

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE VERSION 1.0











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WhatsThePoint: Working with points and shapes in the Cartesian plane Grade 8

These materials were produced by the Wits Maths Connect Secondary (WMCS) project at the University of the Witwatersrand.

Visit us at www.witsmathsconnectsecondary.co.za

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Whats The Points working with points and shapes in the cartesian plane

Grade 8

VERSION 1.0

About this booklet

This booklet contains 10 worksheets on working with points and shapes in the Cartesian plane. The materials are intended for Grade 8 learners and include solutions for each question.

We provide a 1-page summary of the basics of plotting, translating and reflecting points. The pack is called *WhatsThePoint* precisely because there are many "points" that we want learners to grasp – we want them to learn more than plotting points and reading off coordinates. For example, we introduce them to the idea that there can be a relationship between the *x*-coordinate and the *y*-coordinate which will prepare them for work on functions in Grade 9 and beyond.

Each worksheet begins with reading off the coordinates of points and/or plotting points. From there, the worksheets develop in various ways to provide *practice with a purpose*. By this we mean there is a particular point to each worksheet, and this goes beyond simply plotting points and reading off coordinates. For example, learners will be prompted to try to make generalisations such as:

- All points on the *x*-axis have a *y*-coordinate of zero
- All points on the *y*-axis have an *x*-coordinate of zero
- All points in the same vertical line, have the same *x*-coordinate
- All points in the same horizontal line, have the same y-coordinate
- When you translate a point vertically, the *y*-coordinate changes but the *x*-coordinate remains the same
- When you translate a point horizontally, the *x*-coordinate changes but the *y*-coordinate remains the same
- When you reflect a point across the *x*-axis, the *y*-coordinate changes but the *x*-coordinate is unchanged
- When you reflect a point across the *y*-axis, the *x*-coordinate changes but the *y*-coordinate is unchanged

In addition, we provide opportunity for learners to work with shapes and to make connections between different representations of coordinate pairs.

The 10 worksheets contain several different foci as indicated in the table below. In the case of worksheets 5, 6, 7 and 8, 9, 10 there is lots of repetition which is done with a clear progression in level of difficulty.

Worksheet	Content
	Recap of key terminology and notation; how to plot, read off, translate and reflect points
1	Check understanding of the recap provided, plotting points and reading coordinates
2	Generalising about points that lie in the same horizontal and vertical lines
3	Translating points
4	Reflecting and translating points
5, 6, 7	Identifying points that meet 2 or 3 conditions; moving points by translating and reflecting them
8, 9, 10	Applying rules to determine coordinates of points; working with different representations of coordinates

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WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

Working with points in the Cartesian plane

THE CARTESIAN PLANE (also called the coordinate plane)

- the position of points and shapes in 2-D space. The Cartesian plane is a grid that enables us to describe
- lines at right angles to each other so that they cross at zero on each number line. A Cartesian plane is formed by drawing two number
- This point of intersection is called the origin.
- **vertical** number line is called the **y-axis**. The two number lines divide the plane into four The horizontal number line is called the *x*-axis and the
- an anti-clockwise direction as shown in Diagram 1. sections called quadrants. Quadrants are numbered in
- We use Roman numerals to number the quadrants.

Diagram 1: The Cartesian plane



Points are plotted on the Cartesian plane.

- Each point has two **coordinates** written as (x; y) in that order.
- The *x*-value gives the distance **left** or **right** from 0.
- The *y*-value gives the distance **up** or **down** from 0.
- We label points with capital letters next to the bracket,
- e.g. B (4;−5)
- The origin has coordinates (0; 0).



How to plot points

- To plot the point D(-3; 2): Start at the origin and move left along line with 2 on the y-axis. the x-axis to -3. Then move up the vertical line and place a dot in
- a dot in line with -2 on the *y*-axis. along the x-axis to 3. Then move down the vertical line and place To plot the point E(3; -2): Start at the origin and move right
- x-value is 0 at any point on the y-axis. Place a dot there. There is no horizontal movement because the To plot F(0; -3): Start at the origin, move down the *y*-axis to -3

How to read the coordinates of a point

- Put your 2 index fingers on the point.
- Move one index finger up/down towards the x-axis. The number on the *x*-axis is the *x*-coordinate.
- Move the other index finger sideways (left or right) towards the y-axis. The number on the y-axis is the y-coordinate
- down to 5 which is the x-coordinate. Move left to 3 which is the To read the coordinates of A: Put both index fingers on A. Move y-coordinate.

TRANSFORMATIONS

A transformation is a change. Translations and reflections point or shape. are types of transformations that change the position of a

Translations

number of units in a given direction. A translation is the movement of a point or shape by a given

Look at Diagram 2.

D. We write D'(1; 2) and we say "D prime". from -3 and stop at (1; 2). The new position is the **image** of We want to translate D(-3; 2) four units right. We move 4

Note:

- The *x*-coordinate changed because we translated D horizontally (i.e. left or right).
- If we translate a point up or down (vertically) the y-coordinate will change but the x-coordinate will not.

Reflections

same distance from a fixed straight line. A reflection moves a point so the point and its image are the



This line acts as a mirror.

A', is the same. The arrows show that the distance between A and its image,

To reflect D(-3; 2) across the *x*-axis

- Count how many units D is away from the x-axis
- D is 2 units *above* the *x*-axis
- So D'(-3;-2) The mirror image of D will be 2 units below the x-axis

Note:

- If we reflect across the y-axis, the y-axis acts as a If we reflect across the x-axis, the y-coordinate changes but the x -coordinate stays the same.
- mirror and the x-coordinate changes but the ycoordinate stays the same.



WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

wits maths connection

Worksheet 1

- In this task you will:
- Answer questions relating to the notes on Working with points in the Cartesian Plane
- 1) Read through the notes again and practice the examples again. Make sure you understand the words typed in **bold**.
- 2) Fill in *positive* or *negative* in the spaces in the table

Quadrant	Sign of <i>x</i> -value	Sign of <i>y</i> -value
I		
II		
IV		

- 3) Diagram 2 shows the position of E(3; -2) on the Cartesian plane.
 - a) Do we move across or up or down for 3?
 - b) What does the negative in front of the 2 tell us to do?
- 4) Explain how to plot P(3; -4). Provide a diagram with your explanation.
- 5) What is the value of *x* for all points on the *y*-axis?
- 6) What is the value of *y* for all points on the *x*-axis?
- 7) Write down the coordinates of B shown in Diagram 2.
- 8) In Diagram 2, point C lies halfway between -4 and -5. Write down the coordinates of C.
- 9) Look at the points D(-3; 2) and F(0; -3) in Diagram 2.
 - a) Translate D(-3; 2) one unit up. Write the coordinates of D'.
 - b) Translate D(-3; 2) one unit down. Write the coordinates of D'.
 - c) Translate F(0; -3) one unit down. Write the coordinates of F'.
 - d) Translate F(0; -3) four units left. Write the coordinates of F'.
 - e) Reflect D(-3; 2) across the *y*-axis. Write the coordinates of D'.
 - f) Reflect F(0; -3) across the *x*-axis. Write the coordinates of F'.

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Worksheet 1

Question	Ansv	wer			
1)					
2)					1
		Quadrant	Sign of <i>x</i> -value	Sign of <i>y</i> -value	
		1	Positive	Positive	
			Negative	Positive	
			Negative	Negative	
		IV	Positive	Negative	
3)					
a)	a)	Across to the rig	ght		
b)	b)	Down			
-					
4)	The	x-coordinate is a	3 so, from the origin	we move across to t	he right 3 units.
	ine	y-coordinate is -	-4 so we move down]	4 units.	
	-1	0 , 1 2 3 4			
	-1				
	-2				
	-4	(3;-4)			
5)					
a)	The value of <i>x</i> for all points on the <i>y</i> -axis is 0.				
b)	The	value of y for all	I points on the <i>x</i> -axis	is 0.	
6)	B(4;	0)			
7)	C(-4	.5;-1)			
8)					
a)	D'(-3	3;3)			
b)	D'(-3	3;1)			
c)	F'(0;	-4)			
d)	F'(-4	;-3)			
e)	D'(3	;2)			
f)	F'(0;	-3)			

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

Worksheet 2

In this task you will:

- Plot points on the Cartesian plane, and compare their positions
- Read off the coordinates of points in the Cartesian plane
- Plot points on the Cartesian plane and write their coordinates
- 1) Plot these 8 points on the Cartesian plane below. Label each point with its capital letter. A(0;3) B(2;1) C(5;0) D(2;-1) E(0; -3) F(-2;-1) G(-5; 0) H(-2; 1)



- 2) Join the points with straight lines. Start at A and work in alphabetical order to H. Describe the shape that is formed.
- 3) Look at points A and E.
 - a) What is the same about the coordinates of A and E?
 - b) What is different about the coordinates of A and E?
 - c) What is the same about the positions of A and E?
 - d) What is different about the positions of A and E?
- 4) Look at points C and G.
 - a) What is the same about the coordinates of C and G?
 - b) What is different about the coordinates of C and G?
 - c) What is the same about the positions of C and G?
 - d) What is different about the positions of C and G?
- 5) Look at points B and D.
 - a) What is the same about the coordinates of B and D?
 - b) What is the different about the positions of B and D?
 - c) Join B to D. Is BD a vertical line or a horizontal line?
- 6) Look at points B and H.
 - a) What is the same about the coordinates of B and H?
 - b) What is the different about the positions of B and H?
 - c) Join B to H. Is BH a vertical line or a horizontal line?



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7) The diagram below shows 8 points that have been plotted. They are labelled A to H. Write down the letter and coordinates of all 8 points. e.g. A(...;...).



- 8) The letters L and H are made up of vertical and horizontal lines. See the grid below.
 - a) Which coordinate remains the same when lines are vertical?
 - b) Which coordinate remains the same when lines are horizontal?



9) Extension

On the Cartesian plane, plot points to make the word ME. You can place it anywhere on the grid. Write the coordinates of each point next to the point.



WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

Worksheet 2

Solutions





WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

Worksheet 3

In this task you will:

- Translate points in one direction and in two directions
- Write coordinates of points knowing the image and translation
- Write translations given the coordinates of a point and its image

This diagram shows 6 points that have been plotted. Use the diagram to answer Q1 and Q2.

6∫у 5÷U Ś 4 3 2 Þ 0 -6 -5 -4 -3 -9 -8 -7 -2 -1 2 3 4 5 6 8 9 R Q

- 1) Write the coordinates of the point and its image after it has been translated.
 - e.g. P is translated 3 units left. P(-6; 1) and P'(-9; 1).
 - a) P is translated 3 units up.
 - b) Q is translated 5 units right.
 - c) R is translated 1 unit down.

- d) S is translated 4 units down.
- e) T is translated 7 units left.
- f) U is translated 2 units right.
- 2) Plot each point after the completed transformation and label it P", Q" etc. on the diagram.
 - a) P is translated 3 units left then 1 unit down. Give the coordinates of P".
 - b) Q is translated 5 units right then 2 units up. Write down the coordinates of Q".
 - c) R is translated 1 unit down then 3 units up. Write down the coordinates of R".
 - d) P is translated 1 unit left then 3 units down. Write down the coordinates of P'''. Are the coordinates of P''' the same as the coordinates of P''? Explain.
- 3) Write down the translation that causes:
 - a) F(3;5) to become F'(3;2).

- c) F(3; 5) to become F'(2; 2).
- b) G(-3; 5) to become G'(-6; 5).
- d) G(-3; 5) to become G'(-6; -3).
- 4) Write down the coordinates of the starting point for each translation.
 - a) A is translated 3 units up. Image is A'(-4; 2).
 - b) B is translated 3 units up. Image is B'(-4; -2).
 - c) C is translated 3 units left. Image is C'(4; 2).
 - d) D is translated 3 units right. Image is D'(-4; -2).
 - e) E is translated 3 units down. Image is E'(-2; -4).





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Worksheet 3



Solutions

Question	Answer
1)	
a)	P(-6;1) and P'(-6;4)
b)	Q(-5;-3) and Q'(-5;-8)
c)	R(3;-2) and R'(3;-3)
d)	S(4;4) and S'(4;0)
e)	T(6;0) and T'(-1;0)
f)	U(0;5) and U'(2;5)
2)	
a)	P''(-9;0)
b)	Q''(0;-1) P'''• 2
c)	R"(3;0)
d)	P'''(-7;-2) No. P'' comes from translating P, 3 units left then 1 unit down. P''' is from translating P, 1 unit left then 3 units down.
3)	
a)	3 units down
b)	3 units to the left
c)	1 unit left and 3 units down
d)	3 units left and 8 units down
4)	
a)	A(-4;-1)
b)	B (-4;-5)
c)	C(7;2)
d)	D(-7;-2)
e)	E(-2;-1)

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

Worksheet 4

In this task you will:

- Reflect points across the *x* or *y*-axis
- Write coordinates of the image knowing the original point and reflection
- Write the reflection given the coordinates of a point and its image
- Write the coordinates of a point and its image after translation and reflection
- 1) Six points are plotted in the diagram.
 - a) Write down the coordinates of A to F.
 - b) Reflect each point across the *x*-axis.
 - i) Plot the images. Label them A' F'.
 - ii) Write down the coordinates A' F'.
 - c) Reflect each point across the *y*-axis.
 - Plot the images and label them A" F".
 Use a different colour.
 - ii) Write down the coordinates of A'' F''.
- Copy and complete the table. You are given a point and the axis which it is reflected across.

Give the:

- quadrant of the point
- name and coordinates of the image
- quadrant of the image



Point	Quadrant	Reflected across	Image	Quadrant
$G\left(2\frac{1}{4};2\frac{1}{4}\right)$	I	<i>x</i> -axis	G'(;)	
H (3; 1)		<i>x</i> -axis		
J (3; 1)		<i>x</i> -axis		
$\mathrm{K}\left(-2\frac{1}{4};-2\frac{1}{4}\right)$		<i>x</i> -axis		
P (3;0)	n/a	<i>x</i> -axis		n/a
$G\left(2\frac{1}{4};2\frac{1}{4}\right)$		<i>y</i> -axis		
Н (3; 1)		y-axis		
J (3; 1)		y-axis		
$\mathrm{K}\left(-2\frac{1}{4};-2\frac{1}{4}\right)$		y-axis		
P (3;0)	n/a	y-axis		n/a

3) Write down the axis that the point is reflected across.

- a) L(5;2) L'(-5;2)
- b) M(-5;-2) M'(-5;2)

Extension

4) The diagram shows A, its image A', and B with its image B'.

- a) Describe a *translation* that will take A to A'. Give the size and direction.
- b) Describe a *reflection* that will take A to A'. Give the line of reflection.
- c) Repeat Q4a and Q4b for B and B'.

c)
$$N(-5;2)$$
 $N'(-5;-2)$
d) $Q(5;-2)$ $Q'(-5;2)$



If a point, P, lies on the reflection line, then its image, P', will also lie on the reflection line when you perform the reflection. i.e. the point and its image will be in the same position.

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Worksheet 4

Question	Answer						
1)							
a)	A(-4;2) B(1,5;1,5) C(5;-2) D(-2;-2) E(-5;-3) F(3;-3)						
b)							
i)	-7 -6 -5 -4 -3 -2 C' F'	• B' 2 • B' 1 0 -1 1 2 3 -1 -2 • D' -3 -4	• A'		ii) A'(4;2) B'(-1,5;: C'(-5;-2) D'(2;-2) E'(5;-3) F'(-3;-3)	L,5)	
c)							
i)	-7 -6 -5 -4 -3	-2 -1 1 2 -2 -1 -1 -2 -3 -3	F" C" 3 4 5 6	x ,	ii) A''(-4;-2 B''(1,5;- C''(5;2) D''(-2;2) E''(-5;3) F''(3;3)) 1,5)	
2)							
2)	Point	Quadrant	Poflocted ac	ross	Imaga	Quadrant	
2)	$\begin{array}{ c }\hline Point \\\hline c(a^1,a^1) \\\hline \end{array}$	Quadrant	Reflected ac	ross	Image $C^{\prime}(2^{1},2^{1})$	Quadrant	
2)	Point $G(2\frac{1}{4}; 2\frac{1}{4})$	Quadrant I	Reflected ac	ross	Image $G'(2\frac{1}{4}; -2\frac{1}{4})$	Quadrant	
2)	Point $G(2\frac{1}{4}; 2\frac{1}{4})$ H(3;1)	Quadrant I	Reflected ac x-axis x-axis	ross	Image $G'(2\frac{1}{4}; -2\frac{1}{4})$ H'(3;-1)	Quadrant IV IV	
2)	Point $G(2\frac{1}{4}; 2\frac{1}{4})$ $H(3;1)$ $J(3;1)$	Quadrant I I	Reflected ac x-axis x-axis x-axis	ross	Image G' $\left(2\frac{1}{4}; -2\frac{1}{4}\right)$ H'(3;-1) J'(3;-1) ::::(Quadrant IV IV IV	
2)	Point $G(2\frac{1}{4}; 2\frac{1}{4})$ $H(3;1)$ $J(3;1)$ $K(-2\frac{1}{4}; -2\frac{1}{4})$	Quadrant I I III	Reflected ac x-axis x-axis x-axis x-axis	ross	Image G' $\left(2\frac{1}{4}; -2\frac{1}{4}\right)$ H'(3;-1) J'(3;-1) H' $\left(-2\frac{1}{4}; 2\frac{1}{4}\right)$	Quadrant IV IV IV II	
2)	Point G($2\frac{1}{4}; 2\frac{1}{4}$) H(3;1) J(3;1) K($-2\frac{1}{4}; -2\frac{1}{4}$) P(3;0)	Quadrant I I II III n/a	Reflected ac x-axis x-axis x-axis x-axis x-axis	ross	Image G' $\left(2\frac{1}{4}; -2\frac{1}{4}\right)$ H'(3;-1) J'(3;-1) H' $\left(-2\frac{1}{4}; 2\frac{1}{4}\right)$ P'(3	Quadrant IV IV IV II n/a	
2)	Point $G(2\frac{1}{4}; 2\frac{1}{4})$ $H(3;1)$ $J(3;1)$ $K(-2\frac{1}{4}; -2\frac{1}{4})$ P(3;0) $G(2\frac{1}{4}; 2\frac{1}{4})$	Quadrant I I II III n/a I	Reflected ac x-axis x-axis x-axis x-axis x-axis y-axis	ross	Image G' $\left(2\frac{1}{4}; -2\frac{1}{4}\right)$ H'(3;-1) J'(3;-1) H' $\left(-2\frac{1}{4}; 2\frac{1}{4}\right)$ P'(3) G' $\left(-2\frac{1}{4}; 2\frac{1}{4}\right)$	Quadrant IV IV IV II II n/a II	
2)	Point $G(2\frac{1}{4}; 2\frac{1}{4})$ $H(3;1)$ $J(3;1)$ $K(-2\frac{1}{4}; -2\frac{1}{4})$ $P(3;0)$ $G(2\frac{1}{4}; 2\frac{1}{4})$ $H(3;1)$	Quadrant I I I I I I I I I I I I I I I I I I I	Reflected ac x-axis x-axis x-axis x-axis x-axis y-axis y-axis	ross	Image G' $\left(2\frac{1}{4}; -2\frac{1}{4}\right)$ H'(3;-1) J'(3;-1) H' $\left(-2\frac{1}{4}; 2\frac{1}{4}\right)$ P'(3) G' $\left(-2\frac{1}{4}; 2\frac{1}{4}\right)$ H'(-3;1)	Quadrant IV IV IV II II II II II	
2)	Point $G(2\frac{1}{4}; 2\frac{1}{4})$ $H(3;1)$ $J(3;1)$ $K(-2\frac{1}{4}; -2\frac{1}{4})$ $P(3;0)$ $G(2\frac{1}{4}; 2\frac{1}{4})$ $H(3;1)$ $J(3;1)$	Quadrant I I I I I I I I I I I I I I I I I I I	Reflected ac x-axis x-axis x-axis x-axis x-axis y-axis y-axis y-axis	ross	Image G' $\left(2\frac{1}{4}; -2\frac{1}{4}\right)$ H'(3;-1) J'(3;-1) H' $\left(-2\frac{1}{4}; 2\frac{1}{4}\right)$ P'(3 G' $\left(-2\frac{1}{4}; 2\frac{1}{4}\right)$ H'(-3;1) J'(-3;1)	Quadrant IV IV IV II II II II II	
2)	$\begin{array}{c c} \hline \textbf{Point} \\ \hline G\left(2\frac{1}{4}; 2\frac{1}{4}\right) \\ \hline H(3;1) \\ \hline J(3;1) \\ \hline K\left(-2\frac{1}{4}; -2\frac{1}{4}\right) \\ \hline P(3;0) \\ \hline G\left(2\frac{1}{4}; 2\frac{1}{4}\right) \\ \hline H(3;1) \\ \hline J(3;1) \\ \hline K\left(-2\frac{1}{4}; -2\frac{1}{4}\right) \\ \hline \end{array}$	Quadrant I I I I I I I I I I I I I I I I I I I	Reflected ac x-axis x-axis x-axis x-axis x-axis y-axis y-axis y-axis y-axis y-axis y-axis	ross	Image G' $\left(2\frac{1}{4}; -2\frac{1}{4}\right)$ H'(3;-1) J'(3;-1) H' $\left(-2\frac{1}{4}; 2\frac{1}{4}\right)$ P'(3 G' $\left(-2\frac{1}{4}; 2\frac{1}{4}\right)$ H'(-3;1) J'(-3;1) K' $\left(2\frac{1}{4}; -2\frac{1}{4}\right)$	Quadrant IV IV IV IV II n/a II II IV	
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WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE



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Worksheet 5

In this task you will:

- Plot points and read off their coordinates
- Identify points that satisfy 2 conditions

NOTE

1) The diagram shows a square with vertices A, B, C and D.

The shape in the diagram is a square because all 4 sides have equal length and the internal angles are 90°. Although it has been rotated, it is still a square.



- b) Points P, Q and R are described below. Plot the points on the Cartesian plane above and label each point clearly.
 - i) Point P has the same *x*-coordinate as A and the same *y*-coordinate as D.
 - ii) Point Q has the same x-coordinate as D and the same y-coordinate as B.
 - iii) Point R lies on the same vertical line as B and the same horizontal line as A.
- 2) You want to move from one point to another but you can only move horizontally and vertically.
 - a) Write down the instructions to get from B to D.
 - b) Write down the instructions to get from A to point T with coordinates (11;-1).
- 3) In this question you must identify points that meet the conditions in the middle column. Each point must meet both conditions. Plot each point on the diagram. Write down the coordinates.

Name of point/s	Description of the point	Coordinates of points
E, F	Lies on the <i>x</i> -axis but outside the square	e.g. E(-5;0) F(;)
G, H	Lies on the y-axis and inside square	e.g. G(0;-2) H (;)
J, K, L	In quadrant II and inside the square	
M, N	On the same horizontal line as B and with positive <i>x</i> -values	
S, U	On the same vertical line as D and with positive y-values	
V	Outside the square and very close to vertex C	
W	Inside the square and very close to vertex A	

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE



Worksheet 5

Question	Answer							
1a	A (-3; 5) B (-5; -2) C (2; -4) D (4; 3)							
1b i-iii)	Points required are in red.							
2a	Point B would move 9 units to right and 5 units up. Any order is correct.							
26	Point A would move 14 units to the right and 6 units down. Any order is correct.							
2	There are many possible positions for points in this question. Different colours are used to							
5	indicate possible positions							
	• F F : red on x-axis							
	• G. H : green on v-axis							
	 J. K. L : orange in quadrant II 							
	 M, N : purple in horizontal line in guadrant IV 							
	• S, U : blue in vertical line in quadrant I							
	• V : pink close to C in quadrant IV							
	• W : brown close to A in quadrant II							
	A							
	В							

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

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Worksheet 6

In this task you will:

- Plot points and read off their coordinates
- Identify and plot points that satisfy 3 conditions
- 1) The diagram shows a pentagon with vertices A, B, C, D and E. Write down the coordinates of each vertex.

NOTE

A *pentagon* is any 5-sided shape. The sides do not have to be the same length.



- 2) Focus now on points C and D.
 - a) Look at the points on the diagram. What is the same about their positions? What is different?
 - b) Look at the coordinates of the 2 points. What is the same about their coordinates and what is different?
 - c) Think about all the points that lie on line CD. What is the same about them? What is different?
- 3) Focus on the points where a side of the pentagon intersects (or cuts) an axis.
 - a) Point F is the point where CD intersects the *y*-axis. Plot F and write down its coordinates.
 - b) Point G is the point where BC intersects the *x*-axis. Plot G and write down its coordinates.
 - c) Point H is the point where AE intersects the *y*-axis. Plot H and estimate its coordinates.
 - d) Point J is the point where DE intersects the *x*-axis. Plot J and estimate its coordinates.
- 4) In this question you must identify points that meet the conditions under the heading *Description of the point*. Each point must meet all 3 conditions. Plot each point on the diagram. Write down the coordinates.

Name of point/s	Description of the point	Coordinates of points
K, L, M	Lies on x -axis, inside pentagon, has a negative x -value	e.g. K(-3;0)
N, P	Lies below x-axis, outside pentagon, y-coordinate > -3	e.g. N(-8;-2)
Q, R, S	In quadrant II, x-coordinate is divisible by 3, y-coordinate < 3	e.g. Q(-9; 1,5)
U	On same horizontal line as E, inside pentagon, x -coordinate > 3	
V, W	Inside pentagon, x-coordinate and y-coordinate have same value,	
	$x \le 0$	

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE



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Worksheet 6

Question	Answer						
1	A (-5; 4); B (-7; 1); C (-3; -3); D (3; -3); E (4; 2)						
2a	Points C and D are the same distance away from the x- and y-axis but lie in different quadrants.						
2b	Both points have the same y-coordinate but different x-coordinates.						
2c	All the points that lie on CD will have the same <i>y</i> -coordinate but will have different <i>x</i> -coordinates.						
3a-d	All coordinates are labelled and are in blue. F (0; -3) G (-6; 0) H (0; 2,9) (estimate) J (3,6; 0) (estimate) B B C C C C C C C C C C C C C						
	 There are many possible positions for points in this question. Different colours are used to indicate possible positions. K, L, M : red on negative x-axis (does not include point at the origin) N, P : green in quadrant III outside pentagon Q, R, S : blue in quadrant II; x-values are -3, -6, -9, etc. U : purple in quadrant I V, W : brown in quadrant III including origin 						
	$\begin{array}{ c c c c c c c c c c c c c c c c c c c$						

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE



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Worksheet 7

In this task you will:

- Plot points and read off their coordinates
- Translate and reflect points
- Identify and plot points that satisfy 2 or more conditions
- 1) The diagram contains 3 shapes: triangle ABC, rectangle DEFG and pentagon HJKLM.
 - a) Write down the coordinates of the vertices of each shape. Label each point.



- 2) Look at points B, E and F.
 - a) What is the same about their positions in the plane? What is different?
 - b) What is the same about their coordinates? What is different?
 - c) True or false: If you reflect B across the *y*-axis, its image will land on F. How do you know?
- 3) Look at points C and J.
 - a) What is the same about their positions in the plane? What is different?
 - b) What is the same about their coordinates? What is different?
- 4) The pentagon intersects the *x*-axis in 2 places and the *y*-axis in 2 places.
 - a) Give the exact point where HM intersects the *x*-axis. Write as a coordinate pair.
 - b) Give the exact point where LM intersects the *y*-axis. Write as a coordinate pair.
 - c) Estimate where KL cuts the *x*-axis. Write as a coordinate pair.
 - d) Estimate where JK cuts the *y*-axis. Write as a coordinate pair.
- 5) Give the coordinates of points that meet the criteria in each of the following. Plot the points.
 - a) 2 points that lie inside the rectangle and have the same *y*-coordinate as A.
 - b) 4 points that lie inside the rectangle but not in the pentagon.
 - c) 2 points that lie in quadrant I, inside the pentagon, outside the rectangle, not on an axis.

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

We have provided another copy of the diagram on this page.



- 6) You want to move from one point to another but you can only move horizontally and vertically.
 - a) Move from A to G.
 - b) Move from K to C.
 - c) Move from M to J to F.
- 7) These questions involve reflecting points across the axes.
 - a) Reflect L across the *y*-axis. Will the image of L lie inside the triangle?
 - b) Reflect H across the *x*-axis. Will the image of H lie inside the triangle?
 - c) You reflect point J across the x-axis and then reflect the image of J (call it J') across the y-axis. Will the image of J' land inside the rectangle? How do you know?



WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE



Worksheet 7

Question	Answer						
1	A (-9; 6); B (-5; 1); C (-1; 4); D (1; 7); E (1; 1); F (5; 1); G (5; 7); H (-4; -2);						
	J (-1; -1); K (2; -3); L (4; 5); M (-2; 2)						
2a	All three points are in the same horizontal line, but different vertical lines.						
2b	All three points have the same <i>y</i> -coordinate but different <i>x</i> -coordinates.						
2c	True. Point B is 5 units away from the <i>y</i> -axis and so is point F but in the opposite direction. So point B						
	will land on point F when reflected.						
3a	Both points lie in the same vertical line, but they lie in different horizontal lines.						
30	Both points have the same x-coordinate but different y-coordinates.						
4-							
4a	(-3; 0) 40 (0; 3) 4C (2,8; 0) 40 (0; -1,8)						
-							
5a-c	ef points for each question						
	5a Points are in red						
	5h. Points are in brown						
	5c. Points are in blue						
	M •(0 5, 2 4) (4, 3, 2)						
	B •(2.5, 0.5) F ×						
	H J 2						
	K						
6a	Move point A, 1 unit up and 14 units to the right.						
6b	Move point K, 7 units up and 3 units to the left.						
6c	Move point M, 3 units down and 1 unit to the right which will then be at J. Then move point M, 2 units						
	up and 6 units to the right.						
7a	No. See position of L'.						
7b	Yes. See position of H'.						
7c	No, it will land on E which is a vertex of the rectangle.						
	I he image of J, i.e. J', will be 1 unit left from the y-axis						
	and I unit above the x-axis, (-1,1). when J is reflected $y = \frac{1}{2}$						
	across the y-dxis, it's inidge (j') will be 1 unit to the						
	high of the y-axis and 1 unit above the x-axis (1,1) which will be on F						

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

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Worksheet 8

In this task you will:

- Learn about 6 different way to represent relationships between numbers
- Make links between the different representations

In this worksheet we will look at 6 different ways to show relationships: words, flow diagrams, algebraic rules, tables and plotting points on the Cartesian plane.

1) Words

All the points in this worksheet must obey the rule: Add 2 to the input value to get the output value For example, if the input value is 5, then the output value will be 5 + 2 = 7

2) Flow diagram

The box containing "+2" in the flow diagram shows that we are adding 2 to each input value, to get each output value. For example: -4 + 2 = -2.



a) Work out the missing output values and write them in the spaces provided.

3) Algebraic rule

We can write the rule in algebraic notation as $y = \dots$

In this worksheet, the rule is: Add 2 to the input value to get the output value

a) Write the algebraic rule: y =_____

4) Table

Tables of values are a useful way to represent rules and to represent coordinate pairs of points. The inputs are the x-values and the outputs are the y-values.

<i>x</i> -coordinate (input)	-4	-1	0	3	6	
y-coordinate (output)	-2			5		

- a) Complete the table by filling in the *y*-coordinate of each point.
- b) Insert 2 more points in the table. You must choose the input values and calculate the output values.

5) Coordinate pairs

You have already learned to write points as coordinate pairs. Remember that the *x*-coordinate always comes first.

a) Write the 7 points as <u>coordinate pairs</u>, labelling them P, Q, R, S, T U and V.

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE



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6) Plotting points in the Cartesian plane

When we plot points, we get a visual representation of where they lie on the plane. Sometimes we call this a *graphical representation*.

Plot all 7 points on the Cartesian plane, and label them. Two points have already been plotted.



- 7) Give the coordinates of 2 more points which obey the rule and which lie in quadrant III. Plot the points.
- 8) Give the coordinates of 2 points which obey the rule and which lie in quadrant II. Plot the points.
- 9) You should notice that all the points lie in a straight line. Imagine there is a line passing through the points. (You might want to join the points with a ruler.) Where will the line cut the *x*-axis? Write down the coordinate pair of this point.
- 10) List the 6 ways that we can represent relationships between numbers.

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE



Worksheet 8

Question	Answer								
1									
2a	Input	Outp	ut						
	$\begin{array}{c} -4 \\ -1 \\ 0 \\ 2 \end{array} + 2 \end{array} + 2 \begin{array}{c} -2 \\ 1 \\ 2 \\ 5 \end{array}$								
	6	► 8	lue.						
	x-value	y-va	lue						
3a	y = x + 2								
4a; b	x-coordinate (input) -	4	-1	0	3	6	6,5	7	
	y-coordinate (output) -	2	1	2	5	8	8,5	9	
									-
5a; 6	P (-4: -2): O (-1: 1): B (0: 2): S (3: 5): T (6: 8): 6 Points are in black								
00,0	$\begin{bmatrix} 1 & (-4, -2), & (-1, -1), & (0, 2), & (3, 3), & (0, 0), \\ 0 & (-1, -1), & (0, 2), & (3, 3), & (0, 0), \\ 0 & (-1, -1), & (0, 2), & (-1, -1), \\ 0 & (-1, -1), & (-1, -1)$								
	- (-/-/-// (/-/							9	T U
								8	
								6	
								6	S
								4	
								2	
								×1	
					-7	-6 -5 -4	-3 -2 -	101	2 3 4 5 6 7 8
								-2	
								-3	
7; 8; 9	7. Possible points shown with gr	een	coordi	nates ir	n II.) ĵ	/ V
	8. Possible points shown with red coordinates in III.							T	
	9. Intersection with <i>x</i> -axis shown in orange at (-2;0)								
								6	
								5	S
								4	
								3	*
								Q 2	(-0.5, 1.5)
						~ =	(-2, 0	(1.5, 0	0.5) x
							,-1) (-2	5, -0.5)	
							∕ (-3.5 , - P	1.5) -2	
								-3	
						V			
10	Words, flow diagrams, algebraic	rule	. table	. coord	inate r	airs.gr	aphical	repres	sentation
		····	., саыс	,		ан <i>3,</i> 61	~pmca	, cpics	

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

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Worksheet 9

In this task you will:

- Work with 6 different ways to represent relationships between numbers
- Make links between the different representations

All the points in this worksheet must obey the rule:

Subtract 3 from the input value to get the output value

For example, if the input value is -2, then the output value will be: -2 - 3 = -5

1) The box containing "3" in the flow diagram shows that we are subtracting 3 from each input value, to get each output value, e.g. 4 - 3 = 1



Work out the missing output values and write them in the spaces provided.

- 2) Write the algebraic rule: $y = _$
- 3) In the table, the inputs are the *x*-values and the outputs are the *y*-values.

x-coordinate (input)	-5	-1	0	4	
y-coordinate (output)				1	

- a) Complete the table by filling in the *y*-coordinate of each point.
- b) Insert 2 more points in the table. You must choose the input values and calculate the output values.
- 4) Write the 6 points as <u>coordinate pairs</u>, labelling them A, B, C, D, E and F.



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5) Plot the points on the Cartesian plane and label them. One point has already been done.



- 6) Give the coordinates of 2 more points which obey the rule and which lie in quadrant I. Plot the points.
- 7) Give the coordinates of 2 points which obey the rule and which lie in quadrant IVI. Plot the points.
- 8) You should notice that all the points lie in a straight line. You might want to draw a line passing through the points (using a ruler). Where will the line cut the *x*-axis? Write down the coordinate pair of this point.

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE



Worksheet 9

Question	Answer			
1	Input Output			
	-5			
	x-value y-value			
`	$y = x^2$			
2	y = x - 3			
20.6				
34,0				
	x-coordinate (input) -5 -1 0 4 6 7,5			
	<i>y</i> -coordinate (output) -8 -4 -3 1 3 4,5			
4	A (-5; -8); B (-1; -4); C (0; -3); 5. Points to plot are in black			
	D (4; 1); E (6; 3); F (7,5; 4,5)			
	-6			
	A			
6; 7; 8	6. Possible points shown with red coordinates in I.			
	7. Possible points shown with pink coordinates in IV.			
	8. Line cuts <i>x</i> -axis at (3; 0) shown in blue.			
	9 Îy			
	(7, 4)			
	(5.2) (5.5, 2.5)			
	(3,0) / (3 ,5,0.5) × × · · · · · · · · · · · · · · · · ·			
	(1.38, -1.62)			
	B 4			
	A -7			

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

Worksheet 10

In this task you will:

- Apply a rule to determine the coordinates of points
- Write points using 6 different representations

In this worksheet you get more practice to make connections between different mathematical representations: words, algebraic symbols, flow diagrams, tables and points on the Cartesian plane.

All the points on this worksheet must obey the rule:

```
To get the output, you must double the input and then subtract 1 \frac{1}{2}
```

e.g. if the input is -1, then the output is: 2(-1) - 1 = -3

- 1) Write the rule as an algebraic statement: $y = _$
- 2) Complete the flow diagram, by calculating the missing output values
- 3) Complete the table by filling in the y-coordinate of each point.
- 4) Write down the coordinate pairs for all the points. We have chosen the labels A G.
- 5) Plot the points on the Cartesian plane. What do you notice?





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We have completed some examples in each representation to show what is expected.

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE



Worksheet 10

