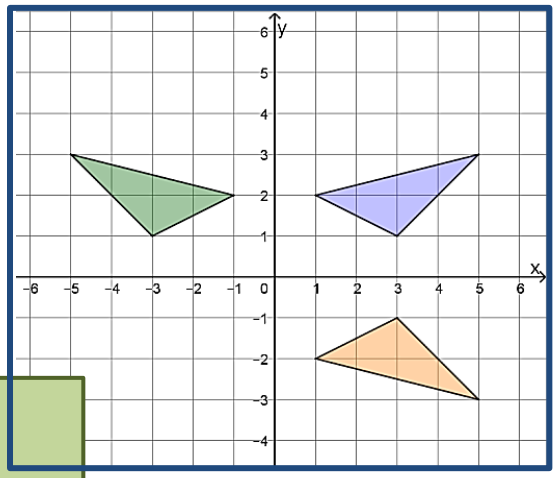
wits maths connect
supporting secondary maths

WhatsThePoint

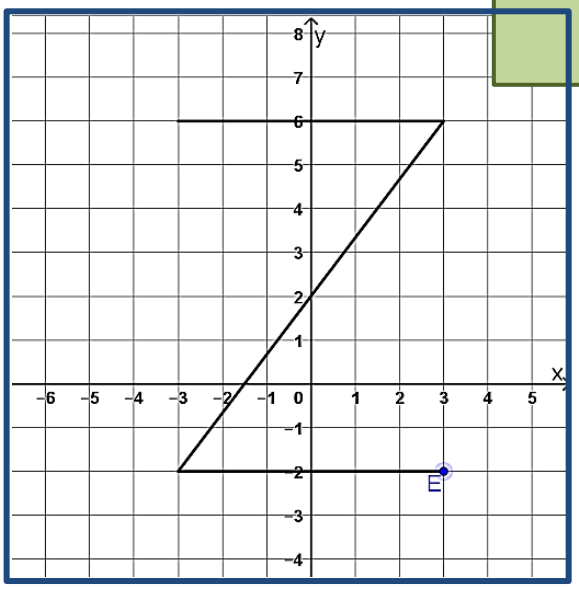
WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

VERSION 1.0

Grade 9



Move the Z-shape so that E moves to (4;1). Where will the Z-shape cross the y-axis?



Reflect the green triangle across the x-axis, and reflect the orange triangle across the y-axis. What image will you see in quadrant III?

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
FIRSTRAND
NRF National Research Foundation
science & innovation
Department:
Science and Innovation
REPUBLIC OF SOUTH AFRICA

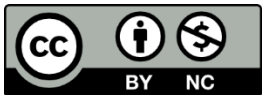
WhatsThePoint: Working with points and shapes in the Cartesian plane

These materials were produced by the Wits Maths Connect Secondary (WMCS) project at the University of the Witwatersrand.
Visit us at www.witsmathsconnectsecondary.co.za

Team members:
Craig Pournara (Project leader and editor)
Micky Lavery, Wanda Masondo, Vasantha Moodley and Yvonne Sanders
With thanks to Hailey Loveland, Sheldon Naidoo, Kate Pournara and Andrew Pournara

The work of the WMCS project is supported financially by the FirstRand Foundation, the Department of Science and Innovation and the National Research Foundation.

This work is licensed under a Creative Commons Attribution-NonCommercial 4.0 International License.



The Creative Commons license means this booklet is freely available to anyone who wishes to use it. It may not be sold on (via a website or other channel) or used for profit-making of any kind.
To view a copy of this license, visit http://creativecommons.org/licenses/by-nc-sa/4.0/

WhatsThePoint

WORKING WITH POINTS AND SHAPES IN THE CARTESIAN PLANE

VERSION 1.0

About this booklet

This booklet contains 10 worksheets on working with points and shapes in the Cartesian plane. The materials are intended for Grade 9 learners and include solutions for each question. The pack is called WhatsThePoint precisely because there are many "points" that we want learners to grasp as they work through the materials.

We assume learners have already been the basic content of plotting, translating and reflecting points. Each worksheet begins with reading off the coordinates of points and/or plotting points. From there, the worksheets develop in various ways to provide *practice with a purpose*. By this we mean that there is a particular point to each worksheet, and this goes beyond simply plotting points and reading off coordinates. Our goal is that these materials should prepare learners for key ideas about graphical representations of functions in Grade 9 and beyond. For example, learners will prompted to try to make generalisations such as:

- All points in the same vertical line, have the same x-coordinate
- All points in the same horizontal line, have the same y-coordinate
- All points on the x-axis have a y-coordinate of zero
- All points on the y-axis have an x-coordinate of zero
- When you translate a point vertically, the y-coordinate changes but the x-coordinate remains the same
- When you translate a point horizontally, the x-coordinate changes but the y-coordinate remains the same
- When you reflect a point across the x-axis, the y-coordinate changes but the x-coordinate is unchanged
- When you reflect a point across the y-axis, the x-coordinate changes but the y-coordinate is unchanged
- If points lie on the same straight line, then they have the same relationship between the x-coordinate and the y-coordinate

In addition, we provide opportunity for learners to work with shapes and to make connections between different representations of coordinate pairs.

The 10 worksheets contain six distinct foci as indicated in the table below. In the case of worksheets 2, 3, 4 and 6, 7, 8, there is lots of repetition together with a clear progression in level of difficulty.

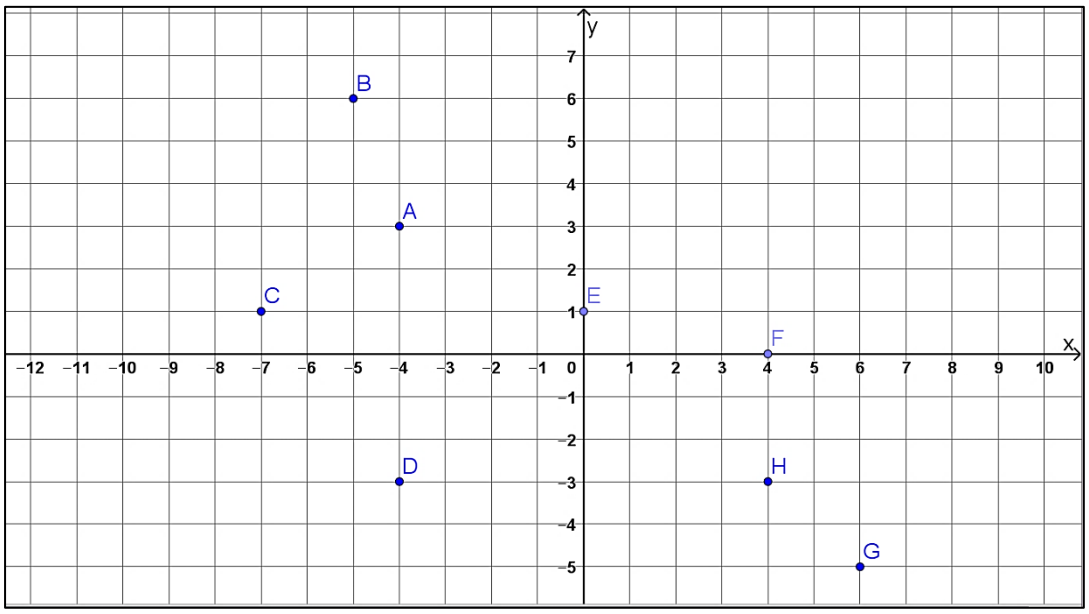
Worksheet	Content
1	Basic practice of the fundamentals in point plotting and reading off coordinates
2, 3, 4	Translating letters L, Z and W in the plane, including calculations of distance, perimeter and area
5	Connecting representations of coordinate pairs: functions diagrams, table, Cartesian plane
6, 7, 8	Translating points onto a given line to recognise the common relationship between coordinates
9	Translating points to generate new lines (preparation for horizontal translations of functions)
10	Reflecting triangles across axes

Worksheet 1: Plotting and comparing positions of points

In this task you will:

- Plot points and read off their coordinates
- Compare the positions of points in the plane and link this to their coordinates

1) The diagram below shows 8 points that have been plotted. They are labelled A to H. Write down the coordinates of all 8 points. Label each one, e.g. A(;).



2) Compare points C and E.
   a) Look at the points on the diagram. What is the *same* about their positions? What is *different* about their positions?
   b) Look at the coordinates of the 2 points. What is the *same* about their coordinates? What is *different* about their coordinates?

3) Look at points A and B – pay attention to their positions on the diagram and their coordinates. What is the *same*? What is *different*?

4) Choose another 3 pairs of points. Give one property that is the same and one property that is different for each pair. Try to use every plotted point at least once.

5) Plot 2 more points on the diagram. Label your new points P and Q.
   a) Say what is different about P and Q.
   b) Find a point (already plotted) which has a property that is the same as P. Say what is the same.
   c) Find a point (already plotted) which has a property that is the same as Q. Say what is the same.

<u>Extension</u>

6) Look at the points B, E, H, G. Notice that they lie in a straight line. You may want to join the points.
   a) Write down the coordinates of the 4 points. What is the relationship between the x-coordinate and y-coordinate of each point? Try to find a relationship that is the same for all 4 points.
   b) Give the coordinates of another (new) point that also has this relationship.
   c) Check that your new point lies on the same line as these 4 points.

Worksheet 1: Plotting and comparing positions of points

Solutions

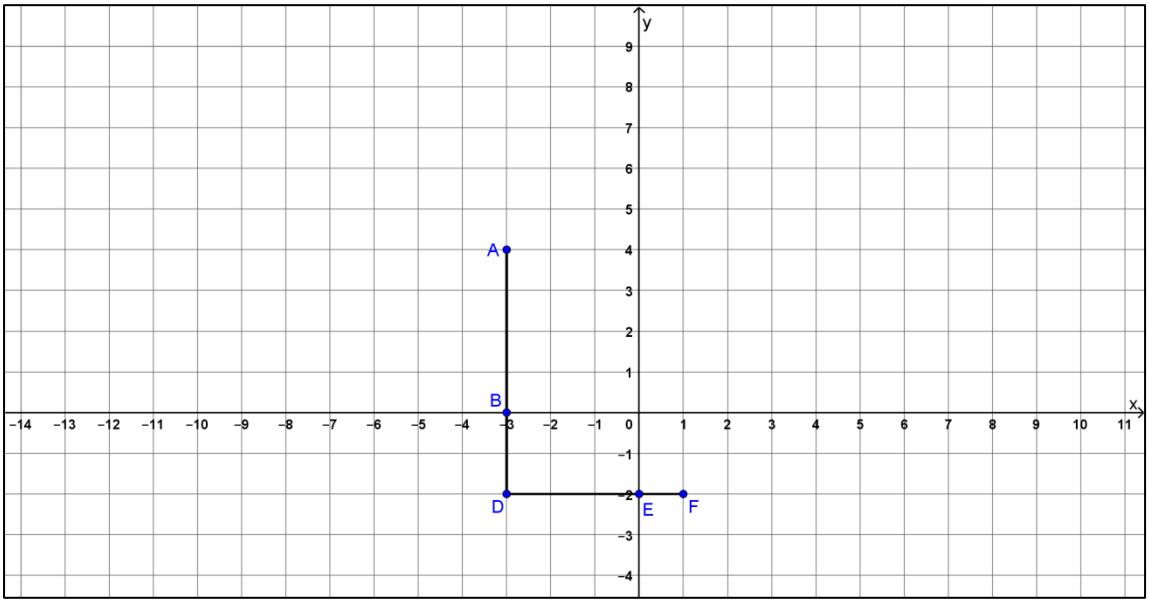
Question	Answer
1	A(-4;3) B(-5;6) C(-7;1) D(-4;-3) E(0;1) F(4;0) G(6;-5) H(4;-3)
2a	Same: Lie in same horizontal line; same distance above x-axis; same side of y-axis Different: Distance from origin, different distance from y-axis.E lies on axis but C doesn't
2b	Same: y-value Different: x-value
3	Same: Lie in quadrant II; negative x-values; positive y-values Different: x-values and y-values are different; A lies closer to x-axis and to y-axis
4	Depends on chosen points. Possible criteria are: • Same quadrant and hence same signs for x-coordinates and y-coordinates, e.g. A, B and C; G and H • Same vertical line, same x-coordinates, e.g. A and D, F and H • Same horizontal line, same y-coordinates, e.g. D and H, C and E • Both points lie on axes, e.g. E and F • Lie below x-axis, e.g. D and H; D and G; G and H Differences will involve aspects such as: quadrant, x-coordinate, y-coordinate, on/off an axis
5a, b, c	Depends on positions of P and Q. See possible criteria in 4.
Extension	
6a	B(-5;6) E(0;1) H(4;-3) G(6;-5) Common relationship: $y = -x + 1$ although learners may express relationship verbally, e.g. they may refer to a "gap of 1 unit" between the values of the coordinates, and that the coordinates have opposite signs.
6b	Even if learners cannot identify an algebraic relationship between the coordinates, it is possible to find other points by drawing the line through B, E, G and H. Additional points: (-2;3) (1;0) (5;-4)
6c	Check this by plotting point or simply by inspection

Worksheet 2: Working with an L-shape

In this task you will:

- Read, plot and move points, and move shapes
- Explore similarities and differences in points that lie on the same horizontal or vertical line

1) Five points have been plotted and joined to make an L-shape.



a) Write down the letter of each point and give its coordinates.
b) What is the same about points A, B and D? And what is different?
c) What is the same about points D, E and F? And what is different?
d) What is the same about points B and E? And what is different?
e) Write down the coordinates of 3 more points that lie on the L-shape. Label them K, M and N.

2) Assume you want to move the L-shape. Focus on points A, B, D, E and F.
a) If you move the L-shape so that F is at (1;0), what will be the coordinates of the other 4 points?
b) If you move the L-shape so that A is at (-1,4), what will be the coordinates of the other 4 points?
c) Move the L-shape so that D is at the origin. What will be the coordinates of the other 4 points?
d) Look at the coordinates of the new points in 2a – 2c. What changed? What stayed the same? Compare with the original position of the L-shape each time.

3) Assume you want to use the original L-shape to make a rectangle.
a) Points A, D and F are 3 vertices (corners) of the rectangle. Plot the 4th point of the rectangle. Call it P. Write down the coordinates of P.

b) Give the coordinates of a new point that lies:
   i) Inside the rectangle
   ii) Outside the rectangle
   iii) On a short side of the rectangle
   iv) On a long side of the rectangle

c) Give the coordinates of a new point that satisfies both conditions in each statement:
   i) Outside the rectangle and on the x-axis
   ii) Inside the rectangle and in quadrant II
   iii) Inside the rectangle and in quadrant I
   iv) On a side of the rectangle and below the x-axis.

Worksheet 2: Working with an L-shape

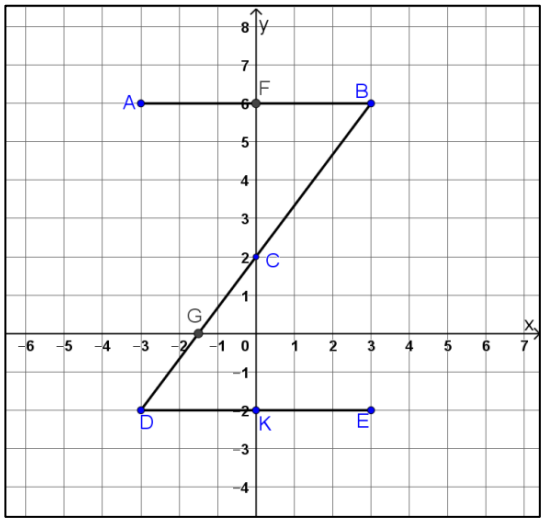
Solutions

Question	Answer
1a	A(-3;4) B(-3;0) D(-3;-2) E(0;-2) F(1;-2)
1b	x-coordinates are the same and all negative (-3), all on left of y-axis y-coordinates are different, one is positive, one is negative, one is zero
1c	y-coordinates are the same and all negative (-2), all below x-axis x-coordinates are different, one is positive, one is negative, one is zero
1d	Both lie on axes and have one coordinate that is zero and one coordinate that is negative B has y-coordinate 0, E has x-coordinate 0
1e	These will depend on learners' choices; all points on AB with have $x = -3$; all points on DF will have $y = -2$
2a	When F(1;0): A(-3;6) B(-3;2) D(-3;0) E(0;0)
2b	When A(-1;4): B(-1;0) D(-1;-2) E(2;-2) F(3;-2)
2c	When D(0;0): A(0;6) B(0;2) E(3;0) F(4;0)
2d	For Q2a: x-coordinates remain the same, y-coordinates increase by 2 For Q2b: x-coordinates increase by 2, y-coordinates remain the same For Q2c: x-coordinates increase by 3, y-coordinates increase by 2 In all cases, the L-shape remained the same size and facing in the same direction
3a	P(1;4)
3b(i)	Coordinates should satisfy the following conditions: $-3 < x < 1;\ -2 < y < 4$
3b(ii)	Coordinates should satisfy the following conditions: $x < -3$ or $x > 1;\ y < -2$ or $y > 4$
3b(iii)	Coordinates should satisfy the following conditions: $y = -2$ or $y = 4;\ -3 \leq x \leq 1$
3b(iv)	Coordinates should satisfy the following conditions: $x = -3$ or $x = 1;\ -2 \leq y \leq 4$
3c(i)	Coordinates should satisfy the following conditions: $y = 0; x < -3$ or $x > 1$
3c(ii)	Coordinates should satisfy the following conditions: $-3 < x < 0;\ 0 < y < 4$
3c(iii)	Coordinates should satisfy the following conditions: $0 < x < 1;\ 0 < y < 4$
3c(iv)	For long sides: $-3 \leq x \leq 1; y = -2$ For short sides: $x = -3$ or $x = 1$; $-2 \leq y < 0$

Worksheet 3: Working with a Z-shape

In this task you will:

- Read, plot and translate points
- Calculate distances between points
- Apply the Theorem of Pythagoras, calculate perimeter and area



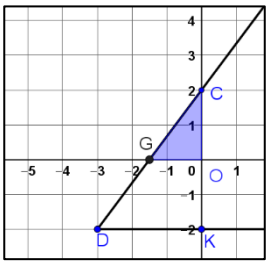
1) The diagram shows a Z-shape drawn on the Cartesian plane.
   a) Write down the coordinate of points A, B, D and E.
   b) Compare the coordinates of points A and B: what is the same, what is different?
   c) Compare the coordinates of points B and E: what is the same, what is different?
   d) Determine the distance from B to E.
   e) Write down the coordinates of the point that is half-way between B and E.

2) The Z-shape intersects the axes in 4 places.
   a) Give the coordinates of each point. Write down the name of the point and its coordinates.
   b) Did you estimate the x-coordinate for point G or did you work it out exactly? How can you be sure of the exact answer for G? Write down an explanation to convince a friend.

3) Imagine you have to walk along the Z-shape from A to B to D and stop at E. How many units will you walk? (Hint: Think about BD as the hypotenuse of a right-angled triangle). Round to 1 decimal place if necessary.

4) A point, P, lies 4 units to the right of E, on the same horizontal line.
   a) Write down the coordinates of P.
   b) What is the distance from D to P?

5) A point, T, lies 5 units to the left of A, on the same horizontal line.
   a) Write down the coordinates of T.
   b) What is the distance from D to T?



6) Look at the shaded area GCO in the diagram.
   a) Determine the area of the triangle
   b) Determine the perimeter of the triangle.
   c) Write down the coordinates of 3 points that lie inside the triangle. (They may not lie on side of the triangle)

7) Now we are going to translate the whole Z-shape so that point D lies at the origin.
   a) Describe the vertical and horizontal translations to move D to the origin.
   b) Write down the new coordinates of A, B and E.

<u>Extension</u>

8) Choose another letter that consists only of straight line segments.
   a) Draw the letter on the Cartesian plane. The letter should lie in at least 3 quadrants.
   b) Make up at least 4 questions similar to the ones on this worksheet. One question must involve applying the Theorem of Pythagoras, and another question must involve translations.
   c) Provide answers to all your questions.
   d) Give your questions to a friend to try.

Worksheet 3: Working with a Z-shape

Solutions

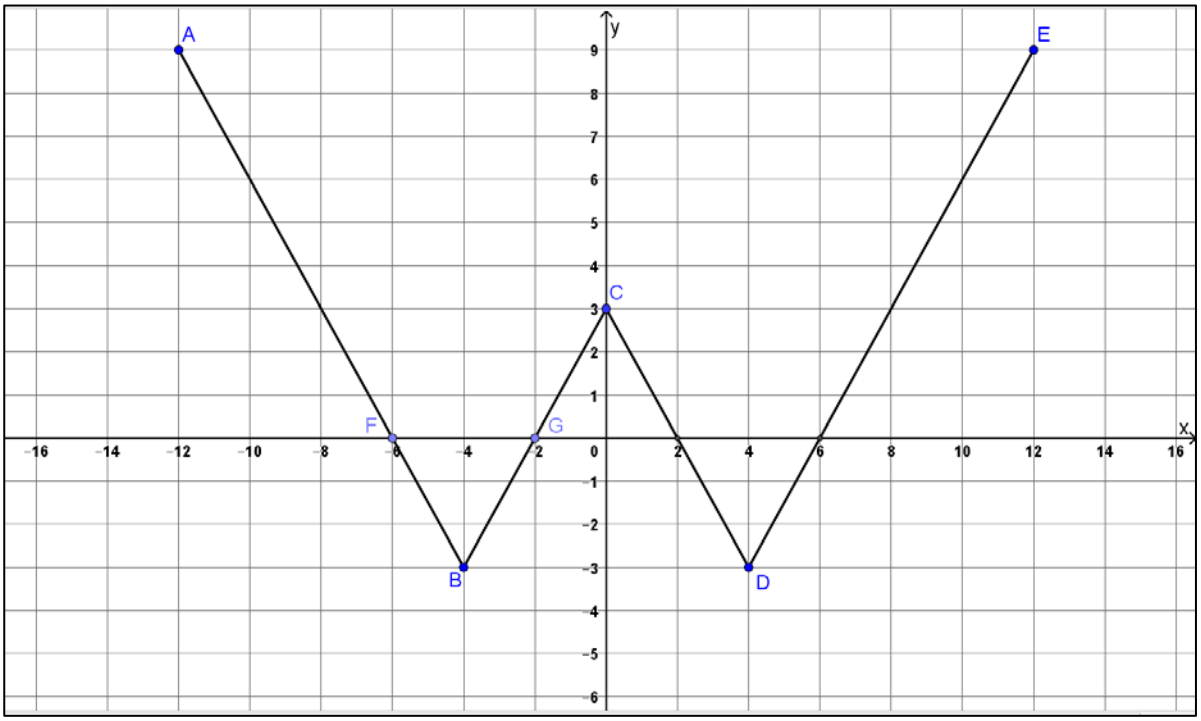
Question	Answer
1a	A(-3;6) B(3;6) D(-3;-2) E(3;-2)
1b	y-coordinates are the same (6) and both are positive x-coordinates are different, one is positive, one is negative
1c	x-coordinates are the same (3) and both are positive y-coordinates are different, one is positive, one is negative
1d	8 units
1e	(3;2)
2a	F(0;6) C(0;2) G(-1½;0) K(0;-2)
2b	We can work it out exactly, e.g. to move from B to C, you go down 4 units and left 3 units. To move from C to G, you move down 2 units (which is half of 4) so you must go left 1,5 units (which is half of 3).
3	From A to B is the same distance as from D to E: 6 units. BD = 10 (using Pythag) so the whole distance is 22 units.
4a	P(7;-2)
4b	10 units
5a	T(-8;6)
5b	Using Pythag: DT=$\sqrt{89}$ = 9,43 (to 2 dp)
6a	OG=1,5 OC=2 ∴ area ΔOCG = 1,5 square units
6b	Using Pythag: GC=$\sqrt{6,25}$ = 2,5 ∴ perim ΔOCG = 6 units
6c	Points should lie inside triangle, not on the sides, e.g. (-½;1) (-½; ½) (-1;$\frac{1}{3}$)
7a	Up 2 units, right 3 units
7b	New coordinates: A(0;8) B(6;8) E(6;0)

Worksheet 4: Working with a W-shape

In this task you will:

- Read, plot, translate and reflect points
- Calculate distances between points
- Apply the Theorem of Pythagoras, calculate area

1) The diagram shows a W-shape drawn on the Cartesian plane. Note the different scales on the axes.



a) Write down the coordinates of points A, B, C, D and E.

b) Compare the coordinates of the following points. Say what is the same and what is different:
i) A and E ii) B and D iii) A and B

2) The x-intercepts are the points where the W-shape intersects (or cuts) the x-axis.
a) Write down the coordinate pairs of all 4 x-intercepts.
b) What is the same about the x-coordinates of each pair?

3) The following points also lie on the W-shape: P (10;6), Q (-8;3), R (-3; -1,5) and S (2,5; -0,75).
a) Plot these 4 points on the diagram. Label each point.
b) Reflect each point across the y-axis. Plot the new points and label them with P', Q' etc.
c) Write down the coordinates of the new points in the table.

Original point	P (10;6)	Q (-8;3)	R (-3; -1,5)	S (2,5; -0,75)
Reflected point	P'	Q'	R'	S'

d) What changes and what stays the same when you reflect the points across the y-axis?

The next 2 questions deal with distances on the diagram.

4) Determine the (shortest) distance from:
   a) A to E
   b) B to D
   c) A to B (Hint: think of AB as the hypotenuse of a triangle). Work to 2 decimal places.

5) Imagine that you walk along the W-shape from A to B to C to D and then to E. How many units will you walk? Show all working.

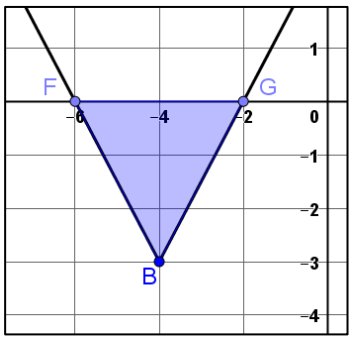
6) Imagine that lines AB and ED are extended downwards until they intersect. Determine the coordinates of the point of intersection. Explain how you got your answer. '

The next 2 questions involve translating the W-shape.

7) The W-shape is translated so C lies at the origin.
   a) Draw the new W-shape on the axes.
   b) What are the new coordinates of points A, B, D and E? Label these points A', B', D' and E' respectively.

8) The <u>original</u> W-shape is translated 1 unit to the left.
   a) What will be the new coordinates of C? Label it C''.
   b) Draw the new W-shape on the axes.
   c) Write down the coordinates of the new positions of A, B, D and E. Label these points A'', B'', D'' and E'' respectively.
   d) Write down the coordinates of the new x-intercepts of the W-shape.

9) The diagram alongside is a section of the original diagram. Look at the shaded region ΔBFG.
   a) Give the coordinates of 3 points that lie *inside* the shaded region.
   b) Determine the area of the shaded region.

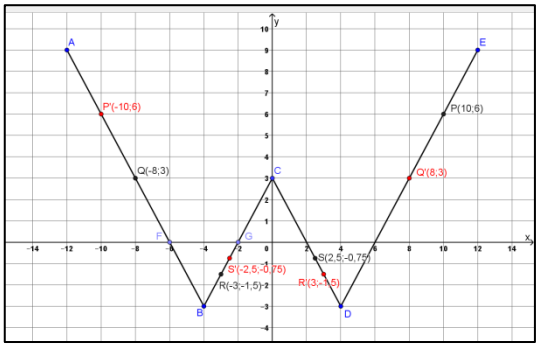
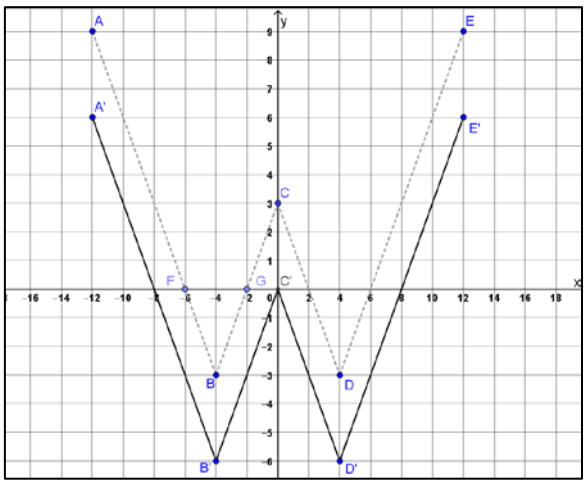


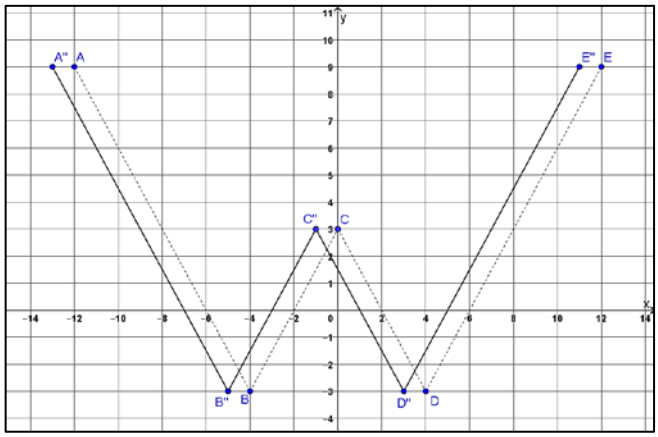
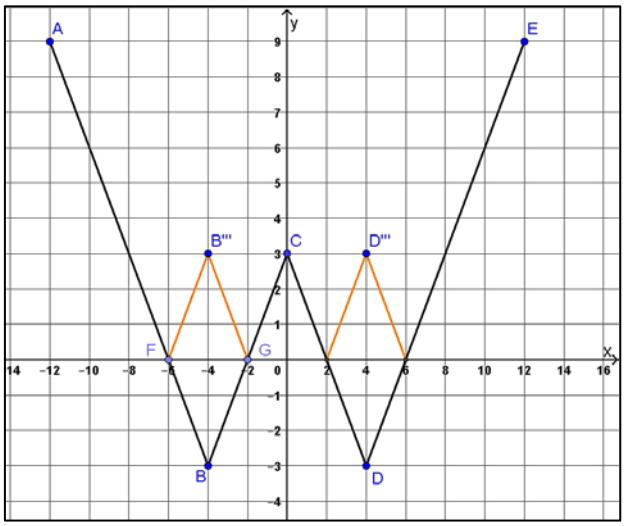
<u>Extension</u>

10) Look at the original W-shape. Now imagine that <u>all</u> points which lie below the x-axis are reflected across the x-axis, so that they are above the axis.
   a) What will be the new coordinates of points B and D? Label them B''' and D'''.
   b) Reflect some more points that are below the x-axis. Plot their new positions, then draw the new shape.
   c) Imagine that you walk <u>along the new shape</u>, starting at B''' to C to D'''. Compare this distance to the distance from B to C to D along the original W-shape. Is the distance *longer*, *shorter* or *the same as* the distance on the original shape?

Worksheet 4: Working with a W-shape

Solutions

Question	Answer
1a	A(-12;9) B(-4;-3) C(0;3) D(4;-3) E(12;9)
1b(i)	The y-coordinates are the same. There is a change in sign of the x-coordinates.
1b(ii)	The y-coordinates are the same. There is a change in sign of the x-coordinates.
1b(iii)	Both points have negative x-coordinates. A has a positive y-coordinate, B has a negative y-coordinate.
2a	(-6;0) (-2;0) (2;0) (6;0)
2b	The y-coordinate of all the points is 0.
3a, 3b	 Reflected points in red.
3c	**Original point** P (10;6) Q (-8;3) R (-3; -1,5) S (2,5; -0,75) **Reflected point** P' (-10;6) Q' (8;3) R' (3;-1,5) S' (-2,5;-0,75)
3d	The x-coordinate changes in sign (to its additive inverse) while the y-coordinate remains the same.
4a	24 units
4b	8 units
4c	$\sqrt{208}$ or 14,42 units
5	AB= $\sqrt{208}$ = 14,42 BC= CD = $\sqrt{52}$ = 7,21 DE= 14,42. Total distance = 43,26 units
6	(0;-9) The lines AB and DE intersect midway between B and D, i.e. on the y-axis.
7a, 7b	A'(-12;6) B'(-4;-6) D'(4;-6) E'(12;6) 

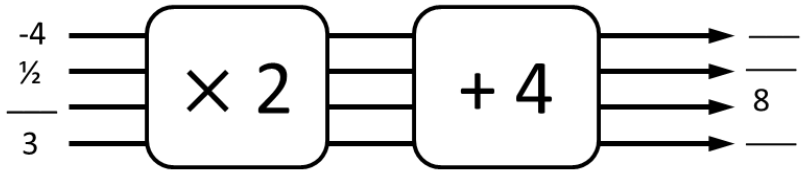
Question	Answer
8a	C'' (-1;3)
8c	A''(-13;9) B'' (-5;-3) D'' (3;-3) E''(11;9)
8d	(-7;0) (-3;0) (1;0) (5;0)
8b	
9a	Many possibilities, e.g. (-4;-1) (-4;-2) (-5; -1,5) (-3;-1,1)
9b	6 sq. units
Extension	
10a	B''' (-4;3) D''' (4;3)
10b	The parts of the W-shape that have changed are shown in orange (and lie above the x-axis). 
10c	The distance is the same, some parts are now above the x-axis but the lengths are the same.

Worksheet 5: Working with points in function diagrams, tables and graphs

In this task you will:

- Determine input and output values in a function machine and enter them in a table
- Plot points from a table onto a set of axes
- Investigate the relationship between x- and y-coordinates for an algebraic rule

1) Complete the function machine with the rule: *Multiply by 2 then add 4*



2) Write the input and output values from the function machine in the table.

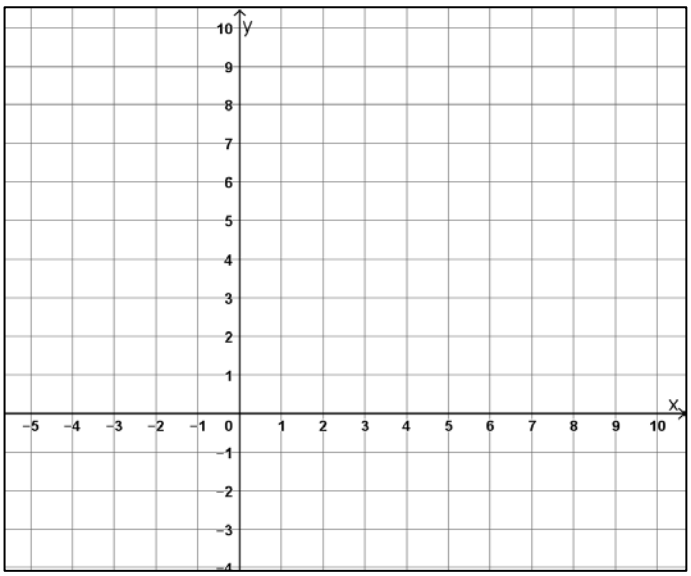
input	-4					
output			8			

Remember: the inputs are the x-values and the outputs are the y-values

3) How do we write rule "multiply by 2 then add 4" as an equation using x and y?

4) Use your algebraic rule to calculate the following:
   a) If the input is 0, what is the output?
   b) If the y-value is 0, what is the x-value?
   c) Write the input and output values of these 2 points in the table.

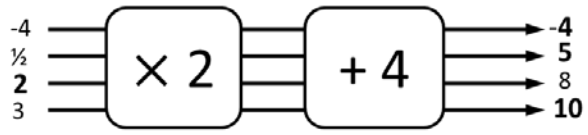
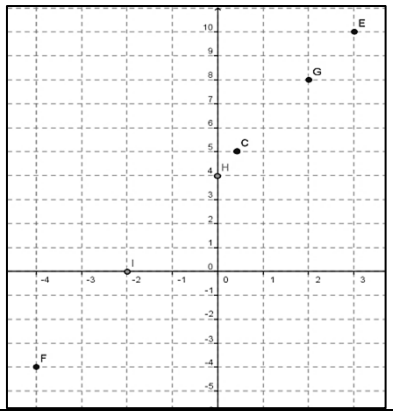
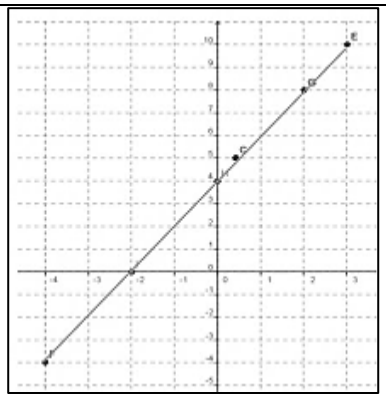
5) You now have 6 points in the table.
   a) Write all 6 points as co-ordinate pairs.
   b) Plot the 6 points on the set of axes.
   c) What do you notice about the points?



6) Use a ruler and join the points to form a straight-line graph. Then answer the following questions:
   a) What is the x-intercept of the graph?
   b) What is the y-intercept of the graph?
   c) How can you be sure that (1;6) lies on the graph? Write an explanation to convince a friend.
   d) The x-axis stops near $x = 10$. Is there a point on the graph with x-coordinate of 20? If so, determine the point's y-coordinate.
   e) Will there be a point on the graph with y-coordinate of -10? If so, determine its x-coordinate.
   f) The point (-4;-4) lies on the graph. Will the point (-8;-8) also lie on the graph? Write an explanation that will really impress your teacher.
   g) Imagine that you plot the point (15;22). Will this point lie *on* the graph? Or a*bove* the graph? Or *below* the graph? Explain how you can be sure.

Worksheet 5: Working with points in function diagrams, tables and graphs

Solutions

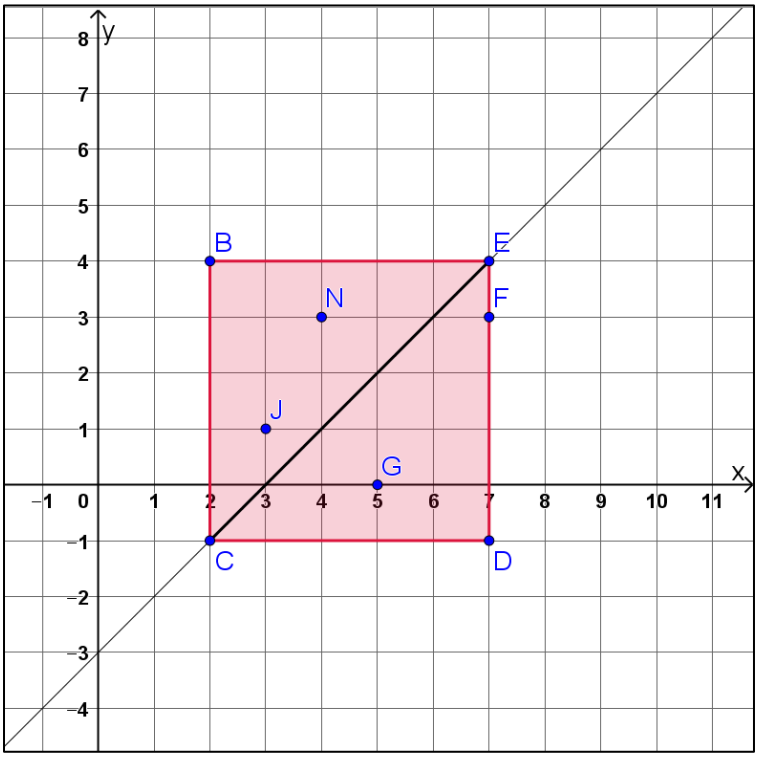
Question	Answer
1	
2	input: -4, ½, 2, 3, 0, -2 output: -4, 5, 8, 10, 4, 0 Note that the last 2 columns come from Q4a and Q4b.
3	$y = 2x + 4$
4a	4
4b	-2
4c	See table above
5a	(-4;-4) (½ ;5) (2;8) (3;10) (0;4) (-2;0)
5b	
5c	They form a straight line
6	
6a	(-2;0)
6b	(0;4)
6c	You can plot the point and see if it lies on the line. When you plot it, it does lie on the line OR Put substitute 1 into the flow diagram and you will get an output of 6. So (1;6) lies on the line.
6d	Yes, $y = 44$
6e	Yes. $x = \frac{-10-4}{2} = -7$
6f	No. When you substitute -8 into the flow diagram, then y = -12, not -8.
6g	$15 \times 2 + 4 = 34$. When x = 15, then y = 34. So the point (15;22) will lie below the graph because 22 is less than 34. I can be sure by extending the axes and plotting the point.

Worksheet 6: Points on a diagonal #1

In this task you will:

- Read, move and plot points
- Investigate the relationships between the coordinates of points that lie on a line

1) Points B, C, D and E form a square as shown in the diagram. Write down the coordinates of B, C, D and E.



2) The points F, G, J and N lie inside or on the side of the square. Write down the coordinates of each point in the table.

Original position of point	New position of point
F	F'
G	G'
J	J'
N	N'

3) CE is a diagonal of the square.
   a) Move points F, G, J and N so that they lie on the diagonal. You may move the points vertically or horizontally but no point may land on a point that is on the line. Label the points F', G' etc. Write down the coordinates in the table above.
   b) What is the relationship between the x-coordinate and the y-coordinate of each point when it lies on the diagonal? Can you find a rule that applies to all 4 points?
   c) Do the coordinates of points B and D also have this relationship? Explain why or why not.

4) The diagonal extends outside the square. Give the coordinates of a point that:
   a) lies on the diagonal, inside the square in quadrant IV.
   b) lies on the diagonal, and in quadrant III.

5) A point T has coordinates (13; p) and lies on the diagonal. What is the value of p?

6) A point V has coordinates (q ; -5) and lies on the diagonal. What is the value of q?

Worksheet 6: Points on a diagonal #1

Solutions

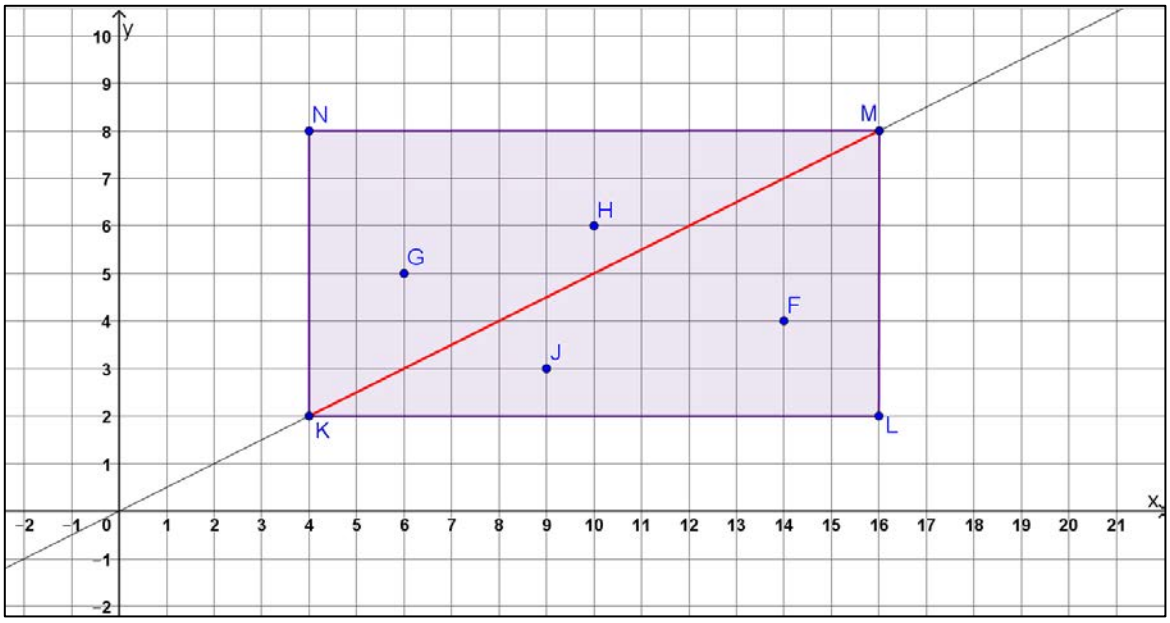
<table>
<tr><th>Question</th><th>Answer</th></tr>
<tr><td>1</td><td>B(2;4) C(2;-1) D(7;-1) E(7;4)</td></tr>
<tr><td></td><td></td></tr>
<tr><td>2</td><td>
<table>
<tr><th>Original position of point</th><th>New position of point</th></tr>
<tr><td>F(7;3)</td><td>F'</td></tr>
<tr><td>G(5;0)</td><td>G'</td></tr>
<tr><td>J(3;1)</td><td>J'</td></tr>
<tr><td>N(4;3)</td><td>N'</td></tr>
</table>
</td></tr>
<tr><td></td><td></td></tr>
<tr><td>3a</td><td>
<table>
<tr><th>Original position of point</th><th>New position of point</th></tr>
<tr><td>F(7;3)</td><td>F'(6;3)</td></tr>
<tr><td>G(5;0)</td><td>G'(5;2)</td></tr>
<tr><td>J(3;1)</td><td>J'(3;0)</td></tr>
<tr><td>N(4;3)</td><td>N'(4;1)</td></tr>
</table>
</td></tr>
<tr><td>3b</td><td>The x-coordinates and the y-coordinates of all the points on the diagonals have a difference of 3 between them. This could be expressed as $y = x - 3$ or $y + 3 = x$ or $x - y = 3$</td></tr>
<tr><td>3c</td><td>No, they do not lie on the diagonal so they have different relationship. The sum of their coordinates is 6. The relationship can be expressed as $x + y = 6$ or $y = 6 - x$.</td></tr>
<tr><td></td><td></td></tr>
<tr><td>4a</td><td>The coordinates must satisfy these conditions: $2 < x < 3$ and $-1 < y < 0$, e.g. (2,5 ; -0,5)</td></tr>
<tr><td>4b</td><td>The coordinates must satisfy these conditions: $x < 0$ and $y < -3$, e.g. (-1;-4)</td></tr>
<tr><td></td><td></td></tr>
<tr><td>5</td><td>$p = 10$ The y-coordinate is 3 less than the x-coordinate ($y = 13 - 3$)</td></tr>
<tr><td></td><td></td></tr>
<tr><td>6</td><td>$q = -2$ The x-coordinate is 3 more than the y-coordinate ($-5 = x - 3$)</td></tr>
</table>

Worksheet 7: Points on a diagonal #2

In this task you will:

- Read, move and plot points.
- Investigate the relationships between the coordinates of points that lie on a line.

1) Points K, L, M and N form a rectangle which is shown in the diagram.
   a) Write down the coordinates of each point.
   b) Determine the perimeter of the rectangle.



2) The points F, G, H and J lie inside the rectangle. Write down the coordinates of each point in the table.

Original position of point	New position of point
F	F'
G	G'
H	H'
J	J'

3) KM is a diagonal of the rectangle.
   a) Move point F horizontally so that it lies on the diagonal. Label the new point F'. Write down its coordinates in the table.
   b) Move points G, H and J horizontally so that they lie on the diagonal. Label the new points G', H' and J' respectively. Write down the new coordinates of each point in the table.
   c) Look at the relationship between the x-coordinate and the y-coordinate of each new point. Describe the relationship in words.
   d) Points K and M also lie on the diagonal. Look at the x- and y-coordinates of these points. Do they have the same relationship as the new points in the table?

4) In the previous questions we moved the points *horizontally* to lie to the diagonal. Now we want to test what happens if we move the points *vertically*.
   a) Move points F, G and H vertically so that they lie on the diagonal. Label the new points F'', G'' and H'' respectively. Write down the coordinates of the new points in the table below.

Original position of point	New position of point
F	F''
G	G''
H	H''

   b) Look again at the relationship between the x-coordinate and the y-coordinate of each new point. You should see that they have the same relationship as when we moved the points horizontally. Confirm that this is the case by checking.
   c) Move point J vertically so that it lies on the diagonal. Label the point J''. Work out the **exact** coordinates of this point.

5) The diagonal extends outside the rectangle. Give the coordinates of a point that:
   a) lies on the diagonal, outside the rectangle and in quadrant I.
   b) lies on the diagonal and in quadrant III.
   c) lies *above* the diagonal and has an x-coordinate larger than 20.
   d) lies *very close to the diagonal but below it* and has an x-coordinate larger than 50.

6) A point P has coordinates (7 ; p) and lies on the diagonal of the rectangle. What is the value of p?

7) A point Q has coordinates (q ; 13,5) and lies on the diagonal. What is the value of q?

Worksheet 7: Points on a diagonal #2

Solutions

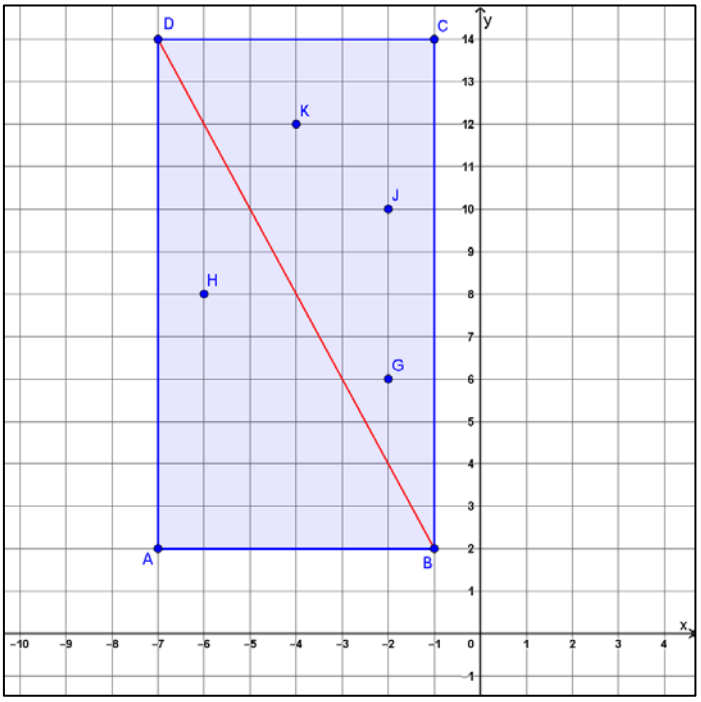
<table>
<tr><th>Question</th><th>Answer</th></tr>
<tr><td>1a</td><td>K(4;2), L(16;2), M(16;8), N(4;8)</td></tr>
<tr><td>1b</td><td>36 units</td></tr>
<tr><td></td><td></td></tr>
<tr><td>2</td><td><table><tr><th>Original position of point</th></tr><tr><td>F (14;4)</td></tr><tr><td>G (6;5)</td></tr><tr><td>H (10;6)</td></tr><tr><td>J (9;3)</td></tr></table></td></tr>
<tr><td></td><td></td></tr>
<tr><td>3a
3b</td><td><table><tr><th>Original position of point</th><th>New position of point</th></tr><tr><td>F (14;4)</td><td>F' (8;4)</td></tr><tr><td>G (6;5)</td><td>G' (10;5)</td></tr><tr><td>H (10;6)</td><td>H' (12;6)</td></tr><tr><td>J (9;3)</td><td>J' (6;3)</td></tr></table></td></tr>
<tr><td>3c</td><td>x-coordinate is 2 times the y-coordinate or y-coordinate is half the x-coordinate</td></tr>
<tr><td>3d</td><td>K(4;2) and M(16;8). Yes</td></tr>
<tr><td></td><td></td></tr>
<tr><td>4a</td><td><table><tr><th>Original position of point</th><th>New position of point</th></tr><tr><td>F (14;4)</td><td>F'' (14;7)</td></tr><tr><td>G (6;5)</td><td>G'' (6;3)</td></tr><tr><td>H (10;6)</td><td>H'' (10;5)</td></tr></table></td></tr>
<tr><td>4b</td><td>x-coordinate is 2 times the y-coordinate</td></tr>
<tr><td>4c</td><td>The x-coordinate of J'' is 9. Therefore the y-coordinate has to be half of 9 which is 4½ because J'' lies on the diagonal. The coordinates of J'' are (9; 4½)</td></tr>
<tr><td></td><td></td></tr>
<tr><td>5</td><td>Many possible answers for these questions</td></tr>
<tr><td>5a</td><td>e.g. (2;1) The coordinates must satisfy these conditions: $x = 2y; 0 < x < 4; x > 16; 0 < y < 2; y > 8$</td></tr>
<tr><td>5b</td><td>e.g. (-2;-1) The coordinates must satisfy these conditions: $x = 2y; x < 0; y < 0$</td></tr>
<tr><td>5c</td><td>e.g. (42;22) The coordinates must satisfy these conditions: $x > 2y; x > 20$</td></tr>
<tr><td>5d</td><td>e.g. (52;25) The coordinates must satisfy these conditions: x is almost equal to $2y; x > 50$</td></tr>
<tr><td></td><td></td></tr>
<tr><td>6</td><td>p =3½ $(7 = 2p)$</td></tr>
<tr><td></td><td></td></tr>
<tr><td>7</td><td>$q = 2(13{,}5) = 27$</td></tr>
</table>

Worksheet 8: Points on a diagonal #3

In this task you will:

- Read, move and plot points
- Investigate the relationships between the coordinates of points that lie on a line

1) Points A, B, C and D form a rectangle. Write down the coordinates of each point.



2) The points G, H, J and K lie inside the rectangle. Write the coordinates of each point in the table.

Original position of point	New position of point
G	G'
H	H'
J	J'
K	K'

3) BD is a diagonal of the rectangle.
   a) Move point G horizontally so that it lies on the diagonal. Label the new point G'. Write its coordinates in the table.
   b) Move H, J and K horizontally to lie on the diagonal. Label the new points H', etc. Write the new coordinates of each point in the table.
   c) Look at the relationship between the x-coordinate and y-coordinate of each new point. Describe the relationship in words.
   d) Points B and D also lie on the diagonal. Do their coordinates have the same relationship as the new points?

4) Plot point F at (-5;5).
   a) Move F horizontally so that it also lies on diagonal BD. Label the point F'.
   b) Work out the **exact** coordinates of F' and say why you are convinced of your answer.
   c) Say you had moved F *vertically* to lie on the diagonal? What would change, what would stay the same? (Compared to moving it horizontally)

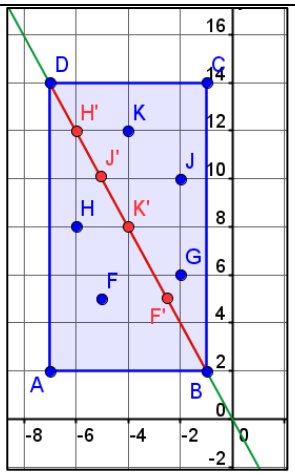
5) The diagonal can be extended outside the rectangle and even beyond the border of the diagram.
   a) Draw in the line that extends outside the rectangle.
   b) Give the coordinates of 2 points that lie on the line but outside the rectangle.
   c) Which of the following points lie on the line?
      (8;-16) (-8;-16) (5;10)
      (-16;8) (7;-14) (0;1)

6) A point P has coordinates (-4 ; p) and lies on the diagonal of the rectangle. What is the value of p?

7) A point M has coordinates (m ; 17) and lies on the line. What is the value of m?

Worksheet 8: Points on a diagonal #3

Solutions

<table>
<tr><th>Question</th><th>Answer</th></tr>
<tr><td>1</td><td>A(-7;2)
B(-1;2)
C(-1;14)
D(-7;14)</td></tr>
<tr><td>2
3a-3b</td><td>
<table>
<tr><th>Original position of point</th><th>New position of point</th></tr>
<tr><td>G(-2;6)</td><td>G'(-2;6)</td></tr>
<tr><td>H(-6;8)</td><td>H'(-4;8)</td></tr>
<tr><td>J(-2;10)</td><td>J'(-5;10)</td></tr>
<tr><td>K(-4;12)</td><td>K'(-6;12)</td></tr>
</table>
</td></tr>
<tr><td>3c</td><td>The y-coordinate is double the x-coordinate and has the opposite sign.</td></tr>
<tr><td>3d</td><td>Yes. B(-1;2) and D(-7;14).</td></tr>
<tr><td></td><td></td></tr>
<tr><td>4a</td><td> 
</td></tr>
<tr><td>4b</td><td>(-2,5;5) because the y-coordinate is double and has opposite sign.</td></tr>
<tr><td>4c</td><td>The y-coordinate would change to 10 and the x-coordinate would be the same, i.e. -5</td></tr>
<tr><td></td><td></td></tr>
<tr><td>5a</td><td>See graph above</td></tr>
<tr><td>5b</td><td>(0;0), (-½;1), (1;-2), (2;-4), (3;-6), (-8;16), (-7,5;15)</td></tr>
<tr><td>5c</td><td>(8;-16) (7;-14)</td></tr>
<tr><td></td><td></td></tr>
<tr><td>6</td><td>$p = -2(-4) = 8$</td></tr>
<tr><td></td><td></td></tr>
<tr><td>7</td><td>$m = -\frac{17}{2} = -8{,}5$</td></tr>
</table>

Worksheet 9: Moving points to make new lines

In this task you will:

- Generate coordinate pairs from a rule, including points with fractions
- Plot points to produce a line
- Move the points in order to move the lines
- Connect different representations: tables, equations and graphs

REMINDER

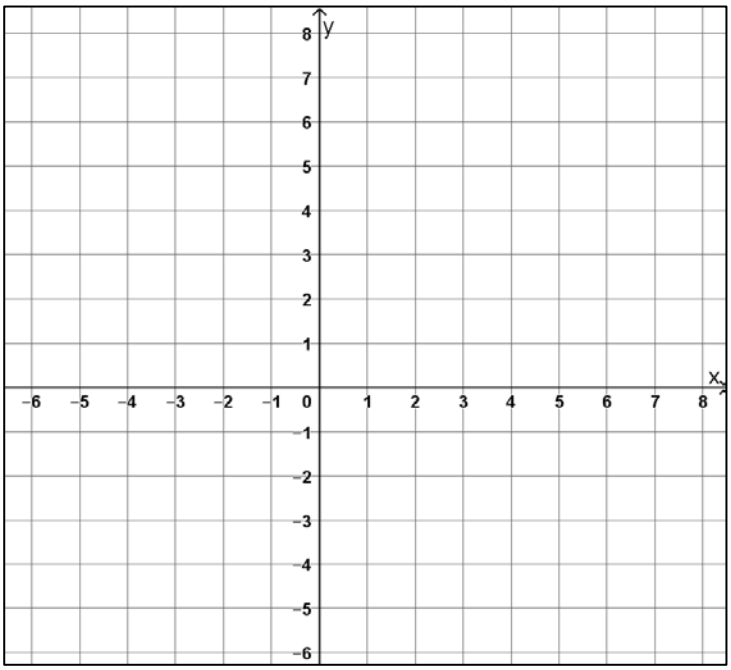
1. We say the y-values are *dependent* on the x-values.
2. Ordered pairs are also called *coordinates.*
3. In the table below, we have chosen consecutive integers from -2 to 2 for the x-values. We can choose any positive or negative number (including fractions) for x and substitute it into the equation to determine the y-value.

1) Given the equation $y = 2x - 1$.
   a) Complete the table to determine the ordered pairs.
   b) Write down the 5 ordered pairs.

x	-2	-1	0	1	2
y					

2) We want to find some points with fractional values.
   a) Show that $(\frac{1}{2}\ ;\ 0)$ satisfies the rule $y = 2x - 1$.
   b) Find another point that also satisfies the rule **and** where the x-coordinate and y-coordinate are improper fractions.

3) You now have 7 ordered pairs.
   a) Use the set of axes and plot all 7 points.
   b) Join the points to make a straight-line graph. Label the graph f.



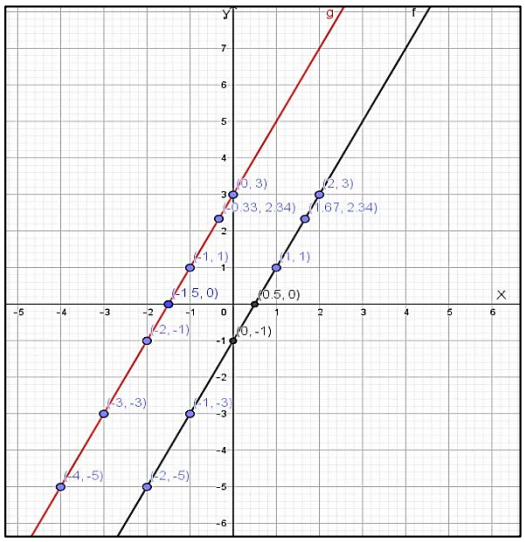
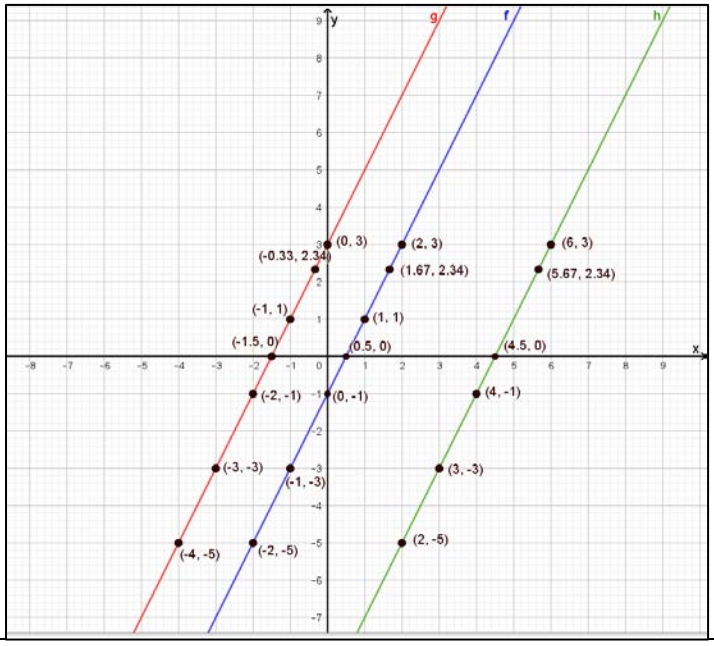
4) Move all 7 points 2 units to the left. For example, (0;-1) will move to (-2;-1).
   a) Plot the new points and write down the new coordinates of each point.
   b) Compare the coordinates of each original point with the coordinates of its new position. What is the same and what is different?
   c) Join your new points to produce another straight line. Label this straight-line graph g.

5) Compare the graphs of f and g. What is the same and what is different about them?

6) Repeat the steps for moving points: move the original points 4 units to the right. Join the points to form a new straight-line graph called h. Compare the coordinates of the original points with the coordinates of the new points, also compare the 3 lines. What changes, what stays the same?

Worksheet 9: Moving points to make new lines

Solutions

<table>
<tr><th>Question</th><th>Answer</th></tr>
<tr><td>1a</td><td>
<table>
<tr><td>x</td><td>-2</td><td>-1</td><td>0</td><td>1</td><td>2</td></tr>
<tr><td>y</td><td>-5</td><td>-3</td><td>-1</td><td>1</td><td>3</td></tr>
</table>
</td></tr>
<tr><td>1b</td><td>(-2;-5) (-1;-3) (0;-1) (1;1) (2;3)</td></tr>
<tr><td></td><td></td></tr>
<tr><td>2a</td><td>Let $x = \frac{1}{2}$ then $y = 2 \times \frac{1}{2} - 1 = 1 - 1 = 0$</td></tr>
<tr><td>2b</td><td>e.g. If $x = \frac{5}{3}$, then $y = 2 \times \frac{5}{3} - 1$, then $y = \frac{7}{3}$. Thus, the point is $(\frac{5}{3};\frac{7}{3})$</td></tr>
<tr><td></td><td></td></tr>
<tr><td>3a</td><td>See the points in 3b.</td><td rowspan="5">3b
 
</td></tr>
<tr><td></td><td></td></tr>
<tr><td>4a</td><td>See the points in 3b.</td></tr>
<tr><td>4b</td><td>The y-coordinates of the new points remain the same as in the original points. For each x-coordinate of the new points, -2 is added to the original values of x-coordinate.
e.g. For (-2;-5), the new point is (-4;-5)</td></tr>
<tr><td>4c</td><td>See the graph in 3b</td></tr>
<tr><td></td><td></td></tr>
<tr><td>5</td><td>Both straight lines have the same gradient, which is 2.
For graph f, the x-intercept is $(\frac{1}{2};0)$ and the y-intercept is (0;-1).
For graph g, the x-intercept is $(-\frac{3}{2};0)$ and the y-intercept is (0;3).</td></tr>
<tr><td>6</td><td>See the graph h.
y-coordinates of new points remain the same. For each x-coordinate, 4 is added to the original value.
The 3 lines have the same gradient of 2, which makes them parallel to each other.
 
</td></tr>
</table>

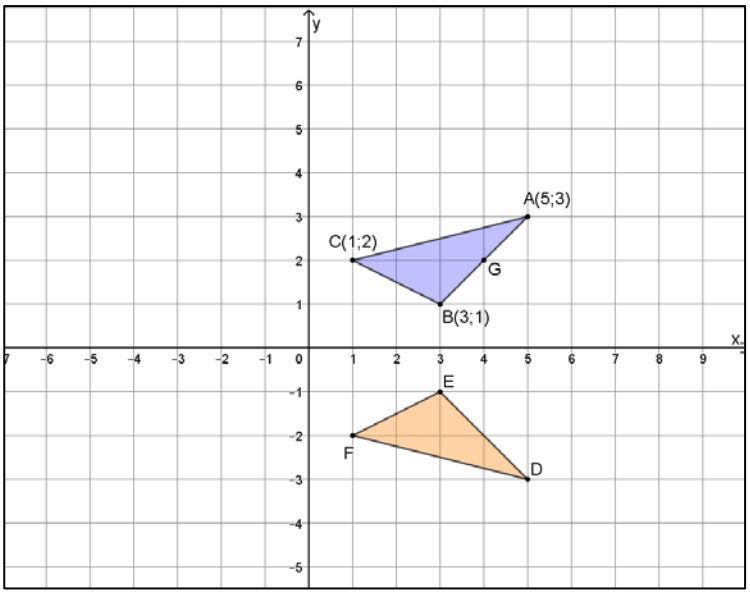
Worksheet 10: Reflecting points and shapes across the axes

In this task you will:

- Read, plot and reflect points
- Generalise observations when we reflect points across the x- and y-axes

NOTE
A reflection of a point (or shape) is the mirror image of that point (or shape) across a specific line.

1) The points A(5;3); B(3;1) and C(1;2) are the vertices of ΔABC. When we reflect ΔABC across the x-axis, we get ΔDEF which is the mirror image of ΔABC.



Coordinates ΔABC	Coordinates ΔDEF
A(5;3)	D
B(3;1)	E
C(1;2)	F

a) Look at the 2 triangles. What is the same and what is different about ΔABC and ΔDEF? Pay attention to the size, shape and position of the triangles.

b) Write the coordinates of D, E and F in the table.

c) Look at the coordinates of the points A to F.
   i) State what is the same and what is different between the pairs of points below:
      (1) A and D (2) B and E (3) C and F
   ii) How does the reflection cause the *sameness* and the *differences* in the coordinates?

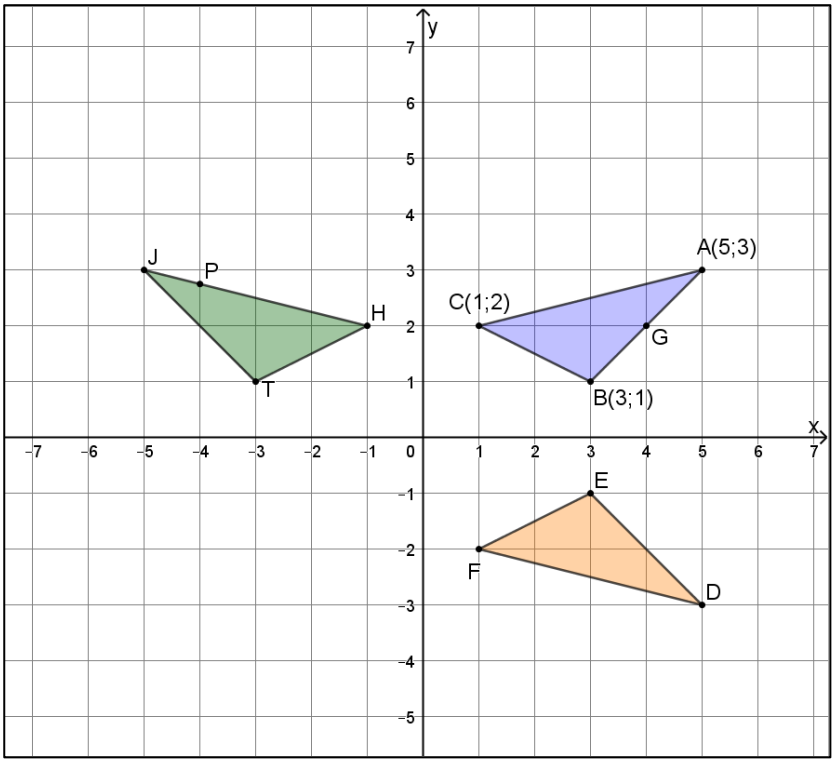
d) The point G(4;2) lies on AB. Reflect G across the x-axis. Label the reflected point G'.
   i) On which side of ΔDEF will G' lie?
   ii) Write down the coordinates of G'.

e) Choose any point inside ΔDEF and name it R.
   i) Write down the coordinates of R.
   ii) Reflect R across the x-axis. Call the new point R'. What are the coordinates of R'?

f) Summarise your observations about what happens to the coordinates of points when they are reflected across the x-axis.

2) ΔJTH is a reflection of ΔABC across the y-axis.

a) Look at the 2 triangles. What is the same and what is different between ΔABC and ΔJTH? Pay attention to the size, shape and position of the triangles.



Coordinates ΔABC	Coordinates ΔJTH
A(5;3)	J
B(3;1)	T
C(1;2)	H

b) Write down the coordinates of J, T and H in the table.

c) Look at the coordinates of the 6 vertices of the 2 triangles.

i) State what is the same and what is different between the pairs of points below:

(1) A and J (2) B and T (3) C and H

ii) How does the reflection cause the *sameness* and the *differences* in the coordinates?

d) Reflect point R' (from Qe ii) across the y-axis. Call the new point R''. What are the coordinates of R''?

e) P$(-4; 2\frac{3}{4})$ lies on JH of ΔJTH. P is reflected across the y-axis to a point P' on ΔABC.

i) On which side of ΔABC will P' lie?

ii) Write down the coordinates of P'.

f) Summarise your observations about what happens to the coordinates of points when they are reflected across the y-axis.

Worksheet 10: Reflecting points and shapes across the axes

Solutions

Question	Answer
1a	The triangles have the same shape and the same size. The positions of the triangles differ.
1b	D(5;-3) E(3;-1) F(1;-2)
1c(i)	The x-coordinates are the same in all pairs i.e. A and D; B and E; C and F. The y-coordinates of A and D change in sign or are additive inverses of each other. The same is true for B and E; C and F.
1c(ii)	The points are reflected across the x-axis therefore the x-coordinates are the same. The y-coordinates have the opposite signs because points above the x-axis have positive y-values while those below the x-axis have negative y-values. The distances between the x-axis and the points on either side are the same but the signs of the y-values differ.
1d(i)	DE
1d(ii)	G′(4;-2)
1e(i)	Any point inside the triangle, e.g. R(3;-2)
1e(ii)	R′(3;2)
1f	The x-coordinates remain the same but the y-coordinates change sign.
2a	The triangles have the same shape and the same size. The positions of the triangles differ.
2b	J(-5;3) T(-3;1) H(-1;2)
2c(i)	For 1, 2 and 3: the y-coordinates are the same in all pairs i.e. A and J; B and T; C and H. For 1, 2 and 3: the x-coordinates of A and J change sign, i.e. they are additive inverses of each other. The same is true for B and T; C and H.
2c(ii)	The points are reflected across the y-axis therefore the y-coordinates are the same. The x-coordinates have the opposite signs because points to the right of the y-axis have positive x-values while those to the left of the y-axis have negative x-values. The distances between the y-axis and the points on either side of the y-axis are the same but the signs of the x-values differ.
2d	R''(-3;2)
2e(i)	AC
2e(ii)	P'(4;$2\frac{3}{4}$)
2f	The y-coordinates remain the same but the x-coordinates change sign.