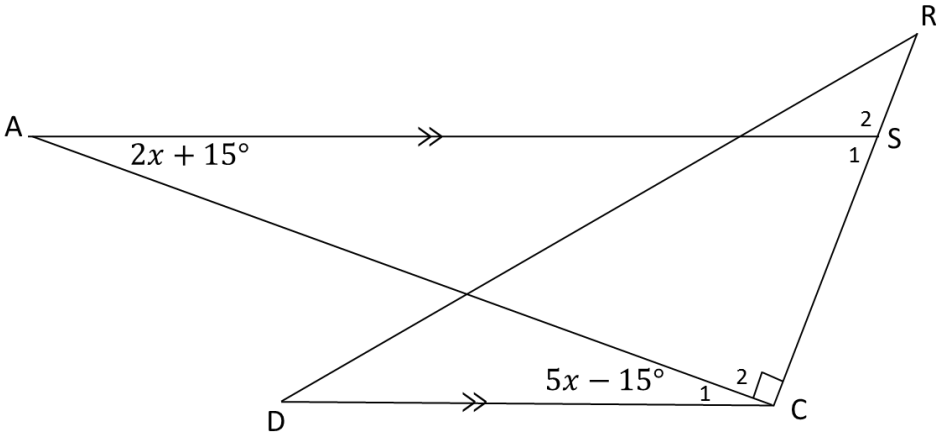
This worksheet focuses on several properties of lines and angles, including parallel lines.

1) HE intersects XY at O, and it intersects YT and P.
   Look at the diagram and say whether the following statements are TRUE or FALSE.
   If TRUE, provide reasons. If FALSE, correct the statement.

   a) $\hat{O}_1$ and $\hat{O}_5$ are adjacent complementary angles.
   b) $\hat{O}_2$ and $\hat{P}_4$ are corresponding angles.
   c) VO and TY are parallel to each other.
   d) UF and OY are parallel to each other.
   e) $\hat{O}_5$ and $\hat{O}_2$ are vertically opposite angles.



2) In the diagram, AC ⊥ CS. AS//DC and S lies on CR.



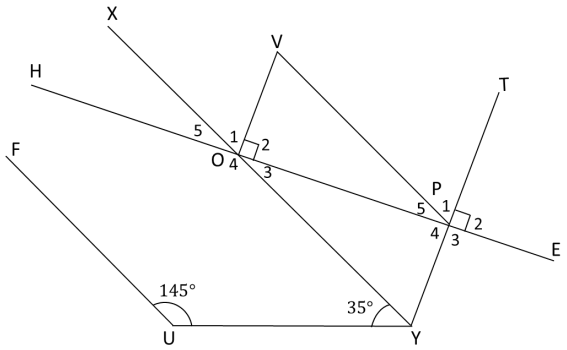
   a) Determine, with reasons, the value of x.
   b) Determine, with reasons, the size of $\hat{C}_1$.
   c) Determine, with reasons, the size of $\hat{S}_1$. Try to do this in TWO different ways.

Worksheet 2.6

Answers

Question

1) HE intersects XY at O, and it intersects YT and P.
Look at the diagram and say whether the following statements are TRUE or FALSE. If TRUE, provide reasons. If FALSE, correct the statement.
   a) $\hat{O}_1$ and $\hat{O}_5$ are adjacent complementary angles.
   b) $\hat{O}_2$ and $\hat{P}_4$ are corresponding angles.
   c) VO and TY are parallel to each other.
   d) UF and OY are parallel to each other.
   e) $\hat{O}_5$ and $\hat{O}_2$ are vertically opposite angles.

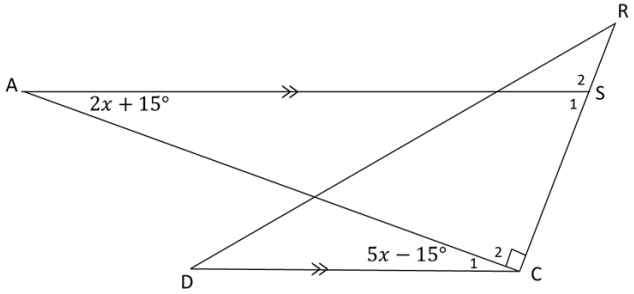


Answer

1)
   a) True $\hat{O}_2 = 90°$ and $\hat{O}_2, \hat{O}_1$ and $\hat{O}_5$ are adjacent $\angle s$ on a str line, so $\hat{O}_1$ and $\hat{O}_5$ are adjacent complementary $\angle s$
   b) False $\hat{O}_2$ and $\hat{P}_4$ are alternate not corresponding angles
   c) True corresp $\angle s$ are equal [$\hat{O}_2$ and $\hat{P}_2$]
   d) True co-int $\angle s$ supp [co-int $\angle s$ are $\hat{U}$ and $X\hat{Y}U$]
   e) False because $\hat{O}_5$ and $\hat{O}_3$ are vertically opposite angles

Question

2) In the diagram, AC ⊥ CS. AS//DC and S lies on CR.



   a) Determine, with reasons, the value of x. .
   b) Determine, with reasons, the size of $\hat{C}_1$.
   c) Determine, with reasons, the size of $\hat{S}_1$. Try to do this in TWO different ways.

Answer

2)

a) $2x + 15° = 5x - 15°$ alt $\angle s$, AS//DC
$15° + 15° = 5x - 2x$
$30° = 3x$
$x = 10°$

b) $\hat{C}_1 = 5x - 15°$ given
$= 5(10°) - 15°$
$= 35°$

c) Method 1:
$\hat{C}_1 + \hat{C}_2 = 35° + 90°$
$= 125°$
$\hat{C}_1 + \hat{C}_2 + \hat{S}_1 = 180°$ co-int $\angle s$, AS//DC
$\hat{S}_1 = 180° - 125°$
$= 55°$

Method 2:
$\hat{C}_1 + \hat{C}_2 = \hat{S}_2$ corresp $\angle s$, AS//DC
$= 55°$

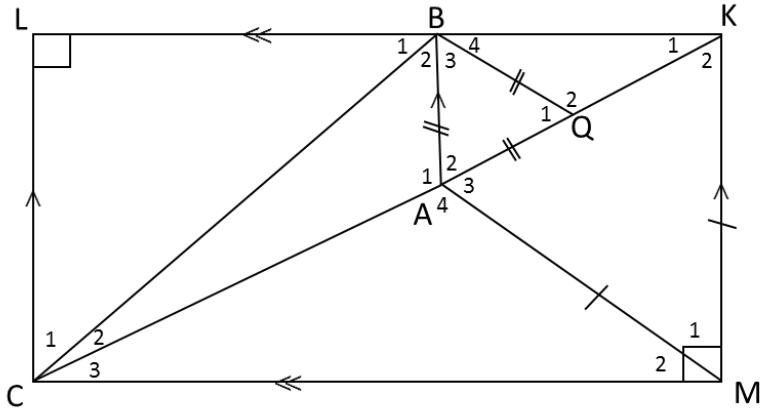
Worksheet 3.1

This worksheet focuses on the sum of the angles of a triangle and types of triangles.

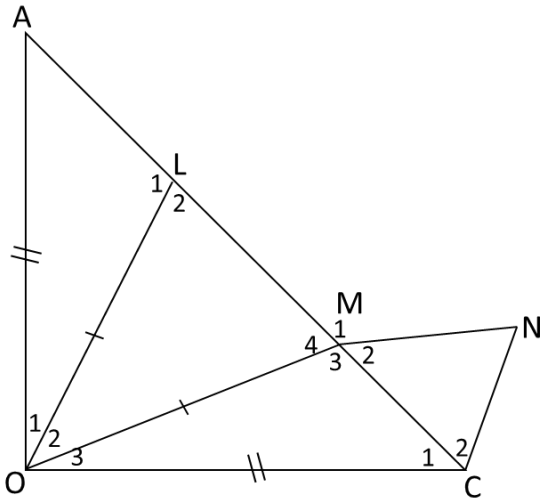
1) Do the angles in the table represent the interior angles of a triangle?
   If they do,
   - make a tick (✓) in the middle column
   - name the type of triangle.

Angles	Is a Δ?	Name of type of Δ
30°, 40° and 120°		
30°, 30° and 120°		
70°, 40° and 70°		
30°, 30° and 30°		
175°, 4° and 1°		
75°, 12° and 80°		
89°, 89° and 89°		
60°, 60° and 60°		

2) The diagram shows rectangle CLKM. CK is a diagonal of the rectangle. Name one of each of the following types of triangles in the diagram.
   a) Acute-angled
   b) Right-angled
   c) Obtuse-angled
   d) Scalene
   e) Isosceles
   f) Equilateral



3) Copy the diagram or fill in your answers on the given diagram.
   a) $OA = OC$ and $\hat{A} = 45°$.
      Determine the size of $\hat{C}_1$ and $A\hat{O}C$.
   b) What kind of triangle is ΔAOC?
   c) Given that $OL = OM$ and $\hat{O}_2 = 50°$, determine the sizes of the other angles of ΔMOL.
   d) Now calculate the sizes of the other angles of ΔAOL.
   e) Determine the sizes of $\widehat{M}_3$ and $\hat{O}_3$.
   f) If you are now told that OL ∥ CN, determine the size of $M\hat{C}N$.



Worksheet 3.1

Answers

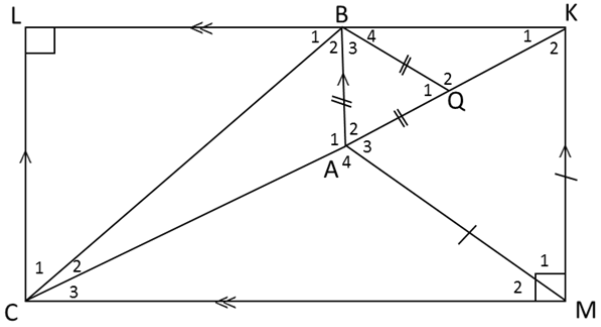
Question

1) Do the angles in the table represent the interior angles of a triangle?
If they do,
- make a tick (✓) in the middle column
- name the type of triangle.

Answer

Angles	**Is a Δ?**	**Name of type of Δ**
30^o, 40^o and 120^o		
30^o, 30^o and 120^o	✓	isosceles
70^o, 40^o and 70^o	✓	isosceles
30°, 30° and 30^o		
175°, 4° and 1°	✓	Obtuse-angled
75°, 12° and 80°		
89°, 89° and 89°		
60°, 60° and 60°	✓	equilateral

2) The diagram shows rectangle CLKM. CK is a diagonal of the rectangle. Name one of each of the following types of triangles in the diagram.



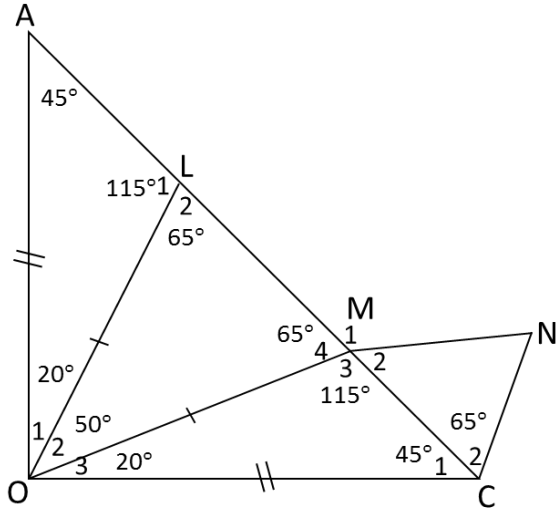
Answers

a) Acute-angled: $\Delta BAQ; \Delta KAM$
b) Right-angled: $\Delta CLB; \Delta CLK;\ \Delta CKM$
c) Obtuse-angled: $\Delta BAC;\ \Delta BKQ$
d) Scalene: ΔBAC (many others)
e) Isosceles: ΔKAM
f) Equilateral: ΔBAQ

Question and answer

3) Copy the diagram or fill in your answers on the given diagram.
a) $OA = OC$ and $\hat{A} = 45°$.
Determine the size of $\hat{C}_1$ and $A\hat{O}C$.
b) What kind of triangle is ΔAOC?
c) Given that $OL = OM$ and $\hat{O}_2 = 50°$, determine the sizes of the other angles of ΔMOL.
d) Now calculate the sizes of the other angles of ΔAOL.
e) Determine the sizes of $\hat{M}_3$ and $\hat{O}_3$.
f) If you are now told that OL ∥ CN, determine the size of $M\hat{C}N$.

Answers: a) to f)



Worksheet 3.2

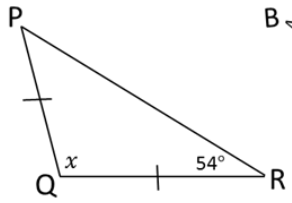
This worksheet focuses on isosceles triangles and includes parallel lines.

1) Write down the properties of an isosceles triangle.
   Draw a diagram and show the properties on the diagram too.

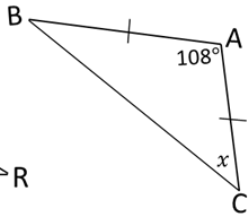
2) Is it possible to have an isosceles triangle with an angle of 95°? Explain.

3) Determine the value of x.

a)

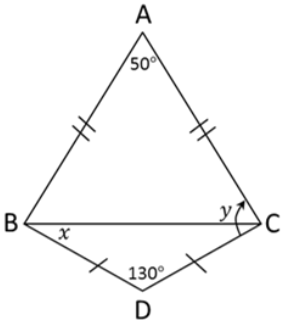


b)

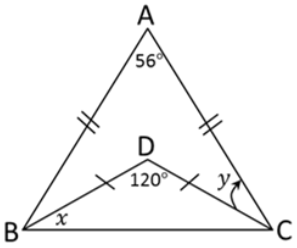


4) Determine the value of x and y.

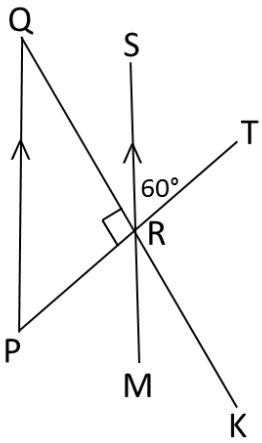
a)



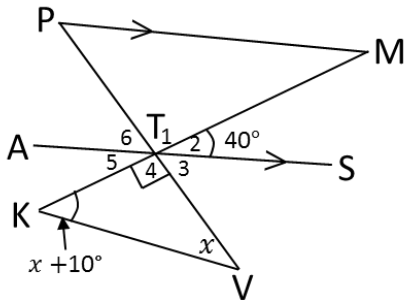
b)



5) PT, KQ and SM intersect at R.
   a) Determine the size of $Q\hat{R}S$, $\hat{P}$ and $\hat{Q}$.
   b) When you join KT, it will be parallel to PQ and RS.
   Determine size of all the angles in ΔRTK.

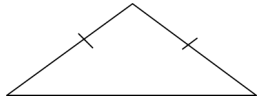
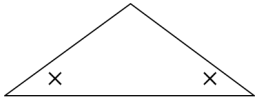
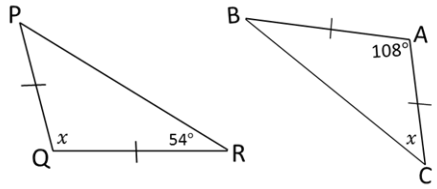
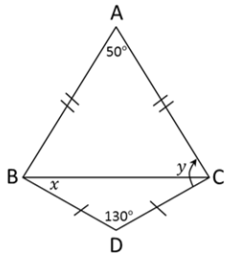
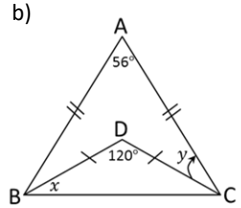


6) PM ∥ AS. PV intersects KM at T.
   a) Determine the sizes of $\hat{T}_1$, $\hat{T}_3$, $\hat{T}_5$, $\hat{T}_6$, $\hat{M}$ and $\hat{P}$.
   Give reasons for each statement. You can find the sizes of the 6 angles in any order.
   b) Determine the value of x.
   Hence determine the size of $\hat{K}$.



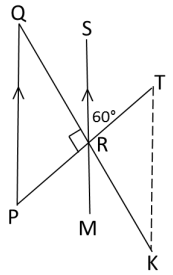
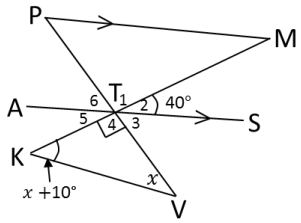
Worksheet 3.2

Answers

Question	Answer
1) Write down the properties of an isosceles triangle. Draw a diagram and show the properties on the diagram too.	1) An isosceles triangle has 2 equal *sides*, the angles opposite the equal sides are equal  An isosceles triangle has 2 equal *angles*, the sides opposite the equal angles are equal 
2) Is it possible to have an isosceles triangle with an angle of 95°? Explain.	2) Clearly we can't have 2 angles of 95° in a triangle. If one angle is 95°, then the two other angles would be $(180° - 95°) \div 2 = 42{,}5°$. Therefore, it is possible to have an isosceles triangle with an angle of 95°.
3) Determine the value of x. a) b)  	3) Reasons are not expected a) $\hat{P} = \hat{R}$ $\angle s$ opp equal sides $= 54°$ $\angle s$ opp equal sides $x = 180° - 2(54°)$ int $\angle s$ Δ $= 72°$ b) $\hat{B} = \hat{C} = x$ $\angle s$ opp equal sides $2x = 180° - 108°$ int $\angle s$ Δ $2x = 72°$ $x = 36°$
4) Determine the value of x and y. a) b)   	4) Reasons are not expected a) $x = 25°$ $\angle s$ opp equal sides; int $\angle s$ Δ $A\hat{C}B = 65°$ $\angle s$ opp equal sides; int $\angle s$ Δ So, $y = 90°$ b) $x = 30°$ $\angle s$ opp equal sides; int $\angle s$ Δ $A\hat{C}B = 62°$ $\angle s$ opp equal sides; int $\angle s$ Δ $y = 32°$

Worksheet 3.2

Answers continued

Question	Answer
5) PT, KQ and SM intersect at R. a) Determine the size of $Q\hat{R}S$, $\hat{P}$ and $\hat{Q}$. b) When you join KT, it will be parallel to PQ and RS. Determine size of all the angles in ΔRTK. 	5) Reasons are not expected a) $Q\hat{R}S = 30°$ $\angle s$ on a str line $\hat{P} = 60°$ corresp ∠s, SR//QP $\hat{Q} = 30°$ alt ∠s, SR//QP b) $T\hat{R}K = 90°$ vert opp ∠s $R\hat{T}K = 60°$ alt ∠s, SR//TK $T\hat{K}R = 30°$ alt $\angle s$, QP//KT
6) PM ∥ AS. PV intersects KM at T. a) Determine the sizes of $\hat{T}_1$, $\hat{T}_3$, $\hat{T}_5$, $\hat{T}_6$, $\hat{M}$ and $\hat{P}$. Give reasons for each statement. You can find the sizes of the 6 angles in any order. b) Determine the value of x. c) Hence determine the size of $\hat{K}$. 	6) a) $\hat{T}_1 = 90°$ vert opp ∠s $\hat{T}_3 = 50°$ $\angle s$ on a str line $\hat{T}_6 = 50°$ vert opp ∠s $\hat{T}_5 = 40°$ vert opp ∠s $\hat{M} = 40°$ alt $\angle s$, PM//AS $\hat{P} = 50°$ alt $\angle s$, PM//AS b) $x + 10° + x + 90° = 180°$ $\angle s$ on a str line $2x = 80°$ $x = 40°$ c) $\hat{K} = x + 10°$ $= 40° + 10°$ $= 50°$ Notice that b and c show that KV ∦ PM.

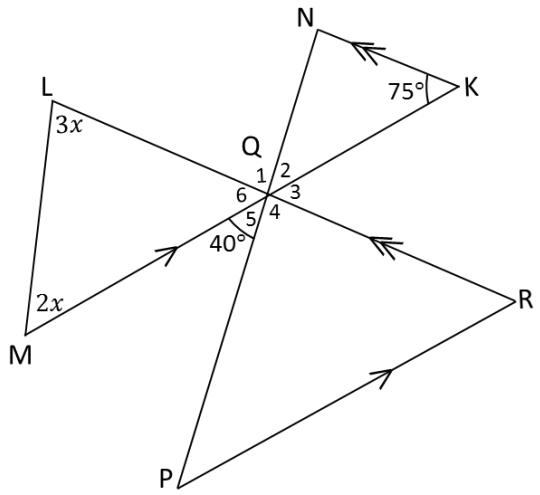
#TRY−ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

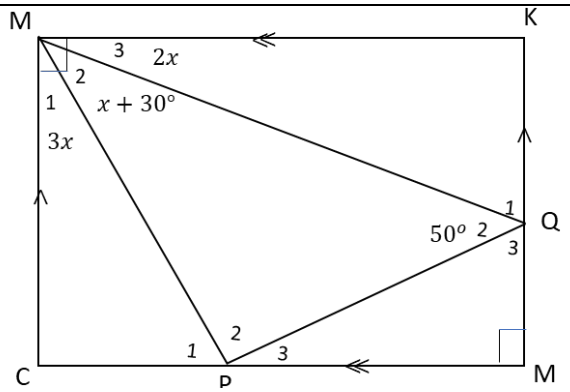
Worksheet 3.3

This worksheet focuses on the sum of the angles of a triangle and includes parallel lines.

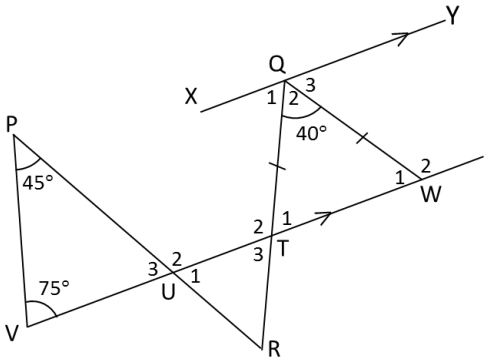
1) Refer to the diagram when answering this question.
   LR, NP and MK intersect at Q.
   a) Write down the pairs of parallel lines shown.
   b) Fill in the sizes of the unknown angles at Q.
   c) Determine the value of x with reasons.
   d) Fill in the size of the unknown angles in ΔLMQ, ΔNKQ and ΔPQR.



2)
   a) Write down the pairs of parallel lines shown in the diagram.
   b) Which angles can be found using the parallel lines?
   c) Determine the value of x.
   d) Fill in the sizes of the unknown angles.



3) In the diagram, XY ∥ VW and TQ = WQ. PR and QR are straight lines. Work out the sizes of the angles in the table, giving reasons.



Angle	Size	Reasons
$\hat{T}_1$		
$\hat{W}_1$		
$\hat{Q}_1$		
$\hat{Q}_3$		
$\hat{T}_2$		
$\hat{U}_3$		
$\hat{U}_1$		
$\hat{R}$		

Worksheet 3.3

Answers

Question

1) Refer to the diagram when answering this question. LR, NP and MK intersect at Q.
   a) Write down the pairs of parallel lines shown.
   b) Fill in the sizes of the unknown angles at Q.
   c) Determine the value of x with reasons.
   d) Fill in the size of the unknown angles in ΔLMQ, ΔNKQ and ΔPQR.

Answer

1)

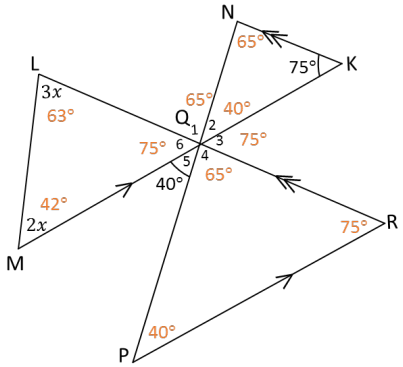
a) NK//LR and MK//PR

c) $2x + 3x + 75° = 180°$ int $\angle s$ **OR** ext $\angle$ of Δ

$5x = 105°$

$x = 21°$

b) and d)



Question

2)
   a) Write down the pairs of parallel lines shown in the diagram.
   b) Which angles can be found using the parallel lines?
   c) Determine the value of x
   d) Fill in the size of the unknown angles.

Answer

2)

a) MK//CM and MC//KM

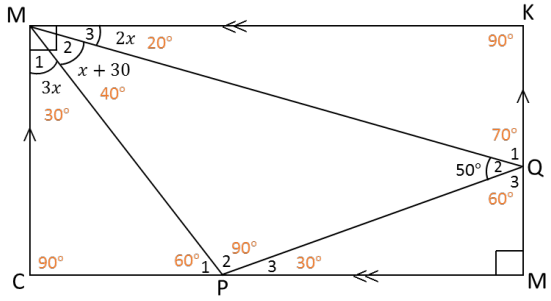
b) $\hat{C}$ from MK//CM and $\hat{K}$ from MC//KM

c) $3x + x + 30° + 2x = 90°$

$6x = 60°$

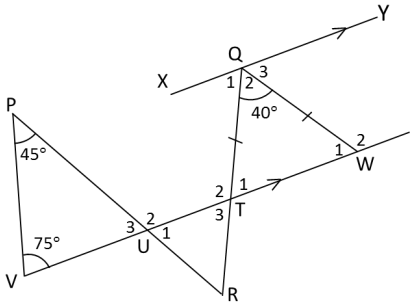
$x = 10°$

d)



Question and answer

3) In the diagram, XY ∥ VW and TQ = WQ. PR and QR are straight lines.

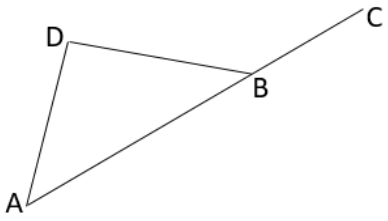


Angle	Measure	Reasons
$\hat{T}_1$	70°	$\angle s$ opp equal sides; int $\angle s$ Δ
$\hat{W}_1$	70°	$\angle s$ opp equal sides; int $\angle s$ Δ
$\hat{Q}_1$	70°	alt $\angle s, XY \parallel VW$
$\hat{Q}_3$	70°	alt $\angle s, XY \parallel VW$ **OR** $\angle s$ on a line; co-int $\angle s, XY \parallel VW$
$\hat{T}_2$	70°	vert opp $\angle s$
$\hat{U}_3$	60°	int $\angle s$ Δ
$\hat{U}_1$	60°	vert opp $\angle s$
$\hat{R}$	50°	int $\angle s$ Δ

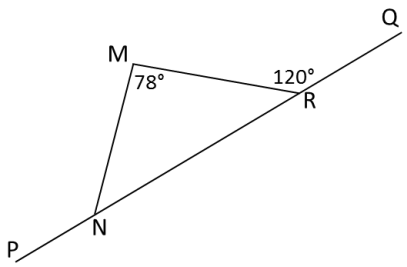
Worksheet 3.4

This worksheet focuses on the exterior angle of a triangle and includes isosceles and equilateral triangles and parallel lines.

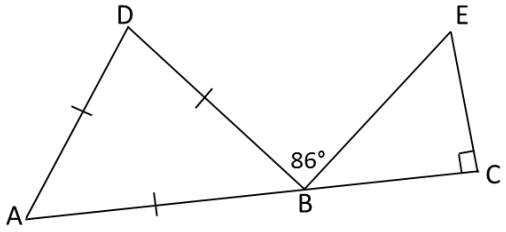
1) Complete the following:
   a) The exterior angle formed when you extend a side of a triangle is equal to _____
   b) $D\hat{A}B + A\hat{D}B$ =___



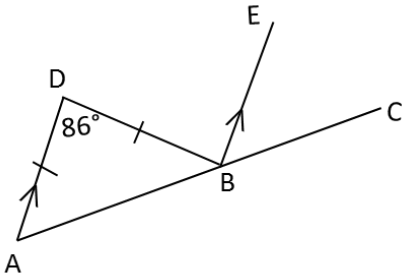
2) Determine the sizes of $M\hat{N}R$, $N\hat{R}M$ and $M\hat{N}P$ in this order.



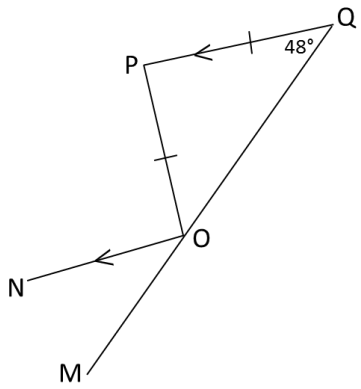
3) ABC is a straight line.
   Determine the size of $E\hat{B}C$ and $B\hat{E}C$.



4) ABC is a straight line.
   Determine the sizes of $D\hat{A}B$, $D\hat{B}E$ and $E\hat{B}C$

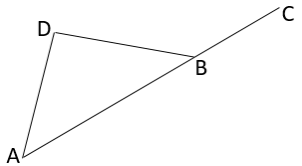
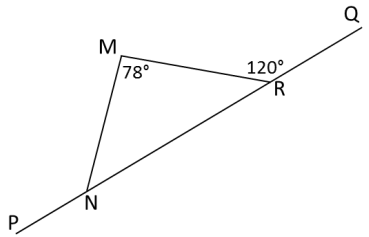
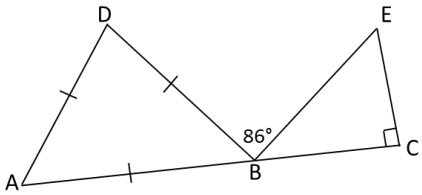
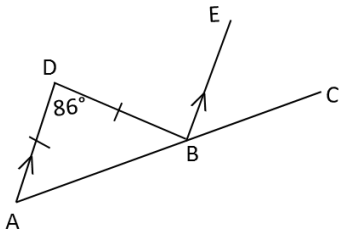
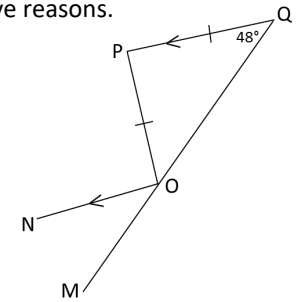


5) MOQ is a straight line.
   Determine the sizes of all the angles in the diagram. Give reasons.



Worksheet 3.4

Answers

<table>
<tr><td>Questions</td><td>1) Complete the following:
a) The exterior angle formed when you extend a side of a triangle is equal to ___
b) $D\hat{A}B + A\hat{D}B$ =___
  </td><td>2) PNRQ is a straight line.
Determine the sizes of $M\hat{N}R$, $N\hat{R}M$ and $M\hat{N}P$ in this order.
  </td><td>3) ABC is a straight line.
Determine the size of $E\hat{B}C$ and $B\hat{E}C$.
  </td></tr>
<tr><td>Answers</td><td>1)
a) the sum of the interior opposite angles
b) $D\hat{B}C$</td><td>2) Reasons are not expected
$M\hat{N}R = 42°$ ext ∠ of Δ
$N\hat{R}M = 60°$ ∠s on a str line
$M\hat{N}P = 138°$ ext ∠ of Δ</td><td>3) Reasons are not expected
$A\hat{B}D = 60°$ int $\angle s$ Δ
$E\hat{B}C = 180° - 60° - 86° = 34°$ ∠s on a str line
$B\hat{E}C = 180° - 90° - 34° = 56°$ int $\angle s$ Δ</td></tr>
<tr><td>Questions</td><td>4) ABC is a straight line.
Determine the sizes of $D\hat{A}B$, $D\hat{B}E$ and $E\hat{B}C$.
  </td><td>5) MOQ is a straight line.
Determine the sizes of all the angles in the diagram. Give reasons.
  </td><td></td></tr>
<tr><td>Answers</td><td>4) Reasons are not expected
$D\hat{A}B = (180° - 86°) \div 2 = 47°$ int $\angle s$ Δ
$D\hat{B}E = 86°$ alt ∠s, AD//BE
$E\hat{B}C = 47°$ corresp ∠s, AD//BE</td><td>5)
$P\hat{O}Q = 48°$ $\angle s$ opp equal sides
$O\hat{P}Q = 180° - (48° + 48°) = 84°$ int $\angle s$ Δ
$M\hat{O}N = 48°$ corresp ∠s, PQ//NO
$P\hat{O}N = 84°$ alt $\angle s$ Δ, PQ//NO</td><td></td></tr>
</table>

#TRY−ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 3.5

This worksheet focuses on the exterior angle of a triangle.

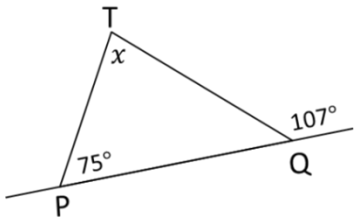
1) If 2 lines are perpendicular, the angle between them is ___

2) What is wrong with this statement:
"The exterior angle of a triangle is any angle outside the triangle".
Use a diagram as part of your explanation.

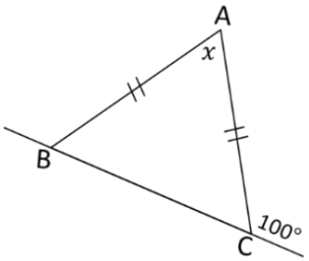
3) The exterior angle of an isosceles triangle is 100°. What is the size of the largest angle in the triangle?

4) Determine the value of x.

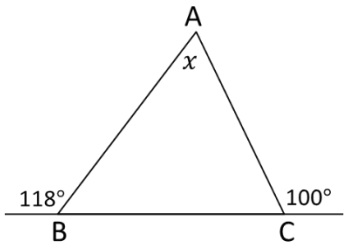
a)



b)



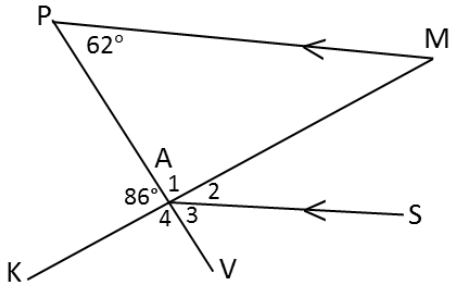
5) Determine the value of x.



6) PV intersects KM at A. PM ∥ AS.

a) Determine the size of $\widehat{M}$.

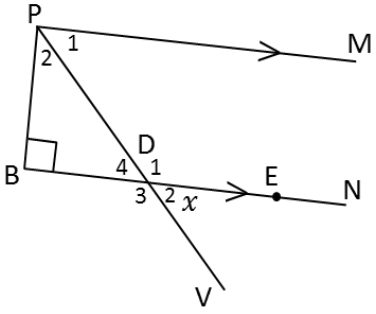
b) Determine the sizes of $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_3$ and $\hat{A}_4$.
Give reasons for each statement. You can find the sizes of the 4 angles in any order.



7) PM ∥ BN. PV intersects BN at D. $x = 50°$

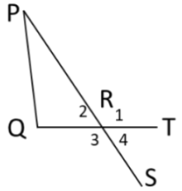
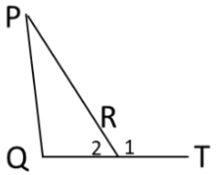
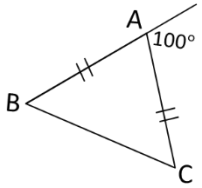
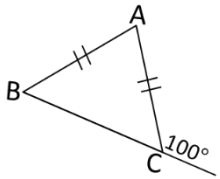
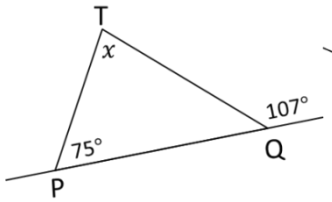
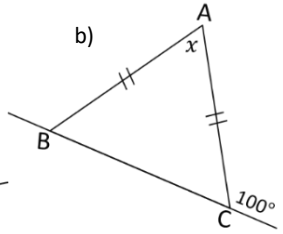
a) Find 2 other angles that have the same value as x.
Give reasons for your answers.

b) MV will intersect BN at E. This will create $P\widehat{M}V = 85°$.
Determine the size of the other angles in ΔDEV, giving reasons for all statements.



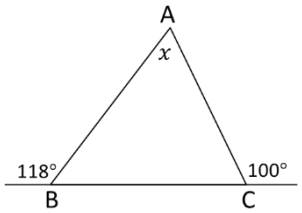
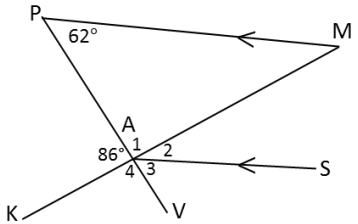
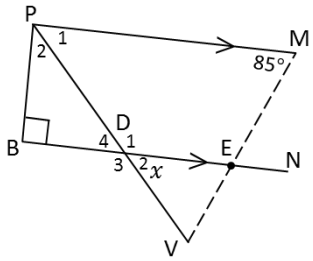
Worksheet 3.5

Answers

Question	Answer
1) If 2 lines are perpendicular, the angle between them is ___	1) $90°$
2) What is wrong with this statement: *"The exterior angle of a triangle is any angle outside the triangle".* Use a diagram as part of your explanation.	2) *The exterior angle of a triangle* is NOT just any *angle outside the triangle.* This would mean $\hat{R}_1$; $\hat{R}_4$ and $\hat{R}_3$ are all exterior angles of ΔPQR.  Only $\hat{R}_1$ and $\hat{R}_3$ are exterior angles of ΔPQR. *The exterior angle of a triangle is* the angle that lies between the <u>extension</u> of one side of the triangle and a side of the triangle. In the diagram below, QR is extended to T and PR is a side of the triangle next to the extension. $\hat{R}_1$ is between the extension and the side. $\hat{R}_1 + \hat{R}_2 = 180°$: the interior angle and the exterior angle are adjacent supplementary angles. 
3) The exterior angle of an isosceles triangle is 100°. What is the size of the largest angle in the triangle? Diagrams for the answer:  Diagram 1  Diagram 2	3) There are 2 possibilities for the position of the exterior angle: i) The extended side is one of the equal sides (diagram 1) ii) The extended side is the non-equal side (diagram 2) In diagram 1, the angles are $80°, 50°, 50°$. So the largest angle is adjacent to the $100°$ angle. In diagram 2, the angles are $80°, 80°, 20°$. So the largest angle is also adjacent to the $100°$ angle but there are 2 angles that are $80°$.
4) Determine the size of x. a)  b) 	4) Reasons are not expected a) $x = 107° - 75°$ ext ∠of Δ $= 32°$ b) $A\hat{C}B = 80°$ $\angle s$ on a str line $A\hat{B}C = 80°$ $\angle s$ opp equal sides; $x = 20°$ int $\angle s$ Δ

Worksheet 3.5

Answers continued

Question	Answer
5) Determine the size of x.  	5) Reasons are not expected $A\hat{C}B = 80°$ $\angle s$ on a str line $x + 80° = 118°$ ext $\angle$of Δ $x = 38°$ **OR** $A\hat{B}C = 62°$ $\angle s$ on a str line $x + 62° = 100°$ ext $\angle$of Δ $x = 38°$ **OR** $A\hat{C}B = 80°$ $\angle s$ on a str line $A\hat{B}C = 62°$ $\angle s$ on a str line $x = 180° - 80° - 62°$ int $\angle s$ Δ $= 38°$
6) PV intersects KM at A. PM ∥ AS. a) Determine the size of $\hat{M}$. b) Determine the sizes of $\hat{A}_1$, $\hat{A}_2$, $\hat{A}_3$ and $\hat{A}_4$. Give reasons for each statement. You can find the sizes of the 4 angles in any order.  	6) a) Reasons are not expected $\hat{M} = 24°$ ext $\angle$of Δ b) Reasons ARE expected $\hat{A}_1 = 94°$ $\angle s$ on a str line **or** int $\angle s$ Δ $\hat{A}_2 = 24°$ alt $\angle$s, PM//AS $\hat{A}_3 + \hat{A}_2 = 86°$ vert opp $\angle$s $\hat{A}_3 = 62°$ $\hat{A}_4 = 94°$ vert opp $\angle$s **or** $\angle s$ on a str line Reasons depend on the order in which the angle sizes are found
7) PM ∥ BN. PV intersects BN at D. $x = 50°$ a) Find 2 other angles that have the same value as x. Give reasons for your answers. b) MV will intersect BN at E. This will create $P\hat{M}V = 85°$. Determine the size of the other angles in ΔDEV, giving reasons for all statements.  	7) a) Reasons ARE expected $\hat{D}_4 = 50°$ vert opp $\angle$s $\hat{P}_1 = 50°$ alt $\angle$s, PM//BN b) Reasons ARE expected $D\hat{E}V = 85°$ corresp $\angle$s, PM//BN $\hat{V} = 45°$... int $\angle s$ Δ **OR** $M\hat{E}B = 85°$ co-int $\angle$s, PM//BN $\hat{V} = 45°$ ext $\angle$of Δ

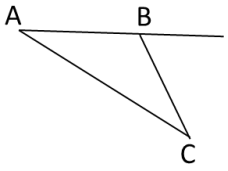
#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

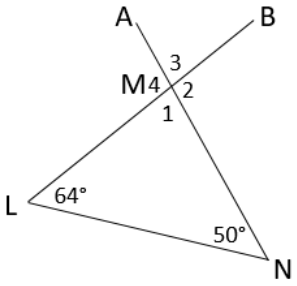
Worksheet 3.6

This worksheet focuses on calculating angle sizes, the effect of different pairs of equal sides and the effect of parallel lines on angle sizes.

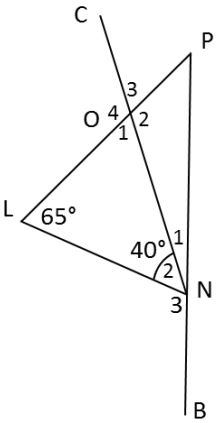
1) Complete: $C\hat{A}B + A\hat{C}B + C\hat{B}A =$ ___



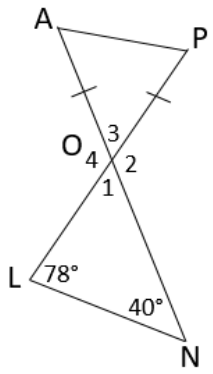
2) AN intersects LB at M.
Determine the sizes of $\hat{M}_1$, $\hat{M}_2$, $\hat{M}_3$ and $\hat{M}_4$



3) LP and CN intersect at O. PNB is a straight line.
$LOP = PN$.
Determine the sizes of
$\hat{N}_1$, $\hat{N}_3$, $\hat{O}_1$, $\hat{O}_2$, $\hat{O}_3$, $\hat{O}_4$ and $\hat{P}$
in any order.



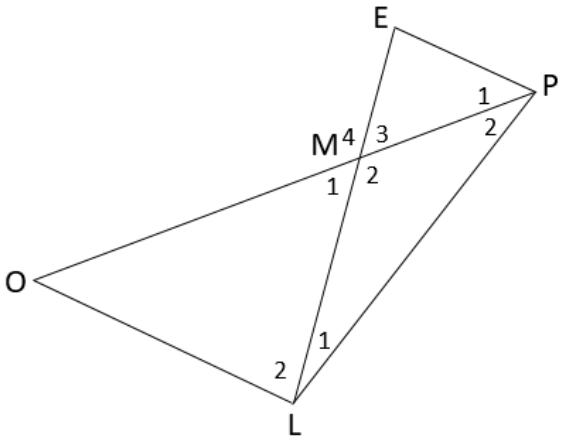
4) AN intersects LP at O.
Determine the sizes of $\hat{A}$, $\hat{O}_1$, $\hat{O}_2$, $\hat{O}_3$, $\hat{O}_4$ and $\hat{P}$ in any order.



5) Treat Q5a and Q5b as entirely separate questions.
OP intersects LE at M.

a) If $EP = EM$, list three angles that are equal.

b) If $EML = PL$, and $EP//OL$ list the angles that are equal.

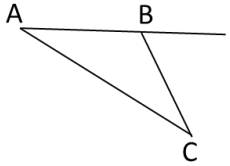


Worksheet 3.6

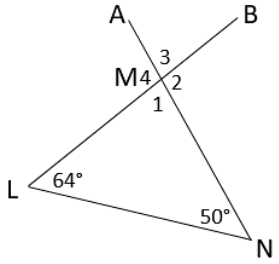
Answers

Questions

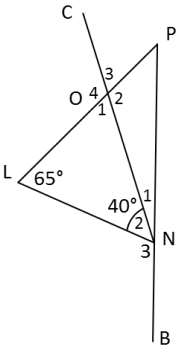
1) Complete: $C\hat{A}B + A\hat{C}B + C\hat{B}A =$ ___



2) AN intersects LB at M.
Determine the sizes of
$\widehat{M}_1$, $\widehat{M}_2$, $\widehat{M}_3$ and $\widehat{M}_4$



3) LP and CN intersect at O.
PNB is a straight line. $LOP = PN$.
Determine the sizes of
$\widehat{N}_1$, $\widehat{N}_3$, $\widehat{O}_1$, $\widehat{O}_2$, $\widehat{O}_3$, $\widehat{O}_4$ and $\hat{P}$ in any order



Answers

1) $180°$

2) Reasons are not expected

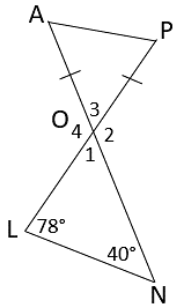
$\widehat{\widehat{M}}_1 = 66°$	int $\angle s\Delta$
$\widehat{M}_2 = 114°$	$\angle$s on a str line **or** ext $\angle$ of Δ
$\widehat{M}_3 = 66°$	vert opp $\angle s$
$\widehat{M}_4 = 114°$	vert opp $\angle s$

3) Reasons are not expected

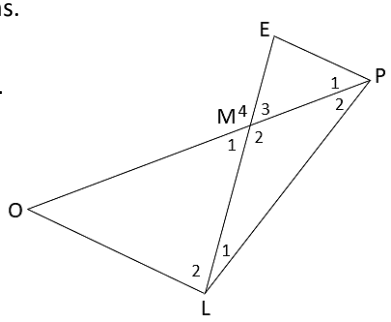
$\widehat{\widehat{N}}_1 = 25°$	$\angle$s opp equal sides	$\hat{O}_3 = 75°$	vert opp $\angle s$
$\widehat{N}_3 = 115°$	$\angle$s on a str line	$\hat{O}_4 = 105°$	vert opp $\angle s$
$\hat{O}_1 = 75°$	int $\angle s$ Δ	$\hat{P} = 50°$	int $\angle s$ Δ
$\hat{O}_2 = 105°$	ext $\angle$ of Δ		

Questions

4) AN intersects LP at O.
Determine the sizes of
$\hat{A}$, $\hat{O}_1$, $\hat{O}_2$, $\hat{O}_3$, $\hat{O}_4$ and $\hat{P}$ in any order.



5) Treat Q5a and Q5b as entirely separate questions.
OP intersects LE at M.
a) If $EP = EM$, list three angles that are equal.
b) If $EML = PL$, and $EP//OL$
list the angles that are equal.



Answers

4) Reasons are not expected

$\hat{O}_1 = 62°$	int $\angle s$ Δ	$\hat{O}_4 = 118°$	vert opp $\angle s$
$\hat{O}_3 = 62°$	vert opp $\angle s$	$\hat{A} + \hat{P} = 118°$	ext $\angle$ of Δ
$\hat{O}_2 = 118°$	$\angle$s on a str line	$\hat{A} = \hat{P} = 59°$	$\angle$s opp equal sides

5) Reasons are not expected

a) $\widehat{M}_3 = \hat{P}_1$ $\angle$s opp equal sides
$\widehat{M}_3 = \widehat{M}_1$ vert opp $\angle s$

b) $\hat{E} = E\hat{P}L$ $\angle$s opp equal sides
$\hat{E} = \hat{L}_2$ alt $\angle$s, EP//OL
$\hat{P}_1 = \hat{O}$ alt $\angle$s, EP//OL

#TRY–ângles

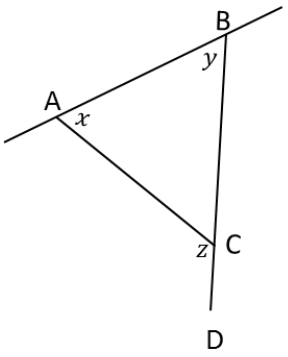
PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 3.7

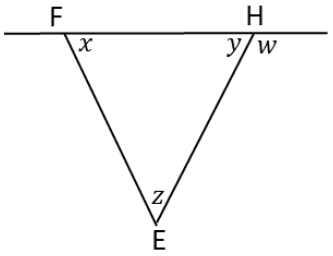
This worksheet focuses on triangles calculations where there are triangles and parallel lines in the diagrams. Reasons are expected.

1) BCD is a straight line. Select the true statement and give a reason for your answer:

A. $x = z$
B. $y = z$
C. $x = y$
D. $z = y - x$
E. $z = x + y$

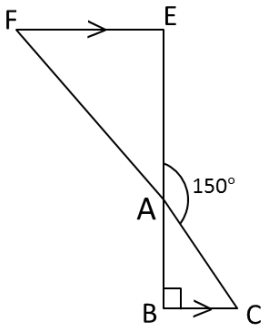


2) ΔFHE is an equilateral triangle. Determine the values of w, x, y and z.

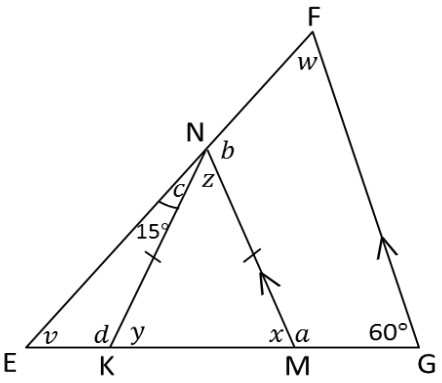


3) FE//BC and $E\hat{A}C = 150°$
Determine, with reasons, the sizes of:

a) $\hat{C}$
b) $\hat{F}$
c) $\hat{E}$

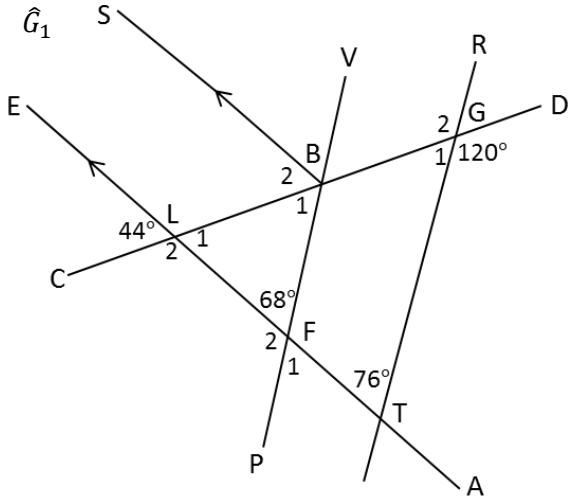


4) In the diagram, K and M lie on EG and N lies on EF. NK=NM, FG‖ NM. $\hat{G} = 60°$ and $c = 15°$. Determine the values of all unknowns, in any order.



5) AE ‖ BS. CD, PV and RT are straight lines. Use the information in the diagram to determine the size of the following angles in TWO DIFFERENT WAYS. Give reasons for all statements.

a) $\hat{B}_1$
b) $\hat{B}_2$
c) $\hat{G}_1$



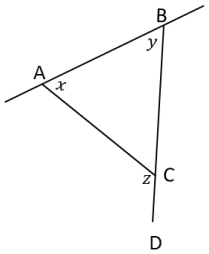
Angle	Value/size	Reason
a		
b		
c		
d		
v		
w		
x		
y		
z		

Worksheet 3.7
Answers

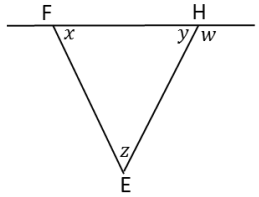
Questions

1) BCD is a straight line. Select the true statement and give a reason for your answer:

A. $x = z$
B. $y = z$
C. $x = y$
D. $z = y - x$
E. $z = x + y$

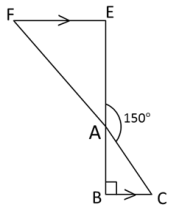


2) ΔFHE is an equilateral triangle, determine the values of w, x, y and z.



3) FE//BC and $E\hat{A}C = 150°$. Determine, with reasons the sizes of:

a) $\hat{C}$
b) $\hat{F}$
c) $\hat{E}$



Answers

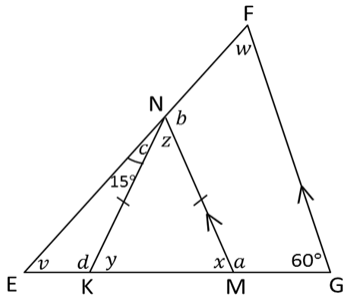
1) E. $z = x + y$ ext ∠ of Δ

2) Reasons are not expected
$x = y = z = 60°$ int $\angle s$ Δ
$w = 120°$ $\angle s$ on a str line

3)
a) $\hat{C} = 60°$ ext ∠ of Δ
b) $\hat{F} = 60°$ alt ∠s, FE//BC
c) $\hat{E} = 90°$ alt ∠s, FE//BC

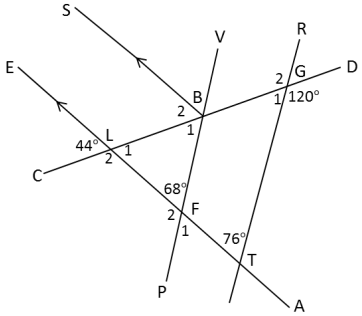
Questions

4) In the diagram, K and M lie on EG and N lies on EF. NK=NM, FG∥ NM. $\hat{G} = 60°$ and $c = 15°$. Determine the values of all unknowns, in any order.



5) Use the information in the diagram to determine the size of the following angles in TWO DIFFERENT WAYS. Give reasons for all statements.

a) $\hat{B}_1$
b) $\hat{B}_2$
c) $\hat{G}_1$



Answers

Angle	Value	Reason
a	120°	Co-int ∠s, NM//FG **or** ∠s on a str line
b	105°	∠s on a str line (need to find z first)
c	15°	given
d	120°	∠s on a str line **or** ext ∠ of Δ or int ∠s Δ (depending on order)
v	45°	ext ∠of Δ **or** int ∠s Δ
w	75°	corresp ∠s or co-int ∠s NM//FG (need b, c, z before w)
x	60°	corresp ∠s, NM//FG **or** ∠s on a str line
y	60°	∠s opp equal sides
z	60°	int ∠s ΔMNK

5)

a) $\hat{L}_1 = 44°$ vert opp ∠s $\hat{B}_1 = 68°$ int ∠s Δ **OR**
$\hat{L}_2 = 136°$ ∠s on a str line $\hat{B}_1 = 136° - 68° = 68°$ ext ∠of ΔBFL

b) $\hat{B}_2 = 44°$ corresp ∠s, SB//EA **OR** $\hat{B}_2 = \hat{L}_1 = 44°$ alt ∠s, SB//EA

c) $\hat{G}_1 = 60°$ ∠s on a str line **OR** $\hat{G}_1 = 180° - 76° - 44° = 60°$ int ∠s ΔLTG

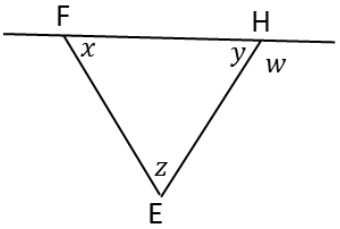
#TRY−ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

Worksheet 3.8

This worksheet deals mainly with isosceles triangles and includes a question on sides opp equal angles.

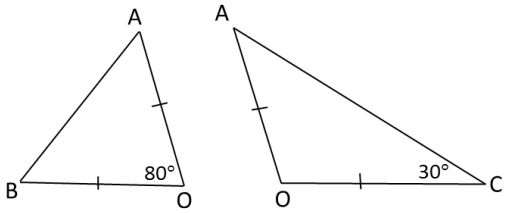
1) Use the diagram to answer the questions:



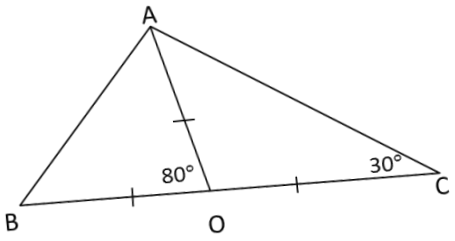
   a) If $FE = FH$:
      i) Which angle values are equal?
      ii) ΔEFH is known as _____
   b) If $FE = FH = HE$, then ΔFEH is _____
   c) If $x = z$, which sides are equal?
   d) If $x = z$, what is the value of w?

2)

   a) Given two triangles. Determine the 4 unknown angles.

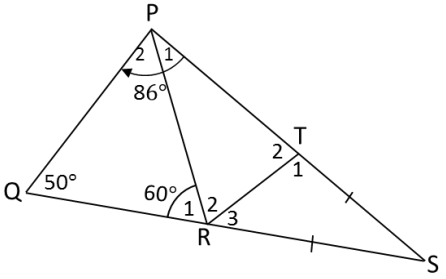


   b) If you now put the two triangles together, you get the following diagram: Show that BOC is **NOT** a straight line.



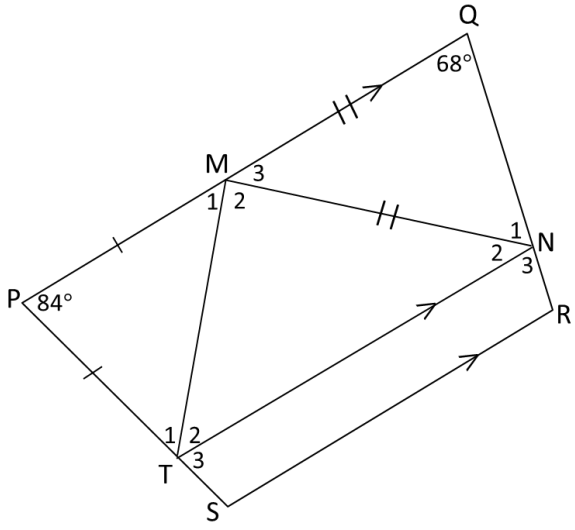
3) Given $\hat{Q} = 50°$; $TS = RS$; $P\hat{R}Q = 60°$ and $T\hat{P}Q = 86°$.

   a) Determine $\hat{S}$
   b) Determine $\hat{R}_3$ and $\hat{T}_1$
   c) Determine $T\hat{R}P$
   d) Is $PQ \parallel TR$? Give a reason for your answer.



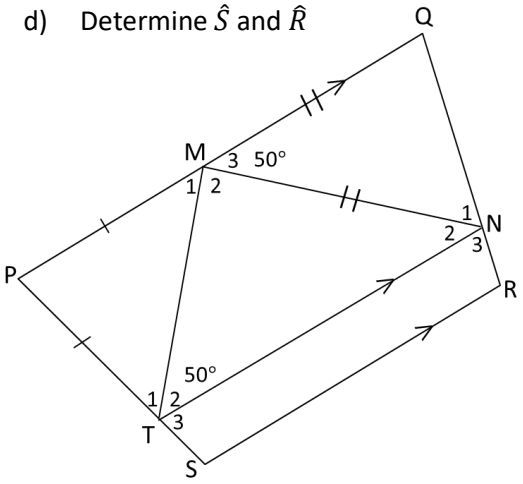
4) $PQ//SR//TN$, $\hat{P} = 84°, \hat{Q} = 68°, PT = PM$ and $MQ = MN$. T lies on PS and N lies on QR.

   a) Determine $\hat{T}_1$, $\hat{N}_3$, and $\hat{M}_3$ in this order.
   b) Now try to determine all other unknown angles.



5) $PQ//ZN//SR$, $\hat{T}_2 = \hat{M}_3 = 50°, PT = PM$ and $MQ = MN$. T lies on PS and N lies on QR.

   a) Show that $MT = MN$
   b) Determine $\hat{M}_2$
   c) What type of triangle is MTN?
   d) Determine $\hat{S}$ and $\hat{R}$

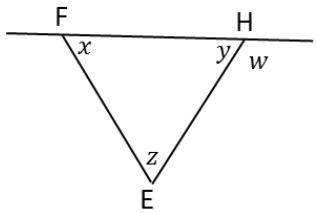
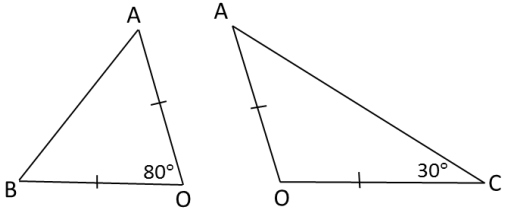
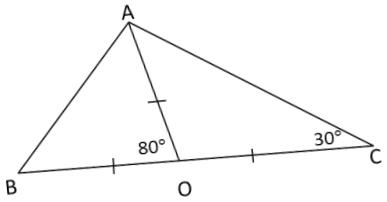
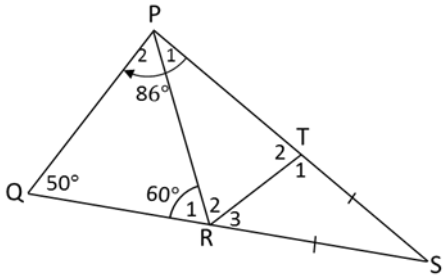


#TRY–ângles

PRACTICE IN SOLVING GEOMETRY PROBLEMS

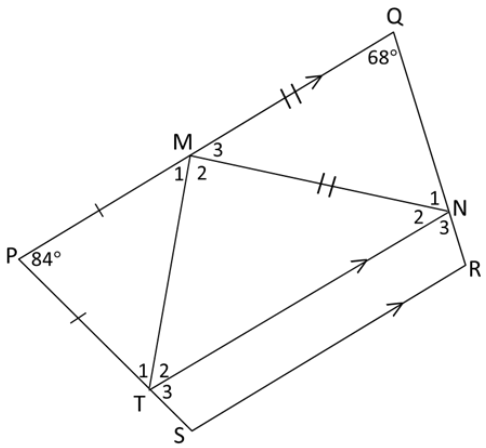
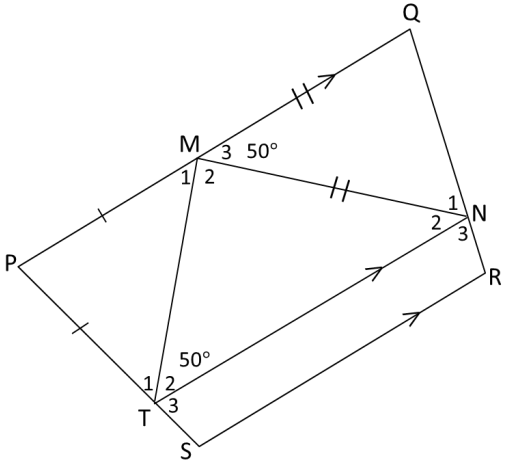
Worksheet 3.8

Answers

Question	Answer
1) Use the diagram to answer the questions a) If $FE = FH$: i) Which angles values are equal? ii) ΔEFH is known as ____ b) If $FE = FH = HE$, then ΔFEH is _____ c) If $x = z$, which sides are equal? d) If $x = z$, what is the value of w? 	1) a) i) $x = y$ ii) An isosceles Δ b) An equilateral Δ c) $FH = HE$ d) $w = 2x$ ***or*** $w = 2z$
2) Given two triangles. a) Determine the 4 unknown angles.  b) If you put the two triangles together, you get the following diagram: Show that BOC is **NOT** a straight line. 	2) Reasons are not expected for a. a) $\hat{A} = \hat{B}$ ∠s opp equal sides $= (180° - 80°) \div 2$ $= 50°$ int∠s Δ $\hat{A} = \hat{C} = 30°$ ∠s opp equal sides $\hat{O} = 120°$ int∠s Δ b) $75° + 80° = 155°$ $\neq 180°$ So BOC is not a straight line. The adjacent ∠s are not supplementary
3) Given that $\hat{Q} = 50°; TS = RS;\ P\hat{R}Q = 60°$ and $T\hat{P}Q = 86°$ a) Determine $\hat{S}$ b) Determine $\hat{R}_3$ and $\hat{T}_1$ c) Determine $T\hat{R}P$ d) Is $PQ \parallel TR$? Give a reason for your answer 	3) Reasons are not expected for a to c. a) $\hat{S} = 180° - 86° - 50°$ int∠s Δ $= 44°$ b) $\hat{R}_3 = \hat{T}_1$ ∠s opp equal sides $= (180° - 44°) \div 2$ $= 68°$ int∠s Δ c) $T\hat{R}P = 180° - 60° - 68°$ $= 52°$ ∠s on a str line d) $PQ \nparallel TR$ Corresponding angles $\hat{R}_3$ and $\hat{Q}$ are not equal in size.

Worksheet 3.8

Answers continued

Question	Answer
4) $PQ//SR//TN,\ \hat{P}=84°, \hat{Q}=68°, PT=PM$ and $MQ=MN$. T lies on PS and N lies on QR. a) Determine $\hat{T}_1$, $\hat{N}_3$, and $\hat{M}_3$ in this order. b) Now try to determine all other unknown angles. 	4) Reasons are not expected a) $\hat{T}_1=\hat{M}_1$ ∠s opp equal sides $=48°$ int∠s Δ $\hat{N}_3=68°$ corresp ∠s, *PQ* ‖*TN* $\hat{M}_3=56°$ ∠s opp equal sides; int∠s Δ b) $\hat{M}_2=56°$ ∠*s* on a str line $\hat{N}_2=56°$ alt ∠s, *PQ* ‖*TN* **OR** int∠s Δ $\hat{R}=112°$ co-int ∠s, *PQ* ‖*SR* **OR** *TN* ‖*SR* $\hat{T}_3=84°$ corresp ∠s, *PQ* ‖*TN* **OR** ∠*s* on a str line $\hat{S}=96°$ co-int ∠s, *PQ* ‖*SR* **OR** *TN* ‖*SR*
5) $PQ//ZN//SR,\ \hat{T}_2=\hat{M}_3=50°$, $PT=PM$ and $MQ=MN$. T lies on PS and N lies on QR. a) Show that $MT=MN$ b) Determine $\hat{M}_2$ c) What type of triangle is MTN? d) Determine $\hat{S}$ and $\hat{R}$ 	5) Reasons are not expected for b to d a) $\hat{M}_3=\hat{N}_2$ alt ∠s, PQ ‖TN $\hat{M}_3=\hat{T}_2=50°$ given So $\hat{N}_2=\hat{T}_2$ and $MT=MN$ sides opp equal∠s b) $\hat{M}_2=80°$ ∠s opp equal sides; int∠s Δ c) MTN is an isosceles Δ d) $\hat{M}_1=50°$ alt ∠s, $PQ \parallel TN$ $\hat{T}_1=50°$ ∠s opp equal sides $\hat{P}=80°$ int ∠s Δ $\hat{S}=100°$ co-int ∠s, $PQ \parallel SR$ $\hat{Q}=\hat{N}_1$ ∠s opp equal sides $=65°$ int∠s Δ $\hat{R}=115°$ co-int ∠s, $PQ \parallel SR$